THE

COMPLETE PRACTICAL ARITHMETICIAN:

CONTAINING,
SEVERAL NEW AND USEFUL
IMPROVEMENTS.
ADAPTED TO
THE USE OF SCHOOLS
AND PRIVATE TUITION.

By THOMAS KEITH, k
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SECOND EDITION.

> Arithmetic is the eaſieſt, and conſequently the firſt Sort of abſtract Reaſoning which the Mind commonly bears, or accuſtoms itſelf to; and is of ſuch general Uſe in all Parts of Life and Buſineſs, that ſcarce any Thing is to be done without it.
>
> Locke on Education.

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PREFACE.

ARITHMETIC is juſtly conſidered as the baſis of every part of the mathematics; for, even in comparing quantities, or magnitudes, with each other, recourſe is frequently had to numbers. It muſt have originated, as ſoon as mankind began to hold any commercial intercourſe with each other; for when commerce began to be eſtabliſhed, they would ſoon ſee the neceſſity of inquiring into the nature of Numbers, without which no Buſineſs could be carried on; but when, or by whom, it received its Form as an Art, or Science, is very uncertain.

The Phœnicians, the Deſcendants of Noah, who ſettled on the Coaſts of Paleſtine, were the firſt People in the World who made Navigation ſubſervient to Commerce. Hence it is extremely probable that Arithmetic had its Riſe among the Phœnicians, and that they introduced it into Egypt: and this opinion is ſupported by Proclus in his commentary on the firſt book of Euclid. But Joſephus tells us, that, a Famine happening in Canaan, Abraham retired into Egypt, and was the firſt who taught

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the Egyptians the Sciences of Arithmetic and Aſtronomy, (which they were ignorant of before ;) and theſe he brought with him from Chaldea. From Egypt they were tranſmitted to Greece, by Pythagoras and others, and thence to the Romans.

The firſt ſtep neceſſary towards rendering Mens ideas and knowledge of Numbers intelligible, and uſeful, would be to eſtabliſh a method of Notation, upon which they might found their calculations.

The Greeks, Hebrews, Romans, and ſeveral other nations, uſed a Notation by the letters of the alphabet. The beſt method made uſe of by the Greeks was that wherein the firſt nine letters of their alphabet repreſented the numbers from One to Nine; the ſecond Nine any number of Tens from Nine, as Ten, Twenty, &c. to Ninety. Any number of Hundreds they expreſſed by other letters, ſupplying what was wanting with ſome other marks; and, in this order, they proceeded, uſing the ſame letter again, with different marks, to repreſent Thouſands, Tens of Thouſands, &c. No particular treatiſe of their art of computation has been tranſmitted to us. There is a Commentary, by Eutocius, upon Archimedes' Treatiſe of the Dimenſions of a Circle, in Dr. Wallis's Works, and ſome fragments of Pappus, which relate particularly to Multiplication, and ſufficiently ſhew us the difficulty attending their practice, owing to their imperfect Notation.

The ſimple characters made uſe of by the Romans were taken out of their alphabet of capital letters, and were the ſeven following, viz. I. One; V. Five; X. Ten;

L. Fifty; C. one Hundred; D. five Hundred; M. one Thouſand. The intermediate numbers between theſe were expreſſed by a repetition of the ſame, and the ſum of their values repreſented the number, the character of the greateſt value being ſet to the left hand; as II. Two; III. Three; VI. Six; VII. Seven; XII. Twelve; XV. Fifteen; XXI. Twenty-one; LX. Sixty; DX. five Hundred and Ten; DC. ſix Hundred; DCCCC. nine Hundred; DCCCCLXXXXVIIII. nine Hundred and Ninety-nine, &c. But, to prevent too great a repetition of the ſame characters, they ſometimes ſet the leſs character before the greater, and then the difference of their values repreſented the number; as IV. Four; IX. Nine; XL. Forty; CD. four Hundred; CM. nine Hundred. When a number was expreſſed by more than two characters, they diſtinguiſhed it from the character on the left hand by a point; thus, C.XL. one Hundred and Forty; CD.XC.IX. four Hundred and Ninety-nine, and ſo on for numbers greater than a Thouſand. Beſides theſe, they had other expreſſions for numbers greater (and ſome leſs) than a Thouſand; thus, for D. five Hundred, they wrote IↃ; and then, by adding another Ↄ, it gradually increaſed tenfold; as IↃↃ, five Thouſand; IↃↃↃ, fifty Thouſand, &c. Again, for M. one Thouſand, they wrote CIↃ; and by joining Ↄ and C to the right and left, it expreſſed ten times the value; thus, CCIↃↃ, ten Thouſand, &c. or, by drawing a line over any number leſs than one Thouſand, it expreſſed as many thouſands as the letter, or letters, contained units; thus, $\overline{\text{V}}$. five Thouſand; $\overline{\text{VI}}$. ſix Thouſand; $\overline{\text{LX}}$. ſixty Thouſand; $\overline{\text{C}}$. a hundred Thouſand; $\overline{\text{M}}$. a Million, &c. Thus we ſee the difficulty the ancient Greeks and Romans laboured under for want of a more perfect method of Notation.

Archimedes invented a peculiar ſcale and Notation of his own, which he employed in his Arenarius, to calculate the number of the ſands. In the ſecond century of Chriſtianity, to remedy the difficulty of the common method of Notation, particularly with regard to fractions, Claudius Ptolemy is ſaid to have invented the ſexageſimal diviſion of numbers; which diviſion is ſtill uſed in aſtronomical calculations, and for the ſubdiviſion of circles. Every unit was ſuppoſed to be divided into ſixty parts, and each of theſe parts into ſixty, &c. hence any number of ſuch parts were called ſexageſimal fractions. And, to render the computation in integers more eaſy, he made the progreſſion in theſe likewiſe ſexageſimal; thus, from one to fifty-nine he marked in the common way, then ſixty he called a ſexagena prima, and expreſſed it thus I′; two ſixties, or 120, thus, II′, and ſo on to fifty-nine times ſixty, or 3540, which he wrote thus, LIX′. For ſixty times ſixty, or 3600, he wrote I″, calling it a ſexagena ſecunda; for twice 3600, or 7200, he wrote II″; for three times 3600, or 10800 he wrote III″. &c. For $\frac{5}{60}$ he wrote ′V, or V′; for $\frac{15}{3600}$, ″XV, or XV″, &c. The practice by this Notation would be ſomewhat eaſier than by the common Notation, yet ſtill very difficult, eſpecially in Multiplication and Diviſion, as appears by the work of Barlaamus, called Logiſticia, written in Greek about the year 1350; tranſlated into Latin, and publiſhed in the year 1600.

For the excellent method of Notation now in uſe, called the Arabian, (becauſe the Europeans had it from the Arabians) we are indebted to the genius of the Eaſtern nations. The Indians are acknowledged to be the inventors of it; but, at what time, or how long it was before the Arabians got it, we are quite ignorant. We have ſufficient reaſon to believe that the ancient Greeks and Romans knew nothing of it, as Maximus Planudes, the firſt Greek writer who treated of Arithmetic according to the Arabian Notation, acknowledges it to be his opinion that the Indians were the inventors, from whom the Arabians got it, and the Europeans from the Arabians. Now this writer, according to Voſſius, flouriſhed about the year of Chriſt 1370, or, according to Kircher, 1270, and this was long after the Arabian Notation was known in Europe. For Dr. Wallis proves, by many good authorities, that the Europeans were acquainted with it before the year of Chriſt 1000, and that it was brought into England before the year 1150.

Arithmetic, at this period, we may ſuppoſe, was in a rude and imperfect ſtate. The firſt and moſt conſiderable writer, after the Arabian Notation was known in Europe, was Jordanus, of Namur, who flouriſhed about the year 1200. His Arithmetic, (from which the ingenious Mr. Malcolm acknowledges he has taken ſeveral things) was publiſhed and demonſtrated by Joannes Faber Stapulenſis, in the fifteenth century, ſoon after the invention of printing. The ſame author likewiſe wrote a treatiſe, which he called Algoriſmus Demonſtratus, but it was never printed: the manuſcript, we are informed by Dr. Wallis, is in the Savilian library at Oxford.

To trace out the ſeveral improvements of Arithmetic in a regular gradation would be a difficult taſk, and afford but little amuſement to the reader. The moſt remarkable writers before the ſixteenth century, in Italy, were Lucas de Burgo (whoſe work is particularly recommended by Dr. Wallis) and Nicolas Tartaglia; in France, Clavius and Ramus; in Germany, Stifelius and Heniſchius; in England, Buckley, Diggs, and Record. In or about the year 1629, Mr. Edmund Wingate's Arithmetic was printed; but the Arithmetic now extant under his name, as improved by Mr. J. Dodſon, F. R. S. cannot literally be ſaid to bear any affinity to the original work. Since Mr. Wingate wrote, the bare names of thoſe who have written on the ſubject of Arithmetic, in England only, would fill a moderate volume. Many of theſe writers were men of ſcientific abilities, and it would be impoſſible to mention a few without doing injuſtice to a greater number.

It remains now to point out the moſt material improvements made in Arithmetic ſince the Arabian Notation was known in Europe. Progreſſion, arithmetical and geometrical, the nature of powers, the extraction of roots, and the combination of numbers, &c. have received conſiderable improvements from ſeveral authors at different periods. About the year 1464, Regiomontanus introduced decimal parts* in his triangular tables inſtead of ſexageſimals, which, before his time, were uſed in aſtronomical calculations. Ramus, in his Arithmetic, printed in 1550, makes uſe of decimals in his calculations, as do Buckley, and Record, two Engliſh authors (mentioned before) prior to Ramus; but the firſt treatiſe expreſsly written on the ſubject was by Stevinus, about the year 1582 *. Circulating, or repeating, decimals † were firſt taken notice of by Dr. Wallis, or, at leaſt, he was the firſt who diſtinctly conſidered the ſubject. But, for the greateſt and moſt uſeful improvement made in the modern art of computation, we are indebted to Baron Napier, the undiſputed inventor of logarithms ‡.

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* The nature of Decimals is explained Part I. page 81, &c. of the following treatiſe.

In the enſuing treatiſe, the Rules are given in as clear and expreſſive terms as poſſible; and thoſe parts, which are not immediately neceſſary for the ſcholar to tranſcribe, or fix in his memory, are printed on a ſmaller type than the reſt, to be conſulted occaſionally. Likewiſe, all the rules which belong to any one ſubject, ſuch as PRACTICE, &c. are claſſed together, unmixed with any examples; then the examples follow, with references to the ſeveral propoſitions and rules which they are intended to exerciſe: by this mode of proceeding, all the rules, and the notes and obſervations on them, are under the eye of the ſcholar at once, and he of courſe ſees in an inſtant what

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* Chamb. Encyclop. Hutton's Dict. word Decimal.

† For an Explanation of the Nature and Properties of circulating Decimals, ſee page 91, &c. of the enſuing work.

‡ The nature, conſtruction, and uſe, of logarithms, are familiarly explained in the Key to the Complete Practical Arithmetician.

aſſiſtance he is to expect from them. The examples are very numerous, conſiſting of about two thouſand, beſides a variety of Bills of Parcels, &c. Theſe examples are in general divided into two claſſes; in the firſt claſs reference is made to the particular propoſition which the example is intended to exerciſe; in the ſecond claſs, the examples are promiſcuouſly placed, and will ſerve as exerciſes for thoſe who are farther advanced in numbers. The firſt queſtion in each rule is worked at full length, for the encouragement of the learner, ſo that he is led gradually on both by precept and example.

The anſwers to the ſeveral queſtions are not put down in the Complete Practical Arithmetician; a KEY to the work is publiſhed ſeparate, containing all the anſwers, with the ſolutions at full length wherever there is the ſmalleſt appearance of labour or difficulty. This work contains ſeveral uſeful notes and obſervations on Arithmetic; a general view of Logarithmical Arithmetic; Tables of Compound Intereſt and Annuities; general Demonſtration of the principal Rules in the Complete Practical Arithmetician, &c. &c.

Circulating Decimals, which are ſo little underſtood, are, in the following treatiſe, clearly and diſtinctly treated of.

Loſs and Gain, a rule in which the generality of writers have puzzled both themſelves and their readers, is here rendered plain, eaſy, and intelligible. In Fellowſhip, ſeveral new rules are given. Exchange is likewiſe treated of in a different manner to what it uſually has been, and ſeveral uſeful tables are added, which are not to be found in other authors. Theſe tables have, in this edition, been carefully compared with a correct ſet of tables in the library of Hans Sloane, eſq.

The nature of Ratios, and Proportion, ſo far as they relate to commenſurable quantities, is conſidered. Theſe ſubjects are of the higheſt importance. The learned Whiſton, in his Tacquet's Euclid, ſays, "*Si proportionis* "*doctrinam e Matheſi abſtuleris, nihil fere præclarum aut* "*egregium relinques.*"

The work concludes with ſome general obſervations on Numbers, odd and even; Square and Cube Numbers, &c. Theſe will ſerve to raiſe the curioſity of the learner, and give impulſe to his farther enquiries.

Heddon-court, Swallow-ſtreet,
LONDON, 1798.

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xplanation of the Characters made Uſe of in the following Work.

haract.	Names.	Significations.
+	Plus, or more,	the Sign of Addition, as 2+4 ſignifies that 2 and 4 are to be added together.
—	Minus, or leſs.	the Sign of Subtraction, as 8—3 ſignifies that 3 is to be ſubtracted from 8.
×	multiplied into or by,	the Sign of Multiplication, as 7×5 ſignifies that 7 is to be multiplied into or by 5.
÷	divided by,	the Sign of Diviſion, as 9÷3 ſignifies that 9 is to be divided by 3; and $\frac{9}{3}$, or 3)9(, ſignifies the ſame.
=	equal to	the Sign of Equality, as 9=9 ſignifies that 9 is equal to 9; or 5+4—2=7 ſignifies that 5, increaſed by 4 and diminiſhed by 2 is equal to 7.
:	Proportion	as 2 : 4 :: 8 : 16 ſignifies that 2 is to 4 as 8 is to 16.

he other Characters are explained among the Definitions he Work.

Books publiſhed by the ſame Author:

1. A KEY to the Complete Practical Arithmetician; containing anſwers to all the queſtions; ſeveral uſeful notes and obſervations on Arithmetic; a Synopſis of logarithmical Arithmetic, &c. Price Three Shillings bound.

2. The NEW SCHOOLMASTER'S ASSISTANT, or Scholar's Eaſy Guide to Arithmetic; being an abridgment of the Complete Practical Arithmetician, and the Key. Price bound Two Shillings.

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In this work the general principles of Plane Trigonometry, the properties of ſines, tangents, &c. of arches, are clearly explained and demonſtrated; the practical part ſucceeds the theory, and is illuſtrated by a variety of uſeful and important problems.

The Doctrine of the Sphere, and general principles of ſpherical Trigonometry, are plainly and clearly demonſtrated; and the practical parts of ſpherical Trigonometry, are rendered, in every reſpect, as ſimple and eaſy as the practical parts of Plane Trigonometry.

The orthographic and ſtereographic projection of the Sphere are likewiſe given, and their general principles clearly inveſtigated, and applied to practice, &c.

Thoſe Gentlemen who wiſh to encourage the above Work, will oblige the author by ſending their names to their reſpective Bookſellers, or to Mr. C. Law, Ave-Maria-Lane, or Mr. Longman, Paternoſter-row, London.

THE

COMPLETE PRACTICAL ARITHMETICIAN.

PART I.

DEFINITIONS.

1. ARITHMETIC is the art of computing by numbers; and conſiſts of two parts, *viz.* whole numbers, and fractions vulgar or decimal.

2. *Arithmetic in whole numbers* ſuppoſes its numbers to be entire quantities, and not divided into parts.

3. *Arithmetic in fractions* ſuppoſes its numbers to be parts of ſome whole quantity.

4. *Number* is either an *unit* or a *multitude* of units; *viz.* it s the name of that idea, or notion, we conceive of things onſidered as *one*, or *many*. Every multitude has a diſtinct ame; as, two, three, four, &c. and an unit is the begin-ing of number.

Note 1. When we conſider numbers ſimply, without applying them to y particular ſubject, the idea we form of them is called *abſtract*. Thus, we ſpeak of the number three, four, five, or any other number, ab-actedly, we mean three, four, five, &c. units of any thing whatever. t, when we conſider number not in its general nature, but as a number certain particular things, as four yards, five inches, &c. we call it a *concrete* or an *applicate* number. Example, the number four is leſs than five *abſtractedly* conſidered; yet, taking the numbers in an *applicate* ſenſe, it is not always ſo; thus, the quantity of four *yards* is not leſs than five *inches*.

5. *A whole number* is a preciſe number without any part, or parts, annexed.

6. *A mixed number* is a whole number with ſome part, or parts annexed.

7. *An even number* is that which can be divided into two equal whole numbers.

8. *An odd number* is that which cannot be divided into two equal whole numbers.

9. *A prime number* is that which can only be divided by itſelf, or by an unit, without a remainder.

10. *Numbers* are ſaid to be prime to each other when only an unit meaſures, or divides, them both even.

11. *A ſquare number* is the product of a number by itſelf.

12. *A cube number* is the product of a number and its ſquare.

13. *A compoſite number* is that produced by multiplying two, or more, numbers together.

14. *A perfect number* is that which is equal to the ſum of all its aliquot parts.

Note 2. There are ſeveral other numbers which have particular names, as figurate, abundant, deficient, &c. but their chief uſe is in the higher parts of the mathematics.

15. *An aliquot part* is that which is contained a preciſe number of times in another.

16. *An aliquant part* is ſuch as is contained in another a certain number of times, with ſome part, or parts, over.

17. *An integer* is any whole quantity, or number; as, a pound, a yard, &c. or, 1, 2, 3, &c.

18. *Digits*, or *figures*, are the marks by which numbers are expreſſed, and are the nine following, *viz.* 1 one, 2 two, 3 three, 4 four, 5 five, 6 ſix, 7 ſeven, 8 eight, 9 nine; to which we may add the cipher 0, which is of no value when taken by itſelf; yet, when it is placed on the right or left hand of any figure, increaſes or diminiſhes it tenfold.

19. *The nature of all arithmetical operations*, is by ſome quantities that are given, to find out others that are required.

20. *The principal, or fundamental, rules of arithmetic* are notation, and numeration, which informs us in what manner we are to exerciſe and accommodate numbers to the various purpoſes of buſineſs.—*Numeration* conſiſts principally of four parts, *viz.* Addition, Subtraction, Multiplication, and Diviſion.

Note 3. The operations of arithmetic in general are only of two kinds, viz. *increaſing* and *diminiſhing*; for, multiplication is only a compendious method of performing addition, and diviſion performs the work of many ſubtractions.

21. *A propoſition* is ſomething propoſed to be done, or proved.

22. *An axiom* is a ſelf-evident propoſition, and cannot be rendered more plain by demonſtration.

23. *A theorem* is a demonſtrative propoſition, wherein he nature and property of a thing is propoſed to be roved.

NOTATION.

Definition. *Notation* is the art of expreſſing numbers by gures; and teaches us to read, or write down, any num-er, and to have a clear and diſtinct idea of every figure it.

NOTATION TABLE.

Units	1	1	1	1	1	1	1	1	1
Tens		2	2	2	2	2	2	2	2
Hundreds			3	3	3	3	3	3	3
Thousands				4,	4,	4,	4,	4,	4,
Tens of Thousands					5	5	5	5	5
Hundreds of Thousands						6	6	6	6
Millions							7.	7.	7.
Tens of Millions								8	8
Hundreds of Millions									9
Thousands of Millions									1,
Tens of Thousands of Millions									2
Hundreds of Thousands of Millions									3
Billions, or Millions of Millions									4.
Tens of Billions									5
Hundreds of Billions									6
Thousands of Billions									7,
Tens of Thousands of Billions									8
Hundreds of Thousands of Billions									9
Trillions, or Millions three times repeated									1.
Tens of Trillions									2
Hundreds of Trillions									3
Thousands of Trillions									4,
Tens of Thousands of Trillions									5
Hundreds of Thousands of Trillions									6
Quadrillions, &c.									7.

Note 1. The table above may be extended to any length by continually annexing a period of ſix figures towards the left hand, and writing the word quintillions, ſextillions, ſeptillions, octillions, nonillions, decillions, &c. over the unit's place of each of theſe periods; but the table of nine figures, which is printed on a larger type than the reſt, is ſufficient for common uſe.

Note 2. To write down any number. Rule: write down ciphers to as many periods and places as are named in the given number; then begin at the left hand, and obſerve, at each place, what ſignificant figure is named; take away the cipher, and put the ſignificant figure in its place. Ex. Write down ten million fifty thouſand three hundred and one, thus

$\left\{\begin{matrix}00.000,000\\10.050,301\end{matrix}\right\}$ Proceed in this manner for any other number.

3. To read any number of figures when written down. Rule: divide the figures, from the right-hand to the left, into threes, by a comma and a period alternately, writing *m* over the figure to the left of the firſt pe-

riod, *b* over the figure to the left of the ſecond period, &c. till all the figures are brought down, as in this example.

qu. q. t. b. m.
123,456.123,456.123,456.123,456.578,371.123,875,

or inſtead of m, b, t, q, qu, &c. put 1, 2, 3, 4, 5, &c. to repreſent millions, once, twice, thrice, &c. repeated; and read thus, one hundred and twenty-three thouſand four hundred and fifty-ſix quintillions; one hundred and twenty-three thouſand four hundred and fifty-ſix quadrillions; one hundred and twenty-three thouſand four hundred and fifty-ſix trillions; one hundred and twenty-three thouſand four hundred and fifty-ſix billions; five hundred and ſeventy-eight thouſand three hundred and ſeventy-one millions; one hundred and twenty-three thouſand eight hundred and ſeventy-five.

Examples.

1. Write down in words at length the following numbers.

49
75
1075
378

437
305
1087
47318

17349
10807
314815
107048

149387
1078400
30180070
108374108

2. Write down in proper figures the following numbers.

Eighty-nine. Seven hundred and fifty. Five thouſand and one. Ten thouſand and eighty-ſeven. Twenty thouſand and five.

Six hundred and eighty-five thouſand, three hundred and ſixty.

One million, five hundred thouſand, and one.

Twenty-ſeven million, three hundred and ſixty-five thouſand.

Three hundred and eighty-five millions, ſeven hundred and forty-eight thouſand, three hundred, and five.

Eleven thouſand, eleven hundred, and eleven.

Fifty million, fifty thouſand, fifty hundred, and fifty.

SIMPLE ADDITION.

Definition. Simple Addition is a rule by which ſeveral numbers of one denomination are collected together into one ſum.

Rule.

Place the numbers under each other, *viz.* units under units, tens under tens, &c. add up the figures in the row of units, and carry as many units to the next row as there are tens contained in the ſum: proceed thus till the whole is finiſhed.

For the proof. Divide the numbers to be added into two parts, then add up each part by itſelf, and collect theſe ſums together for the whole.

Note 1. If equal numbers be added to equal numbers, the wholes will be equal.

2. If ſeveral numbers are to be added together, they will amount to the ſame ſum, when placed regularly one under another, whichever line, or row, of figures ſtands uppermoſt.

3. Dr. Wallis, in his Arithmetic, gives the following rule to prove a ſimple-addition ſum. Add the figures in the uppermoſt row together, reject the nines contained in their ſum, and ſet the exceſs directly even with the figures in the row. Do the ſame with each row, and ſet all the exceſſes of nine together in a line, and find their ſum; then, if the exceſs of nines in this ſum (found as before) be equal to the exceſs of nines in the total ſum, the work is right.

Examples.

(1.)	3247	(2.)	14934	(3.)	143716	-	4	Exceſs of nines.
	......		31493		371419	-	7	
	1498		47184		143714	-	2	
	3471		....		171349	-	7	
	4734		37149		371493	-	0	
	8714		14734		471348	-	0	
	4374		34718					
Sum	26038			Sum	1673039	-	2	Proof.
	22791	Sum	180212					
Proof	26038		93611		See note 3.			
			86601					
		Proof	180212					

(4.) Add 1473, 40734, 371049, 40057, 3471473, 5734, 37492, and 4718375, together.

(5.) Collect 371434, 278949375, 67149, 3457143, 714934, 9000987, and 5734747, into one ſum.

(6.) Add 5714329, 4718714, 34983714, 671493, 74987149, 6777894987, and 19, together.

(7.) Add 571493, 40007, 6493497, 4718349, 3714934, 4934938, 174934, and 147349 together.

(8) Suppoſe the diſtance from *London* to *Biggleſwade* be 45 miles, thence to *Peterborough* 36, thence to *Lincoln* 51, and thence to *Hull* 41 miles; how many miles are *Peterborough*, *Lincoln*, and *Hull*, from *London?*

SIMPLE SUBTRACTION.

Definition. Simple ſubtraction teaches to deduct, or ſubtract, a leſs number from a greater of the ſame denomination, whereby the remainder, or difference, is found.

Rule.

Place the leſs number under the greater, ſo that units may ſtand under units, tens under tens, &c. Begin at the unit's place, and ſubtract each figure in the lower line from the figure above it; if the lower figure be greater than the upper, add *ten* to the upper figure, from which ſubtract the lower; ſet down the remainder, and carry *one* to the next lower figure.

For the proof. Add the remainder and leſs number together, and the ſum will be the greater. Or, ſubtract the remainder from the greater number, and the difference will be the leſs.

Examples.

(1.) From	9437149	minuend.	(2.) 473494	(3.) 494871	(4.) 347149
Take	1349348	ſubtrahend.	193487	194985	134948
Diff.	8087801				
Proof	9437149				

(5.) From 47348 take 13456.
(6.) From 194938 take 149542.
(7.) From 5007149 take 171493.
(8.) From 1493487 take 149349.

CLASS

CLASS II.

(9.) From the Creation to the Flood was 1656 years; thence to the building of Solomon's temple 1336 years; thence to Mahomet, who lived 622 years after Chriſt, 1630 years. In what year of the world was Chriſt then born, and how many years is it ſince the creation?

(10.) Sir Iſaac Newton was born in the year 1642, and died in 1727, how old was he at the time of his deceaſe, and how many years is it ſince he died?

(11.) A gentleman has two ſons, the age of the elder added to his makes 126 years, and the age of the younger ſon is equal to the difference between the age of the father and the elder ſon. Now, if the father be 80 years of age, how old are each of his ſons?

(12.) Three boys, A, B, and C, won together 97 marbles at play; now, if the number of marbles B won, be added to the number C won, they will make 60; and, if the number A won be added to the number C won, they will make 62. How many marbles did each boy win ſeparately?

SIMPLE MULTIPLICATION.

Definition 1. *Simple multiplication* is a rule by which we increaſe the greater of two given numbers, of the ſame denomination, as often as there are units in the leſs; being a compendious method of performing addition.

2. The number to be multiplied is called the *multiplicand*; the number you multiply by is called the *multiplier*; and the number produced by multiplication, is called the *product*. Theſe numbers are ſometimes called *factors*, becauſe they are to conſtitute a *factum* or product.

The

The Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Propoſition 1. *To multiply by a ſingle figure, or any number not exceeding* 12.

Rule. Begin at the unit's place of the multiplicand, and multiply each figure in it by the multiplier, writing down the whole of ſuch products as are leſs than 10; but, for ſuch as exceed 10, or a number of tens, write down the exceſs, and carry an unit, for each 10, to the next product.

Prop. 2. *When the multiplier is the product of two, or more, umbers in the table.*

Rule. Multiply the multiplicand by one of the component parts, and that product by the other, &c. for the hole product.

Prop. 3. *When the multiplier conſiſts of ſeveral figures.*

Rule. The multiplicand muſt be multiplied by each gure ſeparately, and the firſt figure of every product muſt nd exactly under the figure you multiply by. Add theſe oducts together for the whole product.

Prop. 4. *When ciphers are intermixed with the figures in e Multiplier.*

Rule. Omit the ciphers, and let the firſt figure of each product be placed under its reſpective multiplier.

Prop. 5. *When there are ciphers at the end of the multiplicand or multiplier.*

Rule. Neglect the ciphers, and multiply as before; then to the right hand of the product annex as many ciphers as were omitted.

For the proof. Multiply the multiplier by the multiplicand, and if the product be the ſame with *that* of the multiplicand by the multiplier, the work is right.

Note 1. If two numbers are to be multiplied together, they will make the ſame product, which ever number you make the multiplier.

2. If ſeveral numbers, as 5, 6, 7, &c. are to be multiplied together, it is the ſame thing whether 5 be multiplied by the product of 6 and 7, or multiplied firſt by 6 and then by 7, &c. And, if ſeveral given numbers are to be multiplied by any number, and the ſum of the products taken; it will be the ſame thing, if you multiply the ſum of thoſe given numbers, by that multiplier.

3. The product of any two numbers can have at moſt but as many places of figures as are in both the multiplier and multiplicand, and at leaſt but one leſs.

4. There is a way of proving multiplication by caſting out the nines as in addition. Thus, caſt the nines out of the multiplier and multiplicand, and ſet down the remainders. Multiply the two remainders together; and, if the exceſs of nines in their product be equal to the exceſs of nines in the total product, the work is *generally* right.

Examples to Propoſition 1.

(1.) Multiply 471347325
by 2

Product 942694650

(2.) Multiply 371493407 by 3.
(3.) Multiply 47048743 by 4.
(4.) Multiply 57134974 by 5.
(5.) Multiply 37180753 by 6.
(6.) Multiply 4900757149 by 7.
(7.) Multiply 3714937187 by 8.
(8.) Multiply 4708714371 by 9.
(9.) Multiply 5714937143 by 10.
(10.) Multiply 3715714936 by 11.
(11.) Multiply 149371574 by 12.

Examples

Examples to Prop. 2.

(12.) Multiply 47134987 by 56.
8

377079896
7

Product 2639559272

(13.) Multiply 47134784 by 21
(14.) Mult. 37149374 by 22.
(15.) Mult. 47187413 by 24.
(16.) Mult. 7493456 by 63.
(17.) Mult. 4194734 by 72.
(18.) Mult. 3175493 by 77.
(19.) Mult. 39007149 by 84.
(20.) Mult. 71340987 by 96.
(21.) Mult. 47154734 by 132.
(22.) Mult. 704134795 by 144.

Examples to Prop. 3.

(23.) Multiply 471493475
by 4395

2357467375
4243441275
1414480425
1885973900

Product 2072213822625

Proof by caſting out the nines
Product
6
ultiplicand 8 X 3 Multiplier.
6

Proof by multiplication.
4395
471493475

21975
30765
17580
13185
39555
17580
4395
30765
17580

2072213822625

(24.) Multiply 430714934 by 743.
(25.) Mult. 37157437 by 14972.
(26.) Mult. 47157149 by 37495.
(27.) Mult. 5714937 by 47159.
(28.) Mult. 47134749 by 371895.
(29.) Mult. 3704957 by 4713759.

Examples

Examples to Prop. 4.

(30.) Multiply 4713457
by 5704008

37707656
18853828..
32994199.
23567285

26885596435656

(31.) Mult. 371493407 by 700505.
(32.) Mult. 57040935 by 5040648.
(33.) Mult. 40750493 by 67100805.
(34.) Mult. 371493471 by 57080507.
(35.) Mult. 4070490385 by 4090805.
(36.) Mult. 5417080574 by 3905008

Examples to Prop. 5.

(37.) Multiply 47150000
by 3980000

37720
42435
14145

Product 187657000000000

(38.) Mult. 471000 by 40700.
(39.) Mult. 507000 by 30500.
(40.) Mult. 4713000 by 6070500.
(41.) Mult. 3075600 by 30500700.
(42.) Mult. 57867000 by 4007500.

CLASS II. *exerciſing all the propoſitions.*

(43.) Mult. 47149 by 7.
(44.) Mult. 371594 by 12.
(45.) Mult. 5719070 by 1440.
(46.) Mult. 70409040 by 371500.
(47.) Mult. 507040500 by 4734050.
(48.) Mult. 37145674 by 3710514.
(49.) Mult. 123456789 by 1234567890.
(50.) Mult. 1234567890 by 987654321.

(51.) Required the continual product of 56, 750, 54730, 64007, and 587504.

(52.) Required the ſum of 157 added 495 times to itſelf.

(53.) Let 954 be added 435 times to itſelf, and ſhew what the laſt ſum total exceeds or falls ſhort of four hundred and fifteen thouſand.

(54.) Required the product of eleven thouſand eleven hundred and eleven by twelve thouſand twelve hundred and twelve.

(55.) What is the difference between thrice ſix and twenty and thrice twenty ſix?

(56.) There are two numbers; the greater is 19 times 508, and their difference is 15 times 112; required the ſum and product of thoſe numbers.

SIMPLE DIVISION.

Definition 1. *Simple Diviſion* is a rule by which we find ow often one number is contained in another of the ſame enomination; being a ſhort method of performing ſub- action.

2. The number to be divided is called the *dividend*, the umber you divide by is called the *diviſor*; and hence will iſe a third number, called the *quotient*, which ſhews how ften the diviſor is contained in the dividend. If the di- iſor does not exactly contain the dividend, a fourth num- er will occur, called the *remainder*, which muſt always be ſs than the diviſor.

Prop. 1. *When the diviſor does not exceed* 12.

Rule. Obſerve how often the diviſor is contained in the ſt, or firſt and ſecond, figure of the dividend, and ſet e quotient figure under it: carry 10 for every unit re- ining after ſubtraction to the next figure of the divi- nd; proceed thus, multiplying and ſubtracting mentally, l you have made uſe of all the figures in the dividend.

Prop. 2. *When the diviſor is a compoſite number.*

Rule. Divide the dividend by one of the component parts, and that quotient by the other, for the required quotient. If there be a remainder to each of the quotients, multiply the laſt remainder by the firſt diviſor, and to that product add the firſt remainder for the true one.

Prop. 3. *When the diviſor conſiſts of ſeveral figures.*

Rule. Find how many times it may be had in as many figures of the dividend as are juſt neceſſary; multiply the diviſor by the quotient-figure, ſubtract the product from that part of the dividend which ſtands above it, and to the right hand of the remainder bring down the next figure in the dividend, which number divide as before; and ſo on till all the figures in the dividend are brought down.

Prop. 4. *When the dividend has ciphers on the right hand.*

Rule. Cut off the ciphers from the diviſor by a daſh of your pen, and alſo cut off as many ciphers, or figures, from the dividend. But when the diviſion is finiſhed, the ciphers omitted muſt be reſtored to their proper places, and the figures cut off in the dividend muſt be placed to the right hand of the remainder.

Note 1. When the ſcholar is pretty ready at diviſion, he may ſubtract each figure of the product as he produces it, and write down only the remainder.

For the proof. Multiply the quotient by the diviſor, to the product add the remainder, if any, and the ſum will be equal to the dividend.

2. There are ſeveral methods of proving diviſion. If you ſubtract the remainder from the dividend, and divide this number by the quotient, the quotient found by this diviſion will be equal to the former diviſor.

3. Or, add the remainder and all the products of the ſeveral quotient-figures by the diviſor together, according to the order in which they ſtand in the work, and the ſum will be equal to the dividend.

4. Another method. Caſt away the nines in the diviſor and quotient, take their product, and caſt away the nines, to which add the exceſs of nines in the remainder after diviſion ; the exceſs of nines in this ſum will be equal to the exceſs of nines in the dividend, when the work is right.

5. An even number cannot divide, or meaſure, an odd number, nor a greater a leſs.

6. Half the ſum of any two numbers, increaſed by half their difference, will give the greater number; and half their ſum, diminiſhed by half their difference, will give the leſs number.

7. The quotient, ariſing from the diviſion of the ſum of two, or more, numbers, is equal to the ſum of the quotients ariſing from the diviſion of the parts, ſeparately, by the ſame diviſor.

8. If any two numbers are ſeparately divided by 9 or 3, and the two remainders multiplied together, and that product divided by 9 or 3, the laſt remainder will be the ſame as if you divided the product of the two firſt numbers by 9 or 3.

9. Any number, divided by 9 or 3, will leave the ſame remainder as the ſum of its digits divided by 9 or 3. Hence, if any number is diviſible by 9 or 3, the ſum of its digits is likewiſe diviſible by 9 or 3, and *vice verſa*. The method of proof by caſting out the nines, in the preceding rules, depends upon this theorem.

Examples to Propoſition 1.

(1.) Divide 1749342345 by 2.

```
             Dividend
Diviſor 2)1749342345
          ----------Rem.
Quotient   874671172 - 1.
          ----------
```

(2.) Divide 471349571 by 3.
(3.) Divide 407104937 by 4.
(4.) Divide 70407143 by 5.
(5.) Divide 170049378 by 6.
(6.) Divide 493740075 by 7.
(7.) Divide 30871050743 by 8.
(8.) Divide 41375714937 by 9.
(9.) Divide 71000571479 by 10.
(10.) Divide 37407184374 by 11.
(11.) Divide 47105713475 by 12.

Examples to Prop. 2.

(12.) Divide 7149347859 by 25.

```
25=5×5)7149347859
       ----------
      5)1429869571—4 ⎫ Rem.
        ---------    ⎬     ___
Quotient 285973914—1 ⎭ 9=1×5+4
         ---------
```

(13.) Divide 7349473857 by 27.
(14.) Divide 749347549 by 144.
(15.) Divide 649305743 by 55.
(16.) Divide 4730715405 by 121.
(17.) Divide 3704095714 by 108.
(18.) Divide 4710437154 by 132.

(19.) Divide 1071540075 by 99.
(20.) Divide 457014374 by 96.

Examples to Prop. 3.

(21.) Divide 34674378 by 95.

```
Diviſor.     Dividend.      Quotient.  Rem.
   95    )   34674378    (   364993 — 43.
             285×                95
             ————           ———————
             . 617          1824965
             . 570×        3284937
               ————              43  Remainder.
             . . 474       ————————
             . . 380×      34674378  Proof by multiplication.
                 ————      ————————
             . . . 943
             . . . 855×
                   ————
             . . . . 887
             . . . . 855×
                     ————
             . . . . . 328
             . . . . . 285×
                       ————
             . . . . . . 43
               ——————————
               34674378  Proof by addition.
               ——————————
```

```
Proof by caſting out the nines.          34674378 Dividend.
             6                                 43 Remainder.
           8+7 Remainder.        Quotient. ————————
Diviſor   5 X 7 Quotient.          364993)34674335(95  Proof by
             6                             3284937          viſion.
        Dividend.                          ———————
                                            1824965
                                            1824965
                                           ———————
```

(22.) Divide 714394756 by 1754.
(23.) Divide 47159407184 by 3574.
(24.) Divide 5719487194715 by 45705.
(25.) Divide 4715714937149387 by 17493.
(26.) Divide 671493471549375 by 47143.
(27.) Divide 571943007145 by 37149.
(28.) Divide 1714347149347 by 57143.
(29.) Divide 4937154714937 5 by 374567.
(30.) Divide 1714937159471 43 by 571007.
(31.) Divide 6754371495671594 by 678957.

Examples to Prop. 4.

(32.) Divide 147159678899 by 145000.

$145\underline{000})147159678\underline{99}(101489\frac{62899}{145000}$ Quotient.

$\underline{145}$

215
$\underline{145}$

709
$\underline{580}$

1296
$\underline{1160}$

1367
$\underline{1305}$

62899 Rem.

Or thus,

$145\underline{000})147159678\underline{99}(101489\frac{62890}{145000}$

215
709
1296
$\underline{1367}$

62899 Rem.

(33.) Divide 571436490075 by 36500.
(34.) Divide 194718490700 by 73000.
(35.) Divide 795498347594 by 47150.
(36.) Divide 1495070807149 by 371500.
(37.) Divide 671493471 4934 by 754000.
(38.) Divide 1071491471430715 by 147500.
(39.) Divide 14714937493714957 by 157900.
(40.) Divide 7149374947194715 by 1749000.

CLASS II. *exercising all the Propoſitions.*

(41.) Divide 714947349 by 90.
(42.) Divide 1714937148 by 14400.
(43.) Divide 7149371478 by 121.
(44.) Divide 71900715708 by 57149.
(45.) Divide 15241578750190521 by 123456789.
(46.) Divide 121932631112635269 by 987654321.
(47.) Divide 69616103498721931800 by 975005700.
(48.) Divide 656458931996524171800 by 700489070.
(49.) Divide 7149437149547 by 3714900.
(50.) Divide 14714937148475 by 123456.
(51.) The ſum of two numbers is 348 and their difference 94, required the numbers.
(52.) What number, multiplied by 365, will produce 15725?

(53.) What

(53.) What number, multiplied by 95, will give the ſame product as 157 by 570?

(54.) What number is that, from which if a twelfth part of 1728 be deducted, and the remainder increaſed by the 95th part of 82175, the ſum will be 1185?

(55.) What number, divided by 1185, will give 497 for the quotient, and leave juſt a fifth part of the diviſor remaining?

(56.) Required the difference between ſix-dozen dozen and half a dozen dozen.

(57.) Subtract 759 out of 171493745 as often as you can, and ſhew what the laſt remainder exceeds or falls ſhort of 500?

TABLES of ENGLISH COIN, WEIGHTS, MEASURES, &c.

Table I. *Money.*

The loweſt piece of money, uſed in England, is a farthing, and all accounts are kept in pounds, ſhillings, pence, and farthings.

2 farthings	-	make	1 halfpenny.
4 farthings	-	—	1 penny.
6 pence	-	—	half a ſhilling.
12 pence	-	—	1 ſhilling.
2 ſhillings and 6 pence		—	half-a-crown.
5 ſhillings	-	—	1 crown.
5 ſhillings and 3 pence		—	a quarter-guinea.
10 ſhillings and 6 pence		—	half-a-guinea.
21 ſhillings	-	—	1 guinea.
20 ſhillings	-	—	1 pound.

Note.

£. denotes pounds, s. ſhillings, and d. pence.
¼ ——— a farthing, or the quarter of any thing.
½ ——— a halfpenny, or the half of any thing.
¾ ——— three farthings, or three-quarters of any thing.

Imaginary Engliſh Coin.

A mark	value	13s.	4d.	An angel	10s.
A noble	—	6	8	A Carolus	23s.
A groat	—		4	A Jacobus	25s.

SHILLINGS

SHILLINGS and PENCE TABLES.

		£.	s.			£.	s.
20	Shillings	1	0	130	Shillings	6	10
30	——	1	10	140	——	7	0
40	——	2	0	150	——	7	10
50	——	2	10	160	——	8	0
60	——	3	0	170	——	8	10
70	——	3	10	180	——	9	0
80	——	4	0	190	——	9	10
90	——	4	10	200	——	10	0
100	——	5	0	210	——	10	10
110	——	5	10	220	——	11	0
120	——	6	0	230	——	11	10

		s.	d.			s.	d.
20	Pence	1	8	80	Pence	6	8
24	——	2	0	84	——	7	0
30	——	2	6	90	——	7	6
36	——	3	0	96	——	8	0
40	——	3	4	100	——	8	4
48	——	4	0	108	——	9	0
50	——	4	2	110	——	9	2
54	——	4	6	120	——	10	0
60	——	5	0	130	——	10	10
70	——	5	10	132	——	11	0
72	——	6	0	144	——	12	0

TABLE II. TROY WEIGHT.

By this weight are weighed gold, ſilver, jewels, amber, and all liquors.

24 grains	make	1 pennyweight, dwt.
20 pennyweights	—	1 ounce, oz.
12 ounces	—	1 pound, lb.

TABLE III. APOTHECARIES WEIGHT.

Apothecaries, chemiſts, &c. uſe this weight in mixing edicines; but buy and ſell their drugs by avoirdupois eight.

20 grains make 1 ſcruple, ℈.
3 ſcruples — 1 dram, ʒ.
8 drams — 1 ounce, ℥.
12 ounces — 1 pound, lb.

TABLE IV. Avoirdupois Weight.

By Avoirdupois Weight are weighed ſuch commodities as are coarſe and droſſy, or ſubject to waſte, as groceries of all kinds, bread, butter, cheeſe, and moſt other common neceſſaries of life; pitch, tar, reſin, wax, tallow, flax, &c. as are likewiſe all metals, ſilver and gold excepted.

16 drams make 1 ounce.
16 ounces — 1 pound.
28 pounds — 1 quarter of an hundred weight.
4 quarters, or 112 pounds, — 1 hundred-weight, cwt.
20 hundred-weight — 1 ton.

There are ſeveral ſorts of ſilk weighed by the great pound of 24 ounces, others by the common pound of 16 ounces. Hence, to reduce great pounds to common, multiply by 3 and divide by 2; and, to bring common pounds into great, multiply by 2 and divide by 3.

Note. A pound avoirdupois is equal to 14 oz. 11 dwts. 15½ grains troy. Hence the avoirdupois pound is to the troy pound as 6999½ is to 5760, or nearly as 17 to 14; and the avoirdupois ounce is to the troy ounce as $437\frac{15}{32}$ is to 410, or nearly as 73 to 80.

A firkin of butter, 56lb.
——— ſoap, 64lb.
A barrel of raiſins, 112lb.
——— ſoap, 256lb.
A puncheon of prunes, 1120lb.
A fother of lead, 19½ cwt. or 2184lb.
A ſtone, horſeman's weight, 14lb.
——— butcher's meat in London, 8lb.
——— ditto in the country, 14lb.
A gallon of train-oil, 7½lb.
A truſs of ſtraw, 36lb.
——— new hay, 60lb.
——— old hay, 56lb.

A load, 36 truſſes.

	lb.	oz.	dr
A peck loaf weighs	17	6	1
A half-peck -	8	11	0
A quartern -	4	5	8

Wool-weight.

A clove, or half-ſtone,	7lb
A ſtone, or 2 cloves,	14lb
2 ſtone, or 1 todd,	28lb
A wey, or 6½ todds,	182lb
A ſack, or 2 weys,	364lb
A laſt, or 12 ſacks,	4368lb

TABL

TABLE V. CLOTH-MEASURE.

Cloth-meaſure is uſed by linen and woollen-drapers. Hollands are meaſured by the Engliſh ell, and tapeſtry by the Flemiſh ell; woollens, linens, wrought ſilks, tapes, &c. by the yard.

2¼ Inches make 1 Nail.
4 Nails — 1 Quarter of a yard.
3 Quarters — 1 Flemiſh ell.
4 Quarters — 1 Yard.
5 Quarters — 1 Engliſh ell.
6 Quarters — 1 French ell.

TABLE VI. LONG MEASURE.

This meaſure is uſed to meaſure diſtances, lengths, breadths, heights, depths, &c. of places or things.

12 Lines, or 3 barley-corns, make 1 Inch.
12 Inches — — 1 Foot.
3 Feet — — 1 Yard.
6 Feet, or 2 yards — — 1 Fathom.
5½ Yards, or 11 half-yards, or 16½ feet } — 1 Rod, pole, or perch.
4 Poles, or 100 links — 1 Chain.
40 Poles, or 10 chains — 1 Furlong.
8 Furlongs, or 80 chains — 1 Mile.
3 Miles - - — 1 League.
60 Geographical miles, or 69½ ſtatute-miles — 1 Degree.

Note. The ſtatute-pole is 5½ yards, but in ſome counties in England they reckon 6 yards to the pole; in the North of England 7 yards are accounted a pole, or rod.

TABLE VII. SQUARE-MEASURE.

Square-meaſure is uſed to meaſure all kinds of ſuperficies; ſuch as land, paving, flooring, plaſtering, roofing, ſlating, tiling, and every thing that has length and breadth.

Square *Square*
144 Inches - make 1 Foot.
9 Feet - — 1 Yard.

30¼ Yards,

Square		*Square*
30¼ Yards, or 272¼ feet	—	1 Pole, rod, or perch.
16 Poles - -	—	1 Chain.
40 Perches - -	—	1 Rood.
4 Roods, or 160 rods, or 4840 yards, or 10 chains	—	1 Acre.
640 Acres - -	—	1 Mile.
100 Feet - - -	—	1 of flooring.

TABLE VIII. Cubic, or Solid, Measure,

Is uſed, in menſuration, to meaſure all kinds of ſolids; or ſuch figures as conſiſt of three dimenſions, *viz.* length, breadth, and depth, or thickneſs.

Cubic		*Cubic*
1728 Inches	make	1 Foot.
27 Feet	—	1 Yard.
166⅜ Yards	—	1 Pole.
64000 Poles	—	1 Furlong.
512 Furlongs	—	1 Mile.

40 Feet of rough timber, or 50 feet of hewn timber, 1 ton, or load.

TABLE IX. Wine-Measure.

By this meaſure, all wines, brandies, rum, ſpirits, diſtilled liquors, cider, perry, mead, vinegar, honey, oil, &c. are meaſured, bought, and ſold.

4 Gills	make	1 Pint.
2 Pints	—	1 Quart.
4 Quarts, or 2 pottles,	—	1 Gallon.
10 Gallons	—	1 Anchor of brandy.
18 Gallons	—	1 Runlet.
31½ Gallons	—	1 Barrel, or half-hogſhead.
63 Gallons	—	1 Hogſhead.
42 Gallons	—	1 Tierce.
84 Gallons	—	1 Puncheon.
2 Hogſheads, or 126 gallons,	——	1 pipe, or butt.
2 Butts, or 4 hogſheads, or 252 gallons,	——	1 ton.

Note. In the North of England, a gill is half a pint; alſo the meaſure of a gill in London, is there called a *jack*.

TABLE X. Ale and Beer-Measure in London.

By this meaſure all malt-liquors are gauged, bought,
nd ſold.

2 Pints - -	make	1 Quart.
4 Quarts - -	——	1 Gallon.
8 Gallons - -	——	1 Firkin of *ale.*
9 Gallons - -	——	1 Firkin of *beer.*
2 Firkins, or 18 gallons	——	1 Kilderkin.
2 Gallons -	——	1 Barrel of *ale.*
6 Gallons - -	——	1 Barrel of *beer.*
8 Gallons -	——	1 Hogſhead of *ale.*
4 Gallons - -	——	1 Hogſhead of *beer.*
2 Hogſheads, or 96 gallons	——	1 Butt of *ale.*
2 Hogſheads, or 108 gallons	——	1 Butt of *beer.*

Note. The above meaſure is uſed only in London for gauging and ſel-
g: in all other places, in England, the following Table is the ſtandard of
and beer meaſure, according to a ſtatute of exciſe made in the year 1689.

ABLE XI. Ale and Beer-Measure in the Country.

2 Pints - -	make	1 Quart.
2 cubic Inches, or 4 quarts	——	1 Gallon.
8½ Gallons - -	——	1 Firkin.
7 Gallons - -	——	1 Kilderkin, or ½ barrel.
4 Gallons - -	——	1 Barrel.
1 Gallons - -	——	1 Hogſhead.

ote. Notwithſtanding the above ſtatute, common brewers, in ſome
s of the country, allow 36 gallons to the publicans for a barrel of ale
eer.

TABLE XII. Dry-Measure.

Dry-meaſure is uſed in meaſuring all *dry* commodities,
wheat, barley, beans, and other grain; fruit, roots,
d, ſalt, coals, oiſters, &c.

2 Pints -	make	1 Quart.
2 Quarts	——	1 Pottle.
2 Pottles, or 8 pints	——	1 Gallon.
2 Gallons -	——	1 Peck.

4 Pecks - - —— 1 Buſhel.
4 Buſhels - —— 1 Coom.
2 Cooms, or 8 buſhels —— 1 Quarter.
4 Quarters —— 1 Chaldron.
5 Quarters - —— 1 Wey.
2 Weys, or 10 quarters, —— 1 Laſt.

For Coals.

4 Pecks make 1 Buſhel.
3 Buſhels — 1 Sack.
36 Buſhels — 1 Chaldron.
21 Chaldrons — 1 Score.

Note. 32 buſhels make a chaldron in the country: 5 pecks make a buſhel water-meaſure: 5 buſhels make a ſack of flour. The ſtandard Wincheſter buſhel is a cylinder of 18½ inches diameter, and 8 inches in depth, and contains 2150$\frac{3}{5}$ cubic inches.—7680 wheat, or barley-corns, are ſuppoſed to fill a pint meaſure.

TABLE XIII. MEASURE OF TIME.

60 Thirds make 1 Second.
60 Seconds — 1 Minute.
60 Minutes — 1 Hour.
24 Hours — 1 Day.
7 Days — 1 Week.
4 Weeks — 1 Month.

13 Months 1 day, or 52 weeks 1 day, or 365 days, a year, for three years together: but every fourth year contains 366 days, and is called leap-year, except thoſe centuries which cannot be exactly divided by four. This is called the *Gregorian year,* being inſtituted by pope Gregory in 1582, and was brought into uſe in England in 1752. Hence, if we conſider the year to conſiſt of 365 days 6 hours, at a medium, one day ought to be ſtruck off the account in 130 years, the ſolar year being only 365 days 5 hours 49 minutes.

The common year is alſo divided into 12 calendar months.

Memorandum.—30 days hath September,
April, June, and November,
February hath 28 alone,
And all the reſt have 31.

In a leap-year, which happens every fourth (except in the odd centuries, as 17, 18, 19, &c.) February has 29 days.

A TABLE, ſhewing the Number of Days from any Day of one Month to the ſame Day of any other Month in the ſame Year.

To the ſame Day.	From any Day of											
	Jan	Feb	Mar.	Apr.	May.	June	July.	Aug.	Sep	Oct.	Nov.	Dec
Jan.	365	334	306	275	245	214	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	123	92	62
Mar.	59	28	365	334	304	273	243	212	181	151	120	90
April	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	123	92	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	61	30	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

Note. In leap-year, if the end of the month of February be in the time, one day muſt be added on that account. To know when it is leap-year, divide the year by 4, and the remainder ſhews how long it is after leap-year; if nothing remains, it is leap-year, excepting the years 1700, 1800, 1900, 2100, &c.

TABLE XIV. Of NUMBER.

12 Units -	make	1 Dozen.
12 Dozen -	—	1 Groſs.
12 Groſs, or 144 dozen	—	1 Great groſs.
20 Units -	—	1 Score.
5 Score -	—	1 ſhort Hundred.
6 Score - -	—	1 long Hundred.
24 Sheets -	—	1 Quire of paper, or parchment.
20 Quires -	—	1 Ream of ditto.
2 Reams - -	—	1 Bundle of ditto.
12 Skins of parchment	—	1 Roll.

COMPOUND ADDITION.

Definition. Compound Addition is a Rule by which ſeveral numbers, of different denominations, are collected together into one ſum.

Rule,

Rule.

Place the numbers ſo that thoſe of the ſame denomination may ſtand directly under each other. Add the firſt row, or loweſt denomination, together, as in ſimple addition, and divide the ſum by as many of the ſame denomination as make one of the next greater: ſet down the remainder, and carry the quotient to the next ſuperior denomination. Proceed thus through all the denominations to the higheſt, which add as in ſimple Addition.

The method of proof is the ſame as in ſimple Addition.

Note. Addition of money may either be performed by the preceding rule, or by the help of the pence-table.

MONEY.—*See Table* I.

(1.)

	£	s.	d.
	174	11	4¼
	74	19	11¼
	64	13	10
	374	19	11½
	64	18	10¾
	105	11	9¼
	74	19	10½
	44	18	11¾
Sum	779	14	7¼
	605	3	3
Proof	779	14	7¼

(2.)

	£	s.	d.
	374	11	5¼
	74	12	7¾
	149	14	10¼
	74	14	11½
	104	13	10
	………	………	………
	105	17	11¼
	74	19	10¼
	16	14	7¾
Sum	976	0	2
	197	12	5¼
	778	7	8¾
Proof	976	0	2

(3.)

£	s.	d.
174	11	4¼
39	18	10½
714	14	7¼
64	19	11¼
108	14	9
74	14	7¼
64	13	10
174	19	4
67	12	5
149	15	9¾
74	14	7
1709.10	1½	

(4.)

£	s.	d.
147	14	9¾
77	11	4½
10	10	10¼
7	7	4
19	4	11¼
19	3	10¼
14	10	11
74	13	9¾
104	14	9¾
74	12	10¼
16	18	5¼
567	4	1½

(5.)

£	s.	d.
149	14	7¼
37	[illegible]	9¾
64	14	10
104	19	11¼
64	13	10
174	19	11¾
47	14	10¼
[illegible]	[illegible]	[illegible]
694	5	7

(6.)

£	s.	d.
14	11	3¼
19	18	10
77	11	3¾
49	14	7
16	18	4½
17	15	10
1	14	9¾
6	18	10¼
295	4	10

(7.)

£	s.	d.
14	19	4¼
17	11	10
39	18	11¼
19	14	9
19	15	11¼
18	19	10
77	19	11¼
14	11	10½
223	12	5¼

(8.)

£	s.	d.
14	10	4¼
77	18	3
[illegible]	13	9½
67	12	4¼
9	11	10
18	10	5
17	19	4
19	10	4
240	6	8½

TROY-WEIGHT.—*See Table* II.

(9.)

lb.	oz.	dwt.
174	11	19
74	10	13
944	9	14
74	11	19
944	10	13
74	11	3
14	9	4
77	10	11
82-1,16		

(10.)

oz.	dwt.	gr.
174	19	23
714	11	14
714	0	18
74	1	22
948	2	21
74	1	12
715	2	14
714	18	16
4129	19	20

(11.)

lb.	oz.	dwt.
71	11	19
64	8	14
77	0	0
14	3	11
64	2	9
74	1	14
77	2	13
19	2	14
4021	11	13

(12.)

oz.	dwt.	gr.
74	19	23
64	14	17
74	19	11
66	13	9
74	14	11
14	10	3
19	11	14
17	10	13
467	14	5

APOTHECARIES'-WEIGHT.—*See Table* III.

(13.)

℔	℥	ʒ
47	11	7
94	10	6
74	10	4
75	9	3
69	0	2
57	1	2
18	2	1
74	1	2

(14.)

℥	ʒ	℈
149	7	2
714	3	0
619	2	1
74	6	2
169	5	2
74	1	2
777	6	1
948	5	2

(15.)

ʒ	℈	gr.
749	2	19
607	1	18
714	2	17
400	0	0
74	1	13
715	2	14
64	1	18
174	2	19

(16.)

℔	℥	ʒ
84	11	7
74	10	6
37	5	4
19	4	3
24	1	2
29	2	6
19	2	4
24	9	5

AVOIRDUPOIS-WEIGHT.—*See Table* IV.

(17.)

T.	cwt.	qr.
174	19	3
74	14	2
714	13	1
718	16	2
34	15	2
14	14	1
70	13	2
165	7	1

(18.)

Cwt.	qr.	lb.
174	3	27
714	2	24
149	1	14
719	2	16
407	1	23
149	2	17
714	2	18
3030	1	9

(19.)

Qr.	lb.	oz.
44	27	15
74	26	14
19	14	13
74	19	14
66	27	13
74	19	10
14	18	11
376	17	6

(20.)

lb.	oz.	drs.
17	15	15
27	14	11
18	13	9
36	14	14
70	0	0
64	13	10
14	14	11
315	7	6

635
5
315

636
378

834
4
336

CLOTH-MEASURE.—*See Table* V.

(21.)			(22.)			(23.)			(24.)		
Yds	qr.	n.	E.E.	qr.	n.	EllsFr.	qr.	n.	EllsFlem.	qr.	n.
74	3	3	77	4	3	749	5	3	714	2	3
64	2	1	14	3	2	704	4	2	615	1	2
74	1	3	74	2	1	108	3	1	714	1	3
49	2	1	49	1	2	705	4	0	724	2	2
74	1	2	74	2	1	708	3	1	149	1	2
44	3	1	74	3	2	474	5	2	718	2	3
74	2	0	44	1	2	174	0	1	419	1	1
14	1	2	74	2	3	194	3	2	710	1	2
481	2	1	484	2	0	3021	0	0	4750	0	2

LONG-MEASURE.—*See Table* VI.

(25.)			(26.)			(27.)			(28.)		
Lea.	m.	f.	F.	p.	yds.	P.	yds.	ft.	Feet.	in.	b.c.
17	2	7	147	39	5½	177	5½	2	174	11	2
14	1	6	614	37	4½	714	4½	1	49	10	1
74	1	7	714	19	3½	714	1½	2	74	11	2
69	2	4	674	17	1½	615	0	1	64	9	1
74	1	0	719	27	2½	714	1½	2	74	10	1
69	2	1	197	19	1½	719	1½	1	64	11	2
74	1	2	714	14	3½	437	2½	1	74	10	0
94	0	3	704	19	4½	614	1½	2	64	9	1
65	1	6									

LAND-MEASURE.—*See Table* VII.

(29.)			(30.)			(31.)			(32.)		
A.	r.	p.	A.	r.	p.	A.	r.	p.	A.	r.	p
77	3	39	714	3	39	14	3	39	174	3	39
64	2	37	619	1	18	74	1	19	714	1	2
74	1	24	714	2	27	64	2	14	618	2	1
64	2	19	619	1	34	74	1	18	719	1	1
74	1	18	719	2	37	47	2	24	734	2	1
64	2	17	719	1	24	18	1	14	715	1	2
14	1	13	615	2	14	74	2	19	639	2	1
74	2	11	74	1	18	34	1	14	714	3	2

9432

427

108

42

16

425

375

393 84
3
430 405

WINE-MEASURE.—*See Table* IX.

(33.)

Tuns	hhd.	gall.
714	3	62
614	2	61
174	1	39
164	2	47
274	1	49
175	2	37
375	1	49
714	2	61
214	0	27

(34.)

Pun.	gall.	qt.
714	83	3
615	81	2
714	74	1
614	18	2
713	75	0
614	17	1
715	14	3
719	28	2
5425	7	2

(35.)

Tierce.	gall.	qt.
74	41	3
64	40	2
74	19	1
64	39	2
74	40	1
69	19	1
17	39	2
18	41	1
450	39	1

(36.)

Gall.	qt.	pt.
14	3	1
74	2	1
39	2	1
17	1	0
19	2	0
77	1	1
39	3	1
14	1	1
217	2	0

ALE and BEER MEASURE.—*See Table* X.

(37.)

B.B.	fir.	gall.
74	3	8
14	2	7
16	1	4
17	1	3
29	2	2
17	1	7
41	2	6
37	1	5

(38.)

A.B.	fir.	gall.
73	3	7
69	2	6
14	1	7
39	2	2
19	1	6
49	2	6
37	1	4
19	1	2

(39.)

A.hhd.	gall.	qt.
714	47	3
614	44	1
374	43	2
157	41	1
719	42	1
374	41	2
174	12	1
19	13	2

(40.)

B.hhd.	gall.	qt.
714	53	3
415	47	2
714	19	1
614	27	1
715	51	2
714	37	2
615	19	1
714	18	2

DRY-MEASURE.—*See Table* XII.

(41.)

h.	b.	p.
14	31	3
74	31	2
64	30	1
74	27	2
64	19	2
74	31	1
64	11	1
5	10	2

(42.)

Ch.	qr.	b.
174	3	7
375	1	6
400	0	5
371	1	4
634	2	3
719	1	2
149	2	1
375	1	3

(43.)

Qr.	b.	p.
149	7	3
715	3	2
649	1	3
479	2	1
675	1	3
149	2	1
375	1	2
649	1	3

(44.)

Score.	ch.	b.
74	20	35
49	19	33
64	17	35
74	14	10
39	13	9
42	16	3
19	17	4
37	18	34

MEASURE of TIME.—*See Table* XIII.

(45.)			(46.)			(47.)			(48.)		
Yrs.	m.	w.	M.	w.	d.	Days.	hrs.	min.	Hrs.	min.	ſec.
737	12	3	64	3	6	714	23	59	647	59	59
347	11	2	74	1	5	74	14	54	137	54	54
618	10	1	34	2	3	64	21	55	375	56	56
374	9	2	74	1	4	74	13	53	714	17	19
375	3	1	63	2	1	69	12	14	615	54	54
714	12	3	74	1	2	74	12	19	714	17	13
415	10	1	64	2	1	37	11	27	613	34	56
714	3	1	74	1	3	16	12	19	624	27	39

CLASS II. *Promiſcuous Examples.*

(49.) A is indebted to B £27 4s. 10d. to C £108 11s. 7¼d. to D £157 0s. 6d. to E £957 11s. 10d. to F £149 11s. 10d. to G £190 10s. 6d. and to H £900 5s. 4d. what is A's whole debt?

(50.) A corn-factor has paid for wheat £49 11s. 10d. for rye £47 13s. 7d. for oats £104 19s. 10d. for barley £77 11s. 3d. for peas £88 11s. 9d. he has alſo paid for carriage and other incidental charges £5 11s. 1½d. for an inſurance 12 guineas; his commiſſion on the whole amounts to 10 guineas; for what ſum muſt he draw upon his employer to clear the account?

(51.) R of *Rotterdam* is debtor to H of *Hull* for fifty firkins of butter, 75 guineas; for 15 pieces of *Yorkſhire* cloth, £215 11s. 10d. for 24 fother of *Derbyſhire* lead, £557 11s. 9d. for cheeſe, £65 11s. 4d. for bar-iron, £100 19s. 7d. for his acceptance of a bill, drawn for £571 11s. 9d. H has alſo paid convoys, inſurances, port-charges, &c. £27 11s. for warehouſe-room, eartage, &c. £7 7s. the factorage of the whole amounts to 100 guineas. For what ſterling money muſt H draw upon R to clear this account?

(52.) A collector of caſh has been out with bills, and gives account that A paid him 50 guineas, B £14 11s. 6d. C £37, D 315 quarter-guineas, E a £50 bank-note, and F 300 guineas. What money had he in charge?

(53.) A nobleman, going out of town, is informed by his ſteward that his butcher's bill comes to £194 17s. his baker's to 49l. 11s. 6d. his brewer's to 95 guineas, his wine-merchant's to £107 11s. 3d. his corn-chandler's to £75, his tallow-chandler's to £27 11s. 6d. his cheeſemonger's to 35 guineas; to his cabinet-maker are owing 315 guineas, alſo for rent, ſervants'-wages, &c. he is indebted £140 11s. 6d. and if he takes 100 guineas with him to defray his expences on the road, for what ſum muſt he ſend to his banker to ſatisfy theſe demands?

(54.) A gentleman bought of a ſilverſmith diſhes to the weight of 16lb. 11oz. 14dwt. plates 42lb. 10oz. 9dwt. ſpoons 14lb. ſalts 12lb. 9oz. waiters 11lb. 5oz. 10dwt. tankards 11 lb. 10oz. and a ſilver tea-board, and other articles, to the weight of 14lb. 11oz. 10dwt. What weight of plate did he buy in all?

(55.) A merchant in *London* bought of a farmer in *Kent* eight bags of hops, No. 1 weighed 3cwt. 2qr. 14lb. No. 2 2cwt. 1qr. 14lb. No 3 4cwt. 1qr. 27lb. No. 4 2cwt. 3qrs. No. 5 4cwt. 1qr 11½lb. No. 6 6cwt. 1qr. 11lb. No. 7 7cwt. 1 qr. 11¼ lb. and No. 8 weighed 5cwt. 3qr. 12lb. the merchant by agreement was to pay the carriage to town; how many cwt. had he to pay for?

(56.) I bought 6 parcels of cloth, the firſt contained 37yds. 1qr.; the ſecond 54yds. 3qrs. 2n. the third 15yds. 1qr. 2n. the fourth 72yds. 2qrs. 1n. the fifth 25½yds. and the ſixth 49¾yds. How many yards did I buy in all?

COMPOUND SUBTRACTION.

Definition. *Compound Subtraction* teaches us to find the ifference of any two numbers of different denominations.

Rule.

Place the leſs number under the greater, ſo that thoſe rts, which are of the ſame denomination, may ſtand rectly under each other. Begin at the loweſt denomi- tion, and ſubtract the under number from the upper: hen any of the lower denominations are greater than the pper, increaſe the upper number by as many as make one of the next ſuperior denomination, from which ſum take the figure in the lower line; ſet down the difference, and carry 1 to the next number in the lower line, and ſubtract as before; and ſo on till you have gone through all the denominations.

The method of proof is the ſame as in ſimple ſubtraction.

MONEY.—*See Table* I.

(1.)

	£	s.	d.
Borrowed	1749	11	9¼
Paid -	948	12	11¾
Remains to pay	800	18	9½
Proof	1749	11	9¼

(2.)

	£	s.	d.
Lent	4749	11	10¼
Received	1494	11	10¾
Due			
Proof			

(3.)

£	s.	d.
149	11	4¼
74	10	7¾

(4.)

£	s.	d.
647	10	7¼
149	19	11¾

(5.)

£	s.	d.
44	11	8¼
17	14	7½

(6.)

£	s.	d.
75	11	10¼
44	19	11¾

(7.)

£	s.	d.
74	11	9¼
39	17	11¾

(8.)

£	s.	d.
747	11	9¾
714	18	8½

(9.)

£	s.	d.
719	11	9¼
614	10	8¾

(10.)

£	s.	d.
613	11	7¼
149	10	4¾

(11.)

	£	s.	d.
Borrowed	71747	11	10½
Paid at different times -	7149	11	4
	675	14	7¾
	714	19	10¼
	147	11	9
	56	19	10¼
	714	11	11¾
	64	18	10¼
Paid in all			
Remains to pay			

(12.)

	£	s.	d.
Received	71437	11	9¼
Laid out at ſundry times	6174	19	10
	734	17	5¼
	615	19	11¾
	375	14	10¼
	74	13	6
	19	18	11½
	77	14	10½
Laid out in all			
Remains in hand			

Required

(13.)

Required the balance of this account.

Dr.			Cr.		
£	s.	d.	£	s.	d.
747	11	10	41	11	10
314	11	9½	74	13	9¼
647	19	10¼	64	11	6¼
374	14	7	77	13	10¼
167	15	9¼	14	15	9
317	11	8	64	15	10
			37	12	4¼

(14.)

Required the balance of this account.

Dr.			Cr.		
£	s.	d	£	s.	d.
34	11	9¼	711	10	4
75	19	11	375	13	10½
67	14	10¾	714	19	11½
47	15	11¼	615	17	10¼
14	19	10	375	14	7
37	15	11½	14	11	6¼
64	12	10¾			

TROY-WEIGHT.—*See Table* II.

(15.)

lb.	oz.	dwt.
14	11	9
11	10	14

(16.)

oz.	dwt.	gr.
74	12	13
64	14	17

(17.)

lb.	oz.	dwt.
175	3	10
159	11	14

(18.)

oz.	dwt.	gr.
17	10	20
14	11	23

APOTHECARIES'-WEIGHT.—*See Table* III.

(19.)

℔	℥	ʒ
144	10	5
64	11	7

(20.)

℥	ʒ	℈
27	4	1
14	7	2

(21.)

ʒ	℈	gr.
27	1	14
14	0	19

(22.)

℔	℥	ʒ
74	10	5
65	11	6

AVOIRDUPOIS-WEIGHT.—*See Table* IV.

(23.)

T.	cwt.	qr.
14	12	2
1	14	3

(24.)

Cwt.	qr.	lb.
17	1	25
14	2	27

(25.)

Qr.	lb.	oz.
143	22	12
74	19	14

(26.)

lb.	oz.	dr.
174	11	10
39	12	13

CLOTH-

Cloth-Measure.—*See Table* V.

(27.)			(28.)			(29.)			(30.)		
Yds	qr.	n.	E.E.	qr.	n.	E.Fr.	qr.	n.	E.Fl.	qr.	n.
174	2	1	174	3	1	171	1	3	12	1	1
39	3	2	49	4	2	74	5	2	10	2	3

Long-Measure.—*See Table* VI.

(31.)			(32.)			(33.)			(34.)		
Lea.	m.	f.	F.	p.	yd.	P.	yd.	ft.	Ft.	in.	b.c.
21	2	4	14	34	4½	14	3¾	1	17	11	2
3	2	6	12	39	5¼	9	4½	2	14	11	1

Land-Measure.—*See Table* VII.

(35.)			(36.)			(37.)			(38.)		
A.	r.	p.	A.	r.	p.	A.	r.	p.	A.	r.	p.
12	1	32	112	1	31	12	1	25	19	1	20
1	3	14	74	2	37	10	3	39	14	2	21

Wine-Measure.—*See Table* IX.

(39.)			(40.)			(41.)			(42.)		
T.	hhd.	g.	Punch.	g.	qt.	Tier.	g.	qt.	Gall.	qt.	pt
27	2	54	147	14	2	14	1	2	24	2	2
19	3	62	79	83	3	12	41	3	18	0	1

Ale and Beer Measure.—*See Table* X.

(43.)			(44.)			(45.)			(46.)		
A.B.	f.	g.	B.B	fir.	g.	A.hhd.	g.	qt.	B.hhd.	g.	qt.
14	3	5	147	1	3	271	1	2	143	1	2
12	3	7	39	3	8	49	47	3	79	52	3

Dry

DRY-MEASURE.—*See Table* XII.

(47.)

Ch.	b.	p.
4	31	3
7	31	2

(48.)

Ch.	qr.	b.
17	3	1
14	3	7

(49.)

Qr.	b.	p.
147	6	2
94	7	3

(50.)

Score.	ch.	b.
47	1	12
14	20	35

MEASURE of TIME.—*See Table* XIII.

(51.)

Yrs	m.	w.
7	11	2
4	12	3

(52.)

M.	w.	d.
147	2	3
19	2	4

(53.)

D.	hrs.	min.
167	21	50
19	23	54

(54.)

Hrs.	min.	ſec.
174	50	19
94	58	57

CLASS II. *Promiſcuous Examples.*

(55.) A horſe in his furniture is worth £52 10s.; out of £24 10s. 6d.; how much does the price of the furniture xceed that of the horſe?

(56.) What ſum added to £11 14s. 9¼d. will make 133 11s. 9½d.?

(57.) A tradeſman, failing, was indebted to A 105 19s. 11d. to B 150 guineas; to C £34 18s. 10d. D £500 19s. to E £700 14s. 9d. When this happened, had caſh by him to the amount of £50; Goods to e amount of £350 14s. 9d. his houſhold furniture was orth £24 11s. his book-debts amounted to £94 14s. 8d. theſe things were faithfully given up to his creditors, at did they loſe by him?

(58.) The great bell at *Oxford*, the heavieſt in *England*, ighs 7T. 11cwt. 3qr. 4lb. *St. Paul's* Bell at *London* weighs . 2cwt. 1qr. 22lb. and *Tom* of *Lincoln* weighs 4T. 10cwt. r. 18lb. How much are theſe bells together inferior in ight to the great bell at *Moſcow*, the largeſt in the rld, which weighs 198T. 2cwt. 1qr.?

(59.) An apprentice, who is 14 years, 11 months, 13 eks, 14 days, 15 hours, 38 minutes old, is to ſerve his ſter till he is 21 years of age. How long has he to ve?

(60.) What is the difference of latitude and longitude between *Calcutta* in the *Eaſt-Indies* (Lat. 22° 34′ N. Long. 88° 34′ E.) and *Lima* in *South America?* (Lat. 12° 1′ S. Long. 76° 44′ W.)

COMPOUND MULTIPLICATION.

Definition. Compound Multiplication is a rule by which we find the amount of any given number, of different denominations, by repeating it any propoſed number of times.

Propoſition 1. *When the multiplier does not exceed* 12.

Rule. Multiply the loweſt denomination by it, divide the product by the number making one of the next higher denomination; ſet down the remainder, and carry the quotient to the product of the next higher denomination: proceed thus till all the denominations are multiplied.

Prop. 2. *If the multiplier exceeds* 12, *and is a compoſite number*

Rule. Multiply ſucceſſively by the component parts inſtead of the whole number at once.

Prop. 3. *When the multiplier cannot be produced by the multiplication of two, or more, ſmall numbers.*

Rule. Find two, or more, numbers that compoſe the *neareſt* number to the multiplier; then multiply by the component parts as before, and add, or ſubtract, the odd parts as you find occaſion.

Prop. 4. *If the multiplier be four, five, or more, hundreds.*

Rule. Multiply the given price, or quantity, by 10, an that product by 10, and ſo on for 10, 100, or 1000, time the price or quantity: then multiply each product by th number of thouſands, hundreds, and tens, and the firſt lin by as many as make up the number of things, or multiplie and the ſum of the products will be the anſwer.

Prop. 5. *If the multiplier be a whole number with pa annexed.*

Rule. When you have multiplied by the whole number, for ½, ⅓, ¼, ⅕, or ⅙, &c. divide the top-line by 2, 3, 4, 5, or 6, &c. but, if the numerator of the fractional part be greater than 1, multiply the top-line by it, and divide the product by the denominator; add this quotient to the product, or value, obtained by multiplying with the whole number.

Note. The upper figure is called the numerator, and the lower one the denominator. Thus 5 *Numerator.*
7 *Denominator.*

Examples to Propoſition 1.

(1.) What coſt 4 yards of cloth
t 7s. 6½d. per yard?
7 6½
4
£1 10 2 anſwer.

2.) 5 buſhels at 5s 10d.

(3.) 6 yards at 6s 9d.
(4.) 7 ells at 5s 11½d.
(5.) 8 oz. at 7s 10d.
(6.) 9 lb. at 7s 5¼d.
(7.) 10 gallons at 16s 4½d.
(8.) 11 cwt. at 1l 9s 10½d.
(9.) 12 ſheep at 1l 17s 9d

(10.) In 9 pieces of Kerſey, each 14yds 3qrs 2n. how nany yards?

(11.) What is the weight of 12 tankards, each weighng 11 oz. 10dwt. 19gr.?

(12.) In 11 pieces pieces of cloth, each 17yds 3 qrs 3n. ow many yards? *Anſwer* 197yds. 1 qr. 1n.

Examples to Prop. 2.

(13.) What coſt 15 gallons of
ine, at 5s. 3½d. per gallon?
5 5×3=15
1 6 5½ price of 5.
3
£3 19 4½ price of 15. Anſ.

4.) 16 hogſheads at 3l 14s 5d

(15.) 24 yards at 7s 5½d.
(16.) 35 cwt. at 1l 14s 8½d.
(17.) 36 tons at 5l 15s 11¼d
(18.) 84 chaldrons at 1l 16s 9¼d.
(19.) 108 buſhels at 7s 9½d.
(20.) 132 ells at 18s 9¼d.
(21.) 144 butts at 5l 13s 9½d

(22.) In 32 wedges of gold, each 2lb. 7oz. 14gr. how ny pounds?

(23.) In 21 fields, each 3a. 2r. 19p. how many acres?

Examples to Prop. 3.

(24.) What coſt 23 yards of cloth at 14s. 9d. per yard?

		s.	d.	
		14	9	$7\times3+2=23$.
			7	
	5	3	3	price of 7.
			3	
	15	9	9	price of 21.
Add	1	9	6	price of 2.
	16	19	3	price of 23.

		s.	d.	
Or thus		14	9	$6\times4-1=23$.
			6	
	4	8	6	price of 6.
			4	
	17	14	0	price of 24.
Subtract		14	9	price of 1.
	16	19	3	price of 23.

(25.) 31 yards at 12s 7¼d.
(26.) 39 dozen at 6s 7½d.
(27.) 139 pair at 4s 9¼d.
(28.) 86 lb. of ſilk at 19s 4d.
(29.) 111 ſacks of flour at 1l 4s 9d.
(30.) 156 cwt. at 4l 9s 6d.

(31.) In 57 years, each 13 m. 1 day, 6 hrs, how many months?

(32.) What is the weight of 29 hhds of ſugar, each 7cwt. 2qr. 18lb.?

(33.) In 67 parcels of tea, each 25lb. 7oz. 13drs. how many cwts. &c.?

Examples to Prop. 4.

(34.) What coſt 394 yards at

		s.	d.	
		17	5½	per yard.
			10	
9×	8	14	7	price of 10.
			10	
	87	5	10	price of 100.
			3	
	261	17	6	price of 300.
	78	11	3	price of 90.
	3	9	10	price of 4.
	343	18	7	price of 394.

(35.) 357 oxen at 7l 10s 5d.
(36.) 549 ſheep at 12s 9½d.
(37.) 754 lb. of tea at 6s 10d
(38.) 198 lb of indigo at 6s 3¼d.
(39.) 754 fother at 20l 5s 10d
(40.) 178 ells at 5s 9¼d.
(41.) 198 barrels at 1l 14s 9d
(42.) 744 chaldron at 1l 16s 8d

Examples

Examples to Prop. 5.

(43.) What coſt $56\frac{1}{2}$ chaldrons

	£	s.	d.	
at	1	14	9	per chaldron.
			7	
	12	3	3	price of 7.
			8	
	97	6	0	price of 56.
		17	$4\frac{1}{4}$	price of $\frac{1}{2}$.
	£98	3	$4\frac{1}{2}$	price of $56\frac{1}{2}$.

(44.) What coſt $4\frac{5}{9}$ yards at

	s.	d.	
	7	6	per yard.
		4	
1	10	0	price of 4.
	4	2	price of $\frac{5}{9}$
£1	14	2	price of $4\frac{5}{9}$.

	s.	d.
	7	6
		5
9)	37	6
	4	2

(45.) $1788\frac{1}{5}$ gallons at 6s 4d.
(46.) $3714\frac{1}{8}$ cwt. at 4l 11s 9d.
(47.) $7149\frac{3}{7}$ chaldrons at 1l 14s 9d.
(48.) $547\frac{5}{8}$ laſts at 5l 5s.
(49.) $1749\frac{1}{2}$ firkins at 14s $9\frac{1}{4}$d.
(50.) $754\frac{3}{4}$ cwt. at 17s $5\frac{1}{4}$d.

Note. Should the above examples be thought inſufficient to complete the ſcholar in this uſeful rule, recourſe muſt be had to the bills of parcels.—If the teacher approve it, he may omit this propoſition till the ſcholar has learnt compound diviſion.

COMPOUND DIVISION.

Definition. *Compound Diviſion* teaches us to find how often one given number is contained in another of different denominations; or, to divide a given compound number into any propoſed number of equal parts.

Rule.

Place the diviſor to the left hand of the dividend. Divide the higheſt denomination of the dividend by the diviſor, and bring the remainder, if any, into the next inferior denomination, adding thereto the parts of that name in the dividend: divide this number as above, and ſo on till the whole is finiſhed. If the diviſor be large, and not a compoſite number, divide after the manner of long diviſion.

The method of proof is by Compound Multiplication.

Note. If the diviſor be a whole number with parts annexed, multiply it by the denominator of the fractional part, adding the numerator of the fractional part to the product; then multiply the dividend by the denominator of the fractional part in the diviſor, and divide as above.

Examples.

(1.) A gentleman's income is 1260l 15s 5d a year, what is that per day, 365 days being in one year?

	£	s.	d.	£	s.	d.
365)	1260	15	5 (	3	9	1 anſwer.
	1095					10
	165			34	10	10×6
	20					10
	3315			345	8	4
	3285					3
	30			1036	5	0
	12			207	5	0
				17	5	5
	365					
	365			1260	15	5 proof.

(2.) Divide 47l 19s 4d by 3.
(3.) Div. 37l 14s 10d by 24.
(4.) Div. 49l 19s $11\frac{1}{4}$d by 66.
(5.) Div. 34l 14s $9\frac{1}{4}$d by 149
(6.) Div. 477l 19s $10\frac{1}{4}$d by $7\frac{1}{4}$
(7.) Div. 149l 11s $3\frac{1}{4}$d by $3\frac{5}{8}$
(8.) Divide 1774l 19s $10\frac{1}{4}$d by 179.
(9.) Div. 47yds 3qr 2n. by 5.
(10.) Divide 375a. 3r. 14p. by 9.
(11.) Div. 714lb 10oz. 12gr. by 89.
12.) Div. 374cwt. 3qr. 10lb. by 48.
(13.) Div. 374EllsE. 2qr. 3n. by 142.
(14.) Div. 3149ch. 21b. 3p. by $374\frac{1}{8}$
(15.) Div. 47oz. 11dwt. 12gr. by $34\frac{5}{7}$.

(16.) If 60 ſheep be ſold for 112l 10s what is the value of 1?

(17.) If 112lb. of cheeſe coſt 2l 18s 8d. what is that per lb.?

(18.) If 17cwt. of lead coſt 15l 5s $7\frac{3}{4}$d what coſts 1?

(19.) Bought 7 yards of cloth for 16s 4d. what is that per yard?

(20.) If 63 oxen coſt 493l. 13s 11$\frac{1}{4}$d. what coſts 1?

(21.) If 66lb. of butter coſt 1l 11s 7$\frac{1}{2}$d. what coſts 1lb.?

(22.) If 528lb. of tobacco coſt 23l 13s. what coſts 1lb.?

(23.) If a tun, or 252 gallons, of wine coſt 60l. what coſts 1 gallon?

(24.) A prize of 1000 guineas is to be divided among 150 ſailors, what is each man's ſhare after deducting $\frac{1}{8}$ part for the officers?

(25.) If 125 ingots of ſilver, each of an equal weight, weigh 1347 oz. 11dwt. 14gr. what is the weight of 1 ingot?

(26.) If 475cwt. 1qr. 14lb. be the weight of 27 hhds. of tobacco, what is the weight of 1?

(27.) Bought 6$\frac{1}{2}$ pieces of tapeſtry, containing 237 ells Flem. 2qr. 2n. what is the length of 1 piece?

CLASS II.

(28.) A common paſture, containing 54. 1r. 35p. another, containing 54$\frac{1}{2}$ acres; and a third, containing 39a. 13 perch. are to be incloſed and divided among 60 pariſhioners; what is each man's ſhare after deducting 21a. 2r. for tithes, admitting the land to be equally good?

(29.) Twenty-ſix wedges of gold, weighing, with a due proportion of alloy, 34lb. 3 oz. 11dwt. 14gr. were brought to the mint to be coined into guineas; what is the weight of each wedge, admitting them equal, and how many guineas may be made out of the whole, ſuppoſing no loſs in the metal, and that an oz. will make 3$\frac{1}{2}$ guineas?

(30.) A perſon ſold a hogſhead of ſugar, weighing 7cwt. 3qr. 14lb. how much pure ſugar was contained in it; thirteen times the weight of the droſs and hhd being equal to the weight of pure ſugar?

(31.) If a *talent* of ſilver be worth 357l 11s 10$\frac{1}{2}$d. what is the value of a *ſhekel*, of which 300 make a *talent*, and what is the weight of a *talent*, a *ſhekel* weighing 9dwt 3gr?

(32.) *Camillus*, the *Roman* general, after conquering the city of *Veii*, and other ſervices done to his country, was, through the enmity and avarice of the tribunes, fined 1500 *as's*, value 4l 13s 9d. Pray, what was the value of an *as* in Engliſh money?

REDUCTION.

Definition. *Reduction* is the method of reducing numbers from one name, or denomination, to another of the ſame value.

Rule.

All great names are brought into ſmall by multiplying with as many of the next leſs as make one of the greater, adding to the product the parts of the leſs name, if the number to be reduced be a compound one: and all ſmall names are brought into great by dividing with as many of the leſs as make one of the next greater.

The method of proof is by reverſing the queſtion.

Note 1. To multiply a whole number by a whole number with a fraction joined to it. When you have multiplied by the whole number, as in ſimple multiplication, if the part be $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, 1-5th, or 1-6th, &c. divide the multiplicand by 2, 3, 4, 5, or 6, &c. but if the numerator of the fractional part be greater than 1, let the multiplicand be multiplied by it, and divide the product by the denominator: add this quotient to the product obtained by multiplying with the whole number.

2. To divide a whole number by a whole number with a fraction joined to it. Multiply the diviſor by the denominator of the fractional part, and add the numerator to the product; let this be a new diviſor: then multiply the dividend by the denominator of the fractional part for a new dividend, which divide by the new diviſor to obtain the true quotient. When the dividend has a fraction joined to it, the rule for obtaining the quotient will be exactly the reverſe of this.

Examples.

MONEY.—*See Table* I.

(1.) In 5l. 5s how many ſhillings, pence, and farthings?

5l. 5s.
20

105 ſhillings.
12

1260 pence.
4

5040 farthings.

Here is a great name brought into a ſmall.

(2.) In 4800 farthings how many pence, ſhillings, and pounds?

4)4800 farthings.

12)1200 pence.

2|0) 10|0 ſhillings.

5 pounds.

Here a ſmall name is brought into a great, by a method of operation directly the converſe of the proceding.

(3.) In 19 pounds, how many ſhillings, pence, and farthings?

(4.) In 55 guineas, how many ſhillings, pence, and farthings?

(5.) Reduce 54l 11s 9¼d. into farthings.

(6.) Reduce 77l 11s 10½d. into halfpence.

(7.) Reduce 94l 14s 8d. into pence.

(8.) Reduce 47l 14s 4d. into two-pences.

(9.) In 34l 11s 9d. how many three-pences and pence?

(10.) In 47l 19s 8d. how many groats, pence, and farthings?

(11.) In 108l 11s 6d. how many ſixpences?

(12.) How many crowns, half-crowns, ſhillings, ſixpences, and pence, are in £54?

(13.) Reduce 74l 13s 9d. into ſhillings, three-pences, and farthings.

(14.) In 11520 farthings how many pence, ſhillings, and pounds?

(15.) In 17880 pence how many pounds?

(16.) Reduce 100800 farthings into guineas.

(17.) In 50400 halfpence how many pounds?

(18.) In 12050 ſhillings how many crowns and pounds?

(19.) Reduce 311040 pence into groats, ſhillings, crowns, and pounds.

(20.) In 102l 16s 3d. how many pieces of coin, each 7s 3½d in value?

(21.) In 7494 dollars, at 4s 6d. each, how many groats, ſhillings, half-crowns, crowns, and pounds?

(22.) In a Jacobus, a Carolus, 5 angels, 3 marks, 5½ nobles, 6 teſters, and 90 groats, how many farthings?

2. TROY-WEIGHT.—*See Table* II.

(23.) In 17lb 5oz. how many grains? *Anſw.* 100320.

(24.) In 6720 grains how many ounces? *Anſw.* 14.

(25.) In 14 ingots of ſilver, each 27oz. 10dwt. how many grains?

(26.) In 474 ſpoons, each weighing 3oz. 10dwt. how many pounds of ſilver?

(27.) How many pints, each 9oz. may be made out of 17lb 6oz 14dwt. of ſilver?

(28.) A gentleman ſent a tankard to his goldſmith weighing 50oz. 8dwt. and ordered him to make it into tea-ſpoons, each weighing $1\frac{1}{3}$oz. how many had he?

3. APOTHECARIES-WEIGHT.—*See Table* III.

(29.) In 25lb. how many ſcruples and grains? *Anſw.* 7200℈. 144000grs.

(30.) In 97920 grains, how many ounces and pounds? *Anſ.* 204℥ 17℔.

(31.) In 15℔ 1℥ 1ʒ 1℈ 2gr. how many grains?

(32.) In 174947 grains how many pounds?

(33.) An apothecary made a compound of 12℥ 1ʒ 2℈ 14gr. into *troches* of 1℈, of $1\frac{1}{3}$℈, and of 14gr. and into *pills* of 11gr. and 13gr. each; he made an equal number of *troches* and *pills*, how many of each had he?

4. AVOIRDUPOIS-WEIGHT.—*See Table* IV.

(34.) In 12 tons of iron how many ℔? *Anſ.* 26880℔.

(35.) In 31360℔ of iron how many tons? *Anſ.* 14 *tons.*

(36.) In 375cwt. 2qr. 15℔ of copper how many ℔?

(37.) Reduce 740900oz. into cwts. and tons.

(38.) In 39 bags of hops, each 3cwt. 1qr. 14℔. how many cwts?

(39.) In 750 fother of lead, each $19\frac{1}{2}$cwt. how many cwts?

(40.) In 135cwt. of raiſins how many parcels, each 90℔?

(41.) In 570 great pounds of ſilk how many common?

(42.) In 525 common pounds of ſilk how many great?

(43.) How many pounds in 54 hhds of tobacco, each weighing $17\frac{1}{2}$cwt.?

(44.) A grocer weighed out a hhd of ſugar, containing 16cwt 3qr 10℔ into parcels of 6℔, of 8℔, of 12℔, of 14℔, and of 28℔, and had an equal number of each; how many of each had he?

5. CLOTH-MEASURE.—*See Table* V.

(45.) In 314 yards how many nails? *Anſw.* 5024 *nails.*

(46.) In 576 French ells how many yards? *Anſw.* 864 *yards.*

(47.) Reduce 97yds 3qrs. into Engliſh ells.

(48.) In 57 pieces of Holland, each 35 ells Flemiſh, how many nails?

(49.) In 14 bales of cloth, each 17 pieces, each piece 56 ells Flemiſh, how many yards?

(50.) In 394 pieces of ſtuff, each 23¼ yards, how many yards?

(51.) In 796 pieces of *Kerſey*, each 45⅞ yards, how many yards?

6. LONG MEASURE.—*See Table* VI.

(52.) In 471 miles how many furlongs and poles? *Anſ.* 3768*f.* 150720 *p.*

(53.) In 123200 yards how many miles? *Anſ.* 70 *m.*

(54.) In 50 miles how many yards, feet, inches, and barley-corns?

(55.) Reduce 37m. 2furl. 37p. 5ft. 6in. into feet.

(56.) In 17400 chains how many furlongs and miles?

(57.) How many barley-corns will reach round the earth, which is 360 degrees, each 69½ miles? and how many quarters of barley are contained in ſuch a number of barley-corns, admitting 9212 barley-corns to fill a pint, and that 512 pints make a quarter?

(58.) How often will a *perambulator*, 2¾ yards in circumference, turn between *London* and *York*, being 198 miles?

7. LAND-MEASURE.—*See Table* VII.

(59.) In 77a. 1r. 14p. how many perches? *Anſ.* 12374*p.*

(60.) In 17280 perches how many acres? *Anſ.* 108 *a.*

(61.) If a piece of ground, containing 14a. 34p. be taken from a field of 50 acres, how many perches will the remainder contain?

(62.) A gentleman has 4 fields, the firſt meaſures 3a. 1r. the ſecond 4½ acres, the third 5a. 30p. and the fourth 4a. 3r. 20p. and theſe he wiſhes to divide into parcels, or ſhares, of 3¾ roods each, for the purpoſe of accommodating his manufacturing tenants with ſmall tenements: how many will he have?

8. WINE-MEASURE.—*See Table* IX.

(63.) Reduce 32 hhds into quarts. *Anſ.* 8064 *qts.*

(64.) In 3276 gallons how many tuns? *Anſ.* 13 *T.*

(65.) How many gallons and pints are in 75 hhds?

(66.) In 77 hhds of brandy how many ½ anchors?

(67.) In 10 tuns 2 hhds 18 gallons of wine how many pipes, puncheons, hhds, tierces, and runlets, and of each an equal number?

9. ALE and BEER-MEASURE.—*See Table* X. *and* XI.

(68.) In 38 hogſheads of ale, in *London*, how many pints? *Anſw.* 14592 *pints.*

(69.) In 38 hogſheads of ale, in the country, how many pints? *Anſw.* 15504 *pints.*

(70.) Reduce 516 barrels of beer, *London* meaſure, into half-pints.

(71.) How many gallons of beer are contained in a back of 50 barrels, country meaſure?

10. DRY-MEASURE.—*See Table* XII.

(72.) In 44 quarters of corn how many pecks? *Anſw.* 1408 *pecks.*

(73.) In 30720 quarts how many laſts? *Anſw.* 12 *laſt*

(74.) In 50 chaldrons of coals how many pecks?

(75.) How many ſacks, of 3 buſhels each, are containe in 193 chald. 12 buſh. of coals?

11. MEASURE OF TIME.—*See Table* XIII.

(76.) In 365d. 5h. 48m. 55ſec. being a *ſolar* year, how many ſeconds? *Anſw.* 31556935 *ſeconds.*

(77.) In 354d. 8h. 48m. 36½ſec. being a *lunar* year, o 12 *lunar* months, how many ſeconds? *Anſw.* 30617316 *ſeconds.*

(78.) How many days, hours, minutes, and ſeconds have elapſed from the creation of the world to Chriſtma 1789, ſuppoſing the creation to have been 4004 years be fore the incarnation of Chriſt?

(79.) If *London* was built 1108 years before Chriſt's na tivity, how many hours is it ſince to Chriſtmas 1789?

(80.) From May 18, 1788, to February 18, 1815, ho many days?

CLAS

CLASS II. *Promiſcuous Examples.*

(81.) A butcher has 22 oxen, each weighing 238½ ſtone, ight pounds to the ſtone, to be cut out for ſea-ſervice into ieces of 14℔, of 26℔, of 22℔, of 30℔, of 16℔, and f 15℔, and to have an equal number of each; how many ieces will theſe oxen produce, allowing nothing for aſte?

(82.) A country gentleman ordered 58l 14s. to be diſtri- uted among the poor inhabitants of 4 villages. Thoſe of the lace of his reſidence were to have 1s. each, thoſe of the next d. the next were to have 6d. and the laſt 4d. each;—four erſons (one out of each village) who ſhared in the bounty, ere appointed to diſtribute the money. Now, admitting e number of indigent perſons in each village to be equal, ow many partook of this charity, the men who diſtri- uted the money being allowed 5s. 4½d. each *extra?*

(83.) A gentleman ſent to his goldſmith 18 ingots of lver, each weighing 3℔ 7oz. 14dwt. 21gr. with orders make it into tankards of 18oz. 14dwt. 10gr. each, ps of 19oz. 15dwt. 11gr. each, ſpoons of 24oz. 10dwt. 4gr. per dozen, ſalts of 4oz. 12dwt. each, forks of 2oz. 11dwt. 14gr. per dozen; for every tankard he was make one cup, a dozen ſpoons, 1 ſalt, and a dozen rks:—how many of each will it make, allowing 7oz. dwt. 14gr. for droſs, and what quantity of ſilver will ere be left?

(84.) How long would 500 people be in counting a *llion* of money, ſuppoſing each of them counted £100 ery minute (without intermiſſion) the year conſiſting of 5 days 6 hours?

(85.) According to the Julian account, which was uſed England before the year 1752, the year conſiſts of 365 ys for three years ſucceſſively, and 366 days every fourth, 365¼ days at a mean; and the ſolar year, according the beſt aſtronomical calculation, conſiſts of 365 days rs. 48m. 55ſec.—Required in how many years the ſea- s of the year would be quite reverſed, *viz.* how many rs would elapſe before *Chriſtmas* would fall upon *Mid- mer?*

(86.) If $44\frac{1}{4}$ guineas make 1 ℔ *Troy*, and 48 halfpence make 1 ℔ *Avoirdupois*, what is the difference between the weight of a guinea and a halfpenny?

(87.) A farmer had 5 ſons, to whom he left £560 in caſh, and 5 bills of £84 10s. 6d. each; he ordered his debts to be paid, amounting to £120, and £20 to be expended at his funeral: the reſidue was to be divided in this manner; the eldeſt ſon was to have a fourth part, and each of the other ſons to have equal ſhares: what was the ſhare of each ſon?

DIRECT PROPORTION, or, *The Rule of Three.*

Definition. Direct Proportion teaches, by three given numbers, to find a fourth, in ſuch proportion to the third, as the ſecond is to the firſt.

Rule.

State the queſtion by placing the numbers in ſuch order that the firſt and third may be of one kind, and the ſecond the ſame as the number required: then bring the firſt and third numbers into one name, and the ſecond into the loweſt denomination mentioned. Multiply the ſecond and third numbers together, divide the product by the firſt, and the quotient will be the anſwer in the ſame denomination as the ſecond number.

The method of proof is by changing the order of the ſtating.

Particular Rules and Obſervations.

1. If the firſt term, and either the ſecond or third, can be divided by any number, without a remainder, let them be divided, and the quotients uſed inſtead of them.

2. Divide the ſecond term by the firſt, multiply the quotient by the third, and the product will be the anſwer.

3. Divide the third term by the firſt, multiply the quotient by the ſecond, and the product will be the anſwer.

4. Divide the firſt term by the ſecond, and the third by that quotient, the laſt quotient will be the anſwer.

5. Divide the firſt term by the third, and the ſecond by that quotient, the laſt quotient will be the anſwer.

6. When any of the above methods can be uſed, they will be found more convenient than the general rule.

7. The greateſt difficulty, in the rule of three, is in ſtating the queſtion, or abſtracting the numbers out of the words in the queſtion, and placing them down in their proper order: to perform which, the following obſervations may aſſiſt the ſcholar.

In all queſtions in the rule of three, there are three given terms: two of ſuppoſition, and one of demand; that of demand muſt always be the third number, and may be known by the words What Coſt? What will? How far? How much? How many? &c. The firſt term muſt always be of the ſame name as the laſt, or term of demand; and the term ſought will be of the ſame kind and denomination as the ſecond term in the ſuppoſition.

8. The method of proof, as has been already obſerved, is by changing the order of the ſtating; the following example will ſhew in what manner it may be varied.—Ex. If 2 yards of cloth coſt four ſhillings, what will 8 yards coſt at the ſame rate?

State it thus:

	1ſt term.		2d term.		3d term.		4th term.
If	2 yards	:	4 ſhillings	::	8 yards	:	16 ſhillings, Anſwer.

Variation 1. As the third term is to the fourth, ſo is the firſt to the ſecond.

2. As the ſecond term is to the firſt, ſo is the fourth to the third.

3. As the fourth term is to the third, ſo is the ſecond to the firſt.

Examples.

(1.) If 2cwt. 3qrs. 14℔ of ſugar coſt 6l. 14s. 2d. what will 12cwt. 3qrs. coſt?

```
        First number.        2d number.          3d number.
    If 2cwt. 3qr. 14lb.  :  6l. 14s. 2d.  ::  12cwt. 3qr.
         4                   20                   4
       ----                 ----                ----
        11 qr.              134 ſhillings.        51 qr.
        28                   12                   28
       ----                 ----                ----
       322 lb.             1610 pence.           408
                                                102
                                                ----
                                                1428 lb.
                                                 1610
                                                -----
                                                14280
                                               8568
                                              1428
                                             -------- 12)
                                         322)2299080( 7140 pence, fourth
 Anſwer 29l. 15s.                            2254   2|0)59|5 ſhill. (numb.
                                             ----
                                              450   £29 15
                                              322
                                              ----
                                              1288
                                              1288
                                              ----
                                              ....0
```

(2.) If 12cwt. 3qr. of ſugar be bought for 29l. 15s. what will 2cwt. 3qr. 14℔ coſt? *Anſ.* 6l. 14s. 2d.

(3.) If 6l. 14s. 2d. be paid for 2cwt. 3qr. 14℔ of ſugar, what quantity may be bought for 29l. 15s? *Anſ.* 12cwt. 3qr.

(4.) If 29l. 15s. will buy 12cwt. 3qr. of ſugar, what quantity will 6l. 14s. 2d. buy? *Anſ.* 2cwt. 3qr. 14℔.

(5.) If a cwt. of tobacco be worth 9l. 16s. what is the worth of 1℔?

(6.) If 1℔ of butter coſt 5¼d. what will a firkin, or 56℔, coſt?

(7.) Bought 3½ yards of cloth for 2l. 16s. 3d. what muſt I give for 28¾ yards at the ſame rate?

(8.) If I buy 56 yards of cloth for 40 guineas, how many ells Flemiſh can I buy for 1135l. 10s?

(9.) A ſailor entered on board a man of war the 14th of May, 1780, and was diſcharged the 11th of December, 1783, what came his wages to at 1l. 5s. per month, reckoning 28 days to a month?

(10.) How long will a perſon be ſaving £100 if he lays by 1s. 6d. per week?

(11.) Bought 55 yards of holland for 11l. 5s. how many Engliſh ells can I buy for 100 guineas at the ſame rate?

(12.) A factor bought 30 quarters of corn for 76l 17s 6d. and 150 quarters of an inferior kind for 361l 11s 8d. to mix with it; how muſt he ſell the mixture per buſhel to gain £20 by the bargain?

(13.) Bought 27 pieces of cloth, each 34 ells, at 7s. 6d. per ell, what is the value of the whole?

(14.) A creditor agrees to receive of his inſolvent debtor after the rate of 10s. 6d. in the pound for a debt of 475l. 10s. how much will he receive in the whole?

(15.) If 18l. 14s. 9¼d. were paid for the carriage of 53cwt. 2qr. 5℔ what was paid for the carriage of 1℔?

(16.) A bankrupt's effects amount to 1000½ guineas. His debts amount to 2547l. 14s. 9d. what will his creditors receive in the pound?

(17.) The rental of a village is 4714l. 11s. 10d. A tax of 117l. 17s. 3½d. is to be made for the ſupport of the poor:—at what rate per pound muſt the aſſeſſment be made to defray the expences?

(18.) A gentleman pays taxes for 350l. 14s.—The rental of the whole village is 4714l. 11s. 10d. upon which a tax is impoſed amounting to 235l. 14s. 7d. What ſum muſt this gentleman pay towards this tax?

(19.) If a tax of 9d. in the pound be impoſed upon a village for the ſupport of the poor, what ſum muſt a gentleman pay towards it, who pays taxes for 350l. 14s?

(20.) Bought 14 hhds of ſugar, each weighing 7cwt. 1qr. 14lb. at 2l. 14s. 9d. per cwt. what do they come to?

(21.) If a pack of wool weighs 2cwt. 2qr. 14lb. what is it worth at 17s. 6d. per todd?

(22.) Bought 157 fother of lead at 5l. 5s. per cwt. paid carriage, &c. 5 guineas; what does the lead ſtand me in per ℔?

(23.) If an ounce of gold be worth £3, what is the worth of 14 ingots, each weighing 3℔ 11oz. 15dwt. 13gr?

(24.) Bought 76 pieces of ſtuff for £722, at 4s. 9d. per yard; how many yards did I buy, and how many Engliſh ells did each piece contain?

(25.) Bought 4 tuns of oil for 247l. 11s.—64 gallons of which being damaged, how muſt I ſell the remainder per gallon ſo as neither to gain nor loſe by the bargain?

(26.) A factor bought a quantity of broad-cloth and baize for £124; the quantity of broad-cloth he bought was $117\frac{1}{2}$ yards, at 17s. 9d. per yard; for every 5 yards of broad cloth he had $1\frac{1}{2}$ yard of baize:—how many yards of baize did he buy, and what did it coſt him per yard?

CLASS II.

(27.) A merchant in *London* bought 59 tons of port-wine for 12 guineas per hhd; the freight thereof from *Oporto* to *London*, coſt 47l. 10s. the loading and unloading 7l. 10s. cuſtom 24l. charges of the cellar 3 guineas;—what was the prime coſt of a gallon of this wine, and how muſt he ſell it per gallon to gain 150l. by his bargain?

(28.) A draper bought 5 packs of cloth, each pack containing 7 parcels, each parcel 15 pieces, and each piece 15 ells E. 2qr. 3n.—For every 5 yards he bought he gave 4l. 7s. 9d. what did the 5 packs of cloth ſtand him in?

(29.) The globe of the earth under the *equinoctial* line, is 360 degrees in circumference, each degree 69$\frac{1}{2}$ miles: —now, if this body turns on its axis in 23hr. 56m. at what rate per hour are the inhabitants upon the *equator*, or the equinoctial, carried from Weſt to Eaſt by this rotation, and at what rate per hour are the inhabitants of *London* carried the ſame way?—The latitude of *London* is 51$\frac{1}{2}$ N. where a degree of longitude meaſures 37m. 2f. 37p. 5$\frac{1}{2}$ft.

(30.) A tax of 225l. 10s. was laid upon four villages, A, B, C, D, for repairing the church:—it has been a cuſtom with theſe villages, time immemorial, that, whenever any taxes were to be levied, as often as A, B, and C, paid each 3d. D paid only 2d. What did each village pay towards the reparation of the church?

(31.) A man bought 120 eggs at 3 for a penny, and afterwards 120 more at two for a penny, how many muſt he ſell for 5d. that he may loſe nothing?

(32.) Required the *exact* time of the day between the hours of 2 and 3, when the hour and minute hands of a clock are both together, when they make an angle of 90 degrees, or are 15 minutes apart; and at what o'clock will they be *exactly* together a ſecond time?

(33.) A hare, purſued by a greyhound, is 144 of her leaps before him at ſetting off; now the hare makes 4 leaps while the greyhound makes 3, but the greyhound leaps as far at twice as the hare does at thrice:—how many leaps muſt the greyhound take to catch the hare?

(34.) Shipped for *Jamaica* 1750 pair of ſtockings at 4s. 5d. per pair, and 1749 yards of *Mancheſter* cotton at 3s. 7d. per yard, and in return I have received 475 gallons of rum at 6s. 9$\frac{1}{2}$d. per gallon, and 27 hhds of ſugar, each weighing 7cwt. 3qr. 15lb. neat, at 3l. 15s. 7d. per cwt.— What is the balance between us, and in whoſe favour?

(35.) A gentleman's yearly income is £3780, his weekly expences amount to 32l. 15s. land-tax, repairs, &c. amount to $\frac{1}{9}$ of his annual income; the charitable donations which he diſtributes amount to $\frac{1}{20}$ part of the remainder, his pocket expences daily amount to 1$\frac{1}{2}$ guinea,—what are his whole expences in a year, and what does he lay up at the year's end?

(36.) Laid out 571l. 1s. 8d. in wine, at 3s. 7d. per gallon, which having received damage, by reaſon of ſome pipes ſtaving, I found my returns no more than 419l. 11s. by ſelling what came to hand in good order, at 7s. 6d. per gallon; pray what quantity of wine was loſt?

(37.) A merchant bought 22½ cwt. of pepper, and 17¼ cwt. of ginger; the pepper coſt him 14l. 19s. 7d. per cwt. the ginger 12l. 17s. 6d.—What is the whole value of the pepper and ginger, and how muſt it be ſold per oz. that he may gain £90 by each ſort?

(38.) Bought a puncheon of rum for 41l. 14s. 6d. to which I put as much water as reduced the prime coſt to 5s. 6d. per gallon; what quantity of water did I put in?

(39.) If, when port wine is at 17½ guineas per hhd, a company of 47 people will ſpend 21 guineas therein, in a certain time, what is wine a pipe when 15 perſons more will ſpend 65 guineas in twice the time, drinking at the ſame rate?

(40.) A grocer delivered 17cwt. 3qr. 10℔ of tobacco in the roll, to be cut and dried; when it came home it weighed 16cwt. 14lb.—How much was loſt in every lb? and, admitting it coſt 8½d. per lb. in the roll, and 1¼d. per lb. cutting, what does the whole now ſtand him in, and what muſt he ſell it for per lb. to gain 10 guineas by it?

INVERSE PROPORTION.

Definition. *Inverſe*, or reciprocal, Proportion, teaches by three given numbers to find a fourth, in ſuch proportion to the ſecond as the firſt is to the third.

Rule.

State the queſtion as in the direct rule. Multiply the firſt and ſecond terms together, and divide the product by the third, the quotient will be the anſwer, and of the ſame denomination as you left the ſecond number.

Note 1. Direct and inverſe proportion are, properly, only parts of the ſame general rule, and, in a mathematical arrangement, it would be beſt to treat of them together. However, as inverſe proportion is not of ſuch extenſive uſe in mercantile affairs as direct proportion, nor abſolutely neceſſary for every ſcholar to learn, I have, according to cuſtom, conſidered them ſeparately.

2. I ſhall here ſpecify, by familiar examples, the difference between direct and inverſe proportion in as clear and conciſe a manner as poſſible. —Obſerve, when the queſtion is ſtated, that, if the third term be *greater* than the firſt, and require the fourth to be *greater* than the ſecond; or, if the third term be *leſs* than the firſt, and require the fourth to be *leſs* than the ſecond, the proportion is direct. But, if the third term be *greater* than the firſt, and require the fourth to be *leſs* than the ſecond; or, if the third term be *leſs* than the firſt, and require the fourth to be *greater* than the ſecond, the proportion is inverſe.

Ex. 1ſt. If 3 yards of cloth coſt 18 ſhillings, what will 24 yards coſt?
If*3 yards : 18s. :: 24 yards : 144s. or £7 4s.

Here it is evident that 24 yards will coſt more than three yards at the ſame rate; hence the proportion is *direct*; for, the third term is *greater* than the firſt, and requires the fourth to be *greater* than the ſecond.

Ex. 2. If 112lb. of ſugar coſt 56s. what will one lb. coſt?
If*112l. : 56s. :: 1lb. : 6d.

Here 1lb. of ſugar will certainly coſt leſs than 112lb. and conſequently the proportion is *direct*; for, the third term is *leſs* than the firſt, and requires the fourth to be *leſs* than the ſecond.

Ex. 3. If 4 men can do a piece of work in 80 days, how many days, of the ſame length, will 16 men require to do the ſame work?
If 4 men : 80 days :: *16 men : 20 days.

Here it is plain that 16 men will do a piece of work ſooner than four men; hence this proportion is *inverſe*; for, the third term is *greater* than the firſt, and requires the fourth to be *leſs* than the ſecond.

Ex. 4. If 21 pioneers make a trench in 18 days, how many days, of the ſame length, will 7 men require to make a ſimilar trench?
If 21 pioneers : 18 days :: *7 pioneers : 54 days.

Here 7 men will evidently require a longer time than 21 men to dig a trench; hence the proportion is *inverſe*; for, the third term is *leſs* than the firſt, and requires the fourth to be *greater* than the ſecond.

Examples.

(1.) If a field of graſs be mowed by 10 men in 12 days, in how many days would it be mowed by 20 men?

1ſt number.		*2d number.*		*3d number.*
If 10 m.	:	12 d.	::	20 m.
		10		
		2|0)12|0		
		6 days, anſwer.		

Note. Such is the quantity of graſs that 10 men would mow it in 12 days, it is therefore obvious that if 20 men were employed, they would mow it in half the time.

(2.) A cer-

(2.) A certain piece of graſs was to have been mowed by 20 men in 6 days; an extraordinary occaſion calls off half the workmen:—it is required to find in what time the reſt will finiſh it? *Anſwer* 12 *days*.

(3.) If the penny-loaf weighs 8 oz. when flour is at 2s. a peck, what ſhould it weigh when flour is ſold for 2s. 6d. the peck?

(4.) Proviſions in a garriſon are found ſufficient to laſt 1800 ſoldiers for three months; but a reinforcement being wanted, that the proviſions may laſt for one month *only*, what number of ſoldiers may be added to the garriſon on this emergency?

(5.) If 3yds. 2qr. of cloth of 1yd. 3qr. wide will make a ſuit of clothes, how many yards of ſtuff, of ¾ yard wide, will make a ſuit for the ſame perſon, allowing the taylor ¼ yard for *cabbage?*

(6.) If I lend my friend £200 for 12 months, on condition of his returning the favour, how long ought he to lend me £150 to requite my kindneſs?

(7.) If a ſtatute-acre be 220 yards long, the breadth will be 22 yards; but if the breadth of an acre be 40 yards, what will the length be then?

CLASS II.

(8.) If 720 men be placed in a garriſon, and have proviſions for ſix months; but hear of no relief at the end of 5 months, how many men muſt depart that the remaining proviſions may laſt 5 months longer?

(9.) If 5 oxen, *or* 7 colts, eat up a certain quantity of graſs in 87 days, in what time will 2 oxen *and* 3 colts eat up the ſame quantity of graſs?

(10.) A regiment of ſoldiers, conſiſting of 1000, are to be new clothed; each coat to contain 2½ yards of cloth of 1¼ yard wide, and to be lined with ſhalloon of ¾ yard wide; how many yards of ſhalloon will line them?

(11.) A lent his friend B 91 guineas from the 11th of December, 1786, till the 10th of May, 1787; B, on another occaſion, let A have 66l. 13s. 4d. from September 3, 1788, to Chriſtmas, 1789; how long ought the perſon *obliged* to lend his friend £40 to retaliate the favour?

(12.) If a ball of 18lb. be ſhot from a cannon with ſuch a force as to ſend it 100 feet in a ſecond, with what velocity would a ball of 24lb. move, were it impelled by the ſame force?

The RULE of FIVE.

Definition. The Rule of Five is ſo called from its being compoſed of five numbers to find a ſixth: it is ſometimes called the *Double Rule of Three,* becauſe all queſtions that can be anſwered by it may be anſwered by two ſtatings in the Single Rule of Three. It is by ſome writers called compound proportion, eſpecially when the terms are more than five.

Rule I. For five Terms.

Let the principal cauſe of gain, loſs, or action, &c. be put in the firſt place; that number which denotes the ſpace of time, or diſtance of place, &c. be put in the ſecond place; and that number which is the gain, loſs, or action, &c. be put in the third place. That done, place the two terms which move the queſtion underneath thoſe of the ſame name. Then, if the blank, or term ſought, fall under the third term, multiply the firſt and ſecond terms together for a diviſor, and the other three for a dividend; but, if the blank fall under the firſt or ſecond term, multiply the firſt, ſecond, and laſt, terms together for a dividend, and the other two for a diviſor, and the quotient will be the anſwer.

Or, Rule II. For any number of terms.

1 Let that term be put in the ſecond place, which is of the ſame denomination with the term ſought.

2. Place the terms of the ſuppoſition, one above another in the firſt place; and the terms of demand one above another in the third place. Then, the firſt and third term of every row will be of one name, and muſt be reduced to the ſame denomination.

3. Examine every row ſeparately, (uſing the ſecond term in common for each row) by ſaying, If the firſt term give the ſecond, does the third require more or leſs? if more, mark the leſs extreme; if leſs, mark the greater extreme for a diviſor.

4. Multiply all the numbers together which are marked, for a diviſor; and thoſe which are not marked for a dividend, and the quotient will be the anſwer.

5. The work may be contracted by throwing out ſuch numbers as occur both in the diviſor and the dividend; or by dividing any two numbers in the diviſor and dividend by their common meaſure, and uſing the quotients inſtead of the original numbers.

Examples.

(1.) If 7 men can reap 126 acres in 12 days, how many acres will 16 men reap in 3 days?

By rule 1ſt.

```
    1ſt term.   2d term.   3d term.
If   7m.    :    12d.    :   126a.
    16m.    :     3d.    :    *
                 126     laſt term.
                   3
                ----
       7         378
      12          16
      --        ----
      84 )      6048 ( 72a.
                ----
                 168
                ----
```

Anſwer 72 acres.

By two ſtatings.

```
If 7m.*  : 126a. :: 16m.  : 288a.
If 12d.* : 288a. ::  3d.  :  72a.
```

Or thus,

```
If 7m.    : 12d.  :: 16m.* : 5¼d.
If 5¼d.*  : 126a. ::  3d.  :  72a.
```

(2.) If 7 men can reap 126 acres in 12 days, how many men will reap 72 acres in 3 days?

By rule 2d.

```
*If 126 acres : 7 men : 72 acres.
     12 days  : ----- :  3 days.*
                  12
                   7
                ----
       126        84
         3        72
      ----      ----
       378 )    6048 (16 m.
                ----
                2268
                ----
```

Anſwer 16 men.

By two ſtatings.

```
If 12d *   : 126a. ::  3d.  : 31½a.
If 31½a.*  :   7m. :: 72a.  : 16m.
```

Or thus,

```
If 12d.    : 7m.  :: 3d*: 28m.
If 126a.*  : 28m. :: 72a. : 16m.
```

The aſteriſms (*) point out the diviſors in the ſingle ſtatings.

(3.) If 7 men in 12 days can reap 126 acres, in how many days will 16 men reap 72 acres? *Anſw. 3 days.*

(4.) A carrier

(4.) A carrier receives 15l. 12s. for the carriage of 4½ tons 18 miles, how much will he carry 72 miles for 20 guineas?

(5.) If £100 principal gain £4 in 12 months, what principal will gain £20 in 19 months?

(6.) The carriage of 11cwt. 2qr. for 150 miles coſts 6l. 14s. 8d. how much muſt be paid for the carriage of 15cwt. 1qr. 22lb. for 64 miles at the ſame rate?

(7.) If a regiment of 1878 ſoldiers conſume 702 quarters of wheat in 336 days, how many quarters will an army of 22536 ſoldiers conſume in 112 days?

(8.) If £100 at intereſt for 1 year, or 365 days, gain £5, how much will 144l. 14s. 9d. gain in 495 days?

(9.) If 12 tailors in 7 days can finiſh 13 ſuits of clothes, how many tailors, in 19 days of the ſame length, can finiſh the clothes of a regiment of ſoldiers conſiſting of 494 men?

(10.) An ordinary of 100 men drank £20 worth of wine at 2s. 6d. per bottle; how many men, at the ſame rate of drinking, will £7 worth ſuffice, when wine is rated at 1s. 9d. per bottle?

The UNIVERSAL RULE of PROPORTION.

1. Set down the terms expreſſing the conditions of the queſtion in one line, taking care to ſeparate the *cauſe* from the *effect*.

2. Under each conditional term ſet its correſponding one in another line, marking the term ſought, or wanting, with an aſteriſm (*).

3. Draw croſs-lines from the *cauſe* term, or terms, in the firſt part of the firſt line, to the *effect* term, or terms, in the ſecond part of the ſecond line; and, from the *effect* term, or terms, in the ſecond part of the firſt line, to the *cauſe* term, or terms, in the firſt part of the ſecond line.

4. Multiply the term, or terms, at that end of the croſs-line, where the ſtar-term is found, into the term, or terms, at the other end of *that* line for a diviſor; then multiply all the terms together, ſtanding at contrary ends

of the other croſs-line, for a dividend. The quotient will be the anſwer.

Note. When a term is only underſtood, and not expreſſed, the place of that term muſt always be ſupplied by an unit.

Examples.

(1.) If four compoſitors, in 16 days of 12 hours long, can compoſe 14 ſheets, of 24 pages in each ſheet, 44 lines in a page, and 40 letters in a line,—in how many days, of 10 hours long, may 9 compoſitors compoſe a volume, to be printed on the ſame letter, conſiſting of 30 ſheets, 16 pages in a ſheet, 48 lines in a page, and 45 letters in a line?—*Heath*.

	Producing cauſe.	compoſe	*Produced effect.*
1 line.	If 4c. : 16d. : 12h.	⤨	14s. : 24p. : 44l. : 40l.
2 line.	9c. : *d. : 10h.		30s. : 16p. : 48l. : 45l.
		compoſe	

4×16×12×30×16×48×45=796262400 dividend.
9×10×14×24×44×40=53222400 diviſor.

Then 796262400, divided by 53222400, gives $14\frac{511488}{532224}$ days, = $14\frac{74}{77}$ days, anſwer.

(2.) If the carriage of 126lb. for 100 miles coſt 6s. how many pounds may I have carried 750 miles for a guinea?

(3.) If 24 meaſures of wine, at 3s. 4d. each, ſerve 16 men for 6 days, how many meaſures, at 2s. 8d. each, will ſerve 48 men for 4 days?

(4.) If a garriſon of 3600 men, in 35 days, at 24oz. per day each man, eat a certain quantity of bread, how many men, in 45 days, at the rate of 14 oz. per day each man, will eat double the quantity?

(5.) A garriſon of 3600 men has juſt bread enough to allow 24 oz. a day to each man for 35 days; but, a ſiege coming on, the garriſon was reinforced to the number of 4800 men: how many oz. of bread a day muſt each man be allowed, to hold out 45 days againſt the ſiege of the enemy?

(6.) If the carriage of 150 feet of wood, that weighs 3 ſtone a foot, comes to £3 for 40 miles, how much will the carriage of 54 feet of free-ſtone, that weighs 8 ſtone a foot, coſt for 25 miles?

(7.) What is the intereſt of 1627l. 10s. for 219 days, at the rate of 5 per cent. per annum?

(8.) If 3yds. 3qr. of Kerſey coſt 8s. 9d. what will 257yds. 2qr. coſt?

(9.) If, when wine is £30 per tun, £20 worth will ſerve a ſhip's company of 336 men for 4 days, at a pint a day for each man,—how long will £500 worth ſerve a crew of 250 men, at $1\frac{1}{2}$ pint a day to each man, when the tun is worth but £24?

(10.) If 336 men, in 5 days of 10 hours each, dig a trench of 5 degrees of hardneſs, 70 yards long, 3 wide, and 2 deep, what length of trench, of 6 degrees of hardneſs, 5 yards wide, and 3 deep, may be dug by 240 men in 9 days of 12 hours each?

(11.) If 96 men can build a wall in 48 days, how many men can build a ſimilar wall in 384 days?

(12.) If 1 cwt. of cheeſe coſt 1l. 17s. 4d. what is that per lb.?

(13.) If 12 pieces of cannon, eighteen-pounders, can batter down a caſtle in an hour, in what time would 9 twenty-four pounders batter down the ſame caſtle, both pieces of canon being fired the ſame number of times, and their balls flying with the ſame degree of velocity?

(14.) If 12 oxen will eat $3\frac{1}{3}$ acres of graſs in 4 weeks, and 21 oxen will eat 10 acres in 9 weeks, how many oxen will eat 24 acres in 18 weeks, the graſs being allowed to grow uniformly?—*Newton.*

VULGAR FRACTIONS.

DEFINITIONS.

1. *Fractions*, or broken numbers, are expreſſions for any aſſignable part of an unit, and are repreſented by two numbers, placed one above the other with a line drawn between them, as $\frac{3}{4}$, three-fourths of an unit, or one-fourth of three units; $\frac{5}{8}$, five-eighths, or one-eighth of five; $\frac{4}{13}$, four-thirteenths, or one-thirteenth of four, &c.—The lower number is called the *denominator*, and ſhews how many parts the integer is divided into; the upper is called

the *numerator*, and ſhews how many of thoſe parts are to be taken; thus, $\frac{2}{3}$, two-thirds, or one-third of two, ſhews that an unit is to be divided into three equal parts, and that two of theſe parts are to be taken; or that *two* units are to be divided into three equal parts, and one of theſe equal parts are to be taken.—Hence it appears that every fraction denotes a diviſion of its numerator by its denominator, and that its value is equal to the quotient obtained by ſuch diviſion.

2. *A proper fraction* is that wherein the numerator is greater than the denominator, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c.

3. *An improper fraction* is that wherein the numerator is greater than, or equal to the denominator, as $\frac{5}{4}$, or $\frac{5}{5}$. Hence, when the numerator is leſs, greater than, or equal to the denominator; the fraction is leſs, greater than, or equal to an unit.

4. *A ſingle, or ſimple fraction*, conſiſts of but *one* numerator and denominator, as $\frac{5}{8}$.

5. *A compound fraction*, or fraction of a fraction, conſiſts of two, or more, fractions connected by the word *of*, as $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{5}{8}$ of $3\frac{1}{5}$, &c.

6. *A mixed number*, is a whole number with a fraction annexed, as $17\frac{1}{2}$, $14\frac{5}{9}$, &c.

7. *A complex fraction*, is a fraction having a fraction or a mixed number for its numerator or denominator, or both, as $\frac{\frac{3}{4}}{14}$, $\frac{\frac{1}{5}}{6}$, $\frac{2}{47\frac{1}{2}}$, $\frac{47\frac{1}{2}}{96}$, or $\frac{\frac{3}{5}}{\frac{4}{7}}$, $\frac{42\frac{5}{8}}{87\frac{4}{5}}$, &c.

8. *The common meaſure* of a fraction, is a number that will divide both the numerator and denominator without a remainder.

9. *Terms* of a fraction, are the numerator and denominator; the numerator being the upper term, and the denominator the lower.

REDUCTION OF VULGAR FRACTIONS.

Definition. The method of changing fractions from one form to another, without altering their value, is called *reduction:* The rules of reduction ſerve to prepare the fractions for addition, ſubtraction, &c.

Propoſition 1. *To find the greateſt common meaſure of a fraction.*

Rule.

Divide the greater term by the leſs, and this diviſor by the remainder, continually, till there is no remainder; then the laſt diviſor will be the greateſt common meaſure of both terms of the fraction, or of any two numbers whatever.

Prop. 2. *To abbreviate, or reduce, fractions to their loweſt terms.*

Rule.

Divide the terms of the given fraction by any number that will divide them without a remainder, and theſe quotients again in the ſame manner; and ſo on till no number greater than one will divide them. Or, divide both the terms of the fraction by their greateſt common meaſure.

Note 1. Any number, ending with an even number, or a cipher, will divide by 2, and leave no remainder.

2. Any number ending with 5 or 0, is diviſible by 5.

3. If any fraction has ciphers at the right-hand of its terms, it may be abbreviated by cutting off the ciphers, as $\frac{4|0}{5|0}=\frac{4}{5}$.

4. If any number ending with 1, 3, 7, or 9, be the numerator or denominator of a fraction, and will not divide by 3, 7, or 9, that fraction is generally in its loweſt terms. The 9th note in ſimple diviſion will be found uſeful here.

Prop. 3. *To reduce a whole number to an equivalent fraction of a given denominator.*

Rule.

Multiply the whole number by the given denominator, and the product will be the numerator required.

Note. Any whole number may be expreſſed like a fraction by writing 1 under it for a denominator. Thus, $5=\frac{5}{1}$.

Prop. 4. *To reduce a mixed number to its equivalent improper fraction.*

Rule.

Multiply the whole number by the denominator of the fraction, and to the product add the numerator; this ſum, written above the denominator, will form the fraction required.

Prop. 5. *To reduce an improper fraction to its equivalent whole or mixed number.*

Rule.

Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.

Prop. 6. *To reduce a complex fraction to a ſimple one.*

Rule.

If the numerator or denominator be whole or mixed numbers, reduce them to improper fractions. Then multiply the denominator of the lower fraction into the numerator of the upper for a new numerator, and the denominator of the upper fraction into the numerator of the lower for a new denominator.

Thus, by the preceding rules,

$$\frac{4}{\frac{7}{8}}=\frac{\frac{4}{1}}{\frac{7}{8}};\quad \frac{\frac{7}{10}}{5}=\frac{\frac{7}{10}}{\frac{5}{1}};\quad \frac{4\frac{7}{8}}{9}=\frac{\frac{39}{8}}{\frac{9}{1}};\quad \frac{7}{5\frac{1}{9}}=\frac{\frac{7}{1}}{\frac{46}{9}};\quad \frac{7\frac{1}{5}}{4\frac{3}{8}}=\frac{\frac{36}{5}}{\frac{35}{8}};$$

and theſe are all the varieties that can poſſibly happen in preparing the fraction.

Prop. 7. *To reduce a compound fraction to a ſimple one.*

Rule.

If any of the propoſed quantities be integers, mixed numbers, or complex fractions, reduce them to their proper terms. Then multiply all the numerators together for a new numerator, and all the denominators for a new denominator. Reduce this new fraction to its loweſt terms.

Note. If you place the ſeveral numerators in a line, with the ſign of multiplication between them; and the denominators underneath them, in a ſimilar manner: you may ſtrike out ſuch figures as are common to both the numerator and denominator, or divide any two of them by their greateſt common diviſor. For it is an univerſal *axiom* in fractions, THAT if you multiply or divide both the numerator and denominator of a fraction by the ſame number, its value is not altered.

Prop. 8. *To reduce fractions of different denominators to others of equal value, having a common denominator.*

1. *General* Rules.

When any of the propoſed quantities are integers, mixed numbers, complex, or compound, fractions, they muſt be reduced to their proper terms by the preceding rules. Then multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators together for a common denominator. Or,

1. Multiply all the denominators, of the given fraction, together for a common denominator.

2. Divide the common denominator by each of the given denominators ſeparately, and multiply the quotients by their ſeveral numerators, the products will be the new numerators.

By the *axiom* in the note to Prop. 7th, ſeveral fractions of different denominators may eaſily be reduced to a common denominator. Thus $\frac{5}{8}$ may be reduced to the ſame denominator as $\frac{3}{16}$, by multiplying its terms by 2, by which it becomes $\frac{10}{16}$. Alſo $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{16}$, may be reduced to a common denominator by multiplying the terms of the firſt fraction by 4, the ſecond by 2, and dividing thoſe of the third by 2; thus $\frac{1}{2}=\frac{4}{8}$, $\frac{3}{4}=\frac{6}{8}$, & $\frac{2}{16}=\frac{1}{8}$.

2 If the leſs denominator of two fractions divide the greater, multiply the terms of that which has the leſs denominator by the quotient. Let $\frac{5}{8}$ and $\frac{9}{16}$ be propoſed; here $16\div8=2$, and $\frac{5}{8}\times\frac{2}{2}=\frac{10}{16}$. And hence the greater of two given fractions may eaſily be diſcovered; for, if we multiply each numerator into the other's denominator, the products will be equal when the fractions are equal; otherwiſe that fraction is the greater which produces the greater product by its numerator; thus $\frac{5}{6}=\frac{10}{12}$, for $5\times12=10\times6$, but $\frac{5}{6}$ is greater than $\frac{7}{10}$, for $5\times10=50$, but $7\times6=42$ only.

3. When ſeveral fractions are propoſed to be reduced to a common denominator, firſt reduce two of them to a common denominator by ſome of the preceding methods, and then theſe and a third, &c.

4. If any number of ſimple fractions be reduced to a common denominator, the ſeveral numerators of the new fractions will be in the ſame proportion to each other as the original fractions; and, if theſe new numerators be divided by their greateſt common meaſure, the quotients will be the leaſt whole numbers in the ſame proportion. This note will be found exceedingly uſeful in ſolving all fractional queſtions where proportion is concerned.

Prop. 9. *To find the proper quantity, or value, of a fraction in the known parts of an integer.*

Rule.

Rule.

Multiply the numerator by the number of parts of the next inferior denomination, which makes one of the denomination of your fraction, and divide the product by the denominator, the quotient will be the value of the fraction. If there be a remainder, multiply it by the next inferior denomination, and divide by the denominator as before: proceed thus till you come to the loweſt denomination.

Prop. 10. *To reduce coins, weights, meaſures, &c. into fractions.*

Rule.

Reduce the coin, weight, meaſure, &c. into the loweſt name mentioned, for a numerator, under which ſet the number of parts contained in an unit of the integer, to which the propoſed fraction is to be reduced for a denominator. Reduce the fraction to its loweſt terms.

Note. This rule is exactly the reverſe of the laſt.

Prop. 11. *To reduce a fraction of one denomination to the fraction of another denomination of equal value.*

Rule.

From a leſs to a greater denomination. Multiply the *denominator* by all the denominations, from that given to that ſought: and, from a greater to a leſs denomination, multiply the *numerator* by all the denominations, from the denomination given to that ſought.

Examples to Propoſition 1.

(1.) Find the greateſt common meaſure to $\frac{216}{408}$. *Or, in other words, find the greateſt number that will divide* 216 *and* 408 *without a remainder.*

```
                  216 ) 408 ( 1
                        216
                        ---
                        192 ) 216 ( 1
                              192
                              ---
 Common meaſure          24 ) 192 ( 8
                              ---
```

(2.) Find the greateſt common meaſure to $\frac{342}{843}$.
(3.) Find the greateſt common meaſure to $\frac{36}{42}$.
(4.) Find the greateſt common meaſure to $\frac{243}{456}$.
(5.) Find the greateſt common meaſure to $\frac{375}{4500}$.

Examples to Prop. 2.

(6.) Reduce $\frac{216}{408}$ to its loweſt terms.

$\overset{\div 3}{\frac{216}{408}}=\overset{\div 4}{\frac{72}{136}}=\overset{\div 2}{\frac{18}{34}}=\frac{9}{17}$ anſwer.—Or the common meaſure, (by example 1) is 24: hence $24)\frac{216}{408}=\frac{9}{17}$ as before.

(7.) Reduce $\frac{374}{1080}$ to its loweſt terms.
(8.) Reduce $\frac{410}{510}$ to its loweſt terms.
(9.) Reduce $\frac{345}{1745}$ to its loweſt terms.
(10.) Reduce $\frac{8343}{9747}$ to its loweſt terms.
(11.) Reduce $\frac{549}{7143}$ to its loweſt terms.

Examples to Prop. 3.

(12.) Reduce 14 to an improper fraction, having 9 for its denominator.

$14\times9=126$ numerator; hence $14=\frac{126}{9}$ the fraction required.

(13.) Reduce 15 to an improper fraction, having 26 for its denominator.

(14.) Reduce 34 to an improper fraction, having 91 for its denominator.

Examples to Prop. 4.

(15.) Reduce $25\frac{3}{8}$ to its equivalent improper fraction.

$25\frac{3}{8}$
8 denominator of the fraction.
———
203 new numerator. Then $25\frac{3}{8}=\frac{203}{8}$.

(16.) Reduce $149\frac{5}{9}$ to an improper fraction.
(17.) Reduce $375\frac{94}{99}$ to an improper fraction.
(18.) Reduce $17494\frac{543}{99999}$ to an improper fraction.
(19.) Reduce $4734\frac{57}{95}$ to an improper fraction.
(20.) Reduce $1789\frac{5}{9}$ to an improper fraction.
(21.) Place 4 ſevens in ſuch a manner that they may be equal to 78.

Examples

Examples to Prop. 5.

(22.) Reduce $\frac{375}{13}$ to its equivalent whole or mixed number.

Every fraction denotes a diviſion of its numerator by the denominator, therefore 375 divided by $13 = 28\frac{11}{13}$, anſwer.

(23.) Reduce $\frac{4790}{25}$ to a whole or mixed number.
(24.) Reduce $\frac{1512}{108}$ to a whole or mixed number.
(25.) Reduce $\frac{375941}{999}$ to a whole or mixed number.
(26.) Reduce $\frac{3745174}{349}$ to a whole or mixed number.

Examples to Prop. 6.

(27.) Reduce $\frac{47\frac{5}{8}}{94}$ to a ſimple fraction.

$\frac{47\frac{5}{8}}{94} = \frac{\frac{381}{8}}{\frac{94}{1}} = \frac{1 \times 381}{8 \times 94} = \frac{381}{752}$ anſwer. (See the note to Prop. 6.)

(28.) Reduce $\frac{34\frac{5}{7}}{84}$ to a ſimple fraction.

(29.) Reduce $\frac{44}{147\frac{5}{9}}$ to a ſimple fraction.

(30.) Reduce $\frac{247}{\frac{5}{7}}$ to a ſimple fraction.

(31.) Reduce $\frac{\frac{147}{314}}{1789}$ to a ſimple fraction.

(32.) Reduce $\frac{394\frac{74}{99}}{894\frac{5}{7}\frac{47}{19}}$ to a ſimple fraction.

Examples to Prop. 7.

(33.) Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $5\frac{3}{8}$ of $\frac{7\frac{5}{9}}{54}$ of 3 to a ſingle fraction.

Firſt, $5\frac{3}{8} = \frac{43}{8}$; $\frac{7\frac{5}{9}}{54} = \frac{68}{486}$; and $3 = \frac{3}{1}$.

Then $\frac{1\times2\times43\times68\times3}{2\times3\times8\times486\times1}=\frac{17544}{23328}=\frac{731}{972}$ anſwer.

Or, rather,

$$\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{43}{2}\times\frac{\overset{17}{\cancel{68}}}{486}\times\frac{\cancel{3}}{1}=\frac{43\times17}{486\times2}=\frac{731}{972}$$ as before. (*See the note to* Prop. 7.

(34.) Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{5}{9}$ of $\frac{27}{101}$ of $\frac{5}{13}$ to a ſingle fraction.

(35.) Reduce $\frac{41}{110}$ of $\frac{3}{19}$ of $\frac{41}{108}$ of $\frac{3}{7}$ to a ſingle fraction.

(36.) Reduce $3\frac{5}{8}$ of $\frac{5}{9}$ of $\frac{27}{301}$ of 49 to a ſingle fraction.

(37.) Reduce $3\frac{4}{7}$ of $\frac{47}{95}$ of $\frac{34}{15\frac{1}{9}}$ of 108 to a ſingle fraction.

(38.) Reduce $\frac{34}{59}$ of $\frac{1}{15}$ of $\frac{51\frac{3}{8}}{37\frac{3}{5}}$ of 34 to a ſingle fraction.

Examples to Prop. 8.

(39.) Reduce $\frac{3}{4}$, $\frac{5}{7}$, and $\frac{3}{11}$, to a common denominator.

$$\left.\begin{array}{l}3\times7\times11=231\\5\times4\times11=220\\3\times7\times\ 4=\ 84\end{array}\right\}\text{ new numerators.}$$

$$4\times7\times11=308 \text{ common denominator.}$$

Hence $\frac{3}{4}=\frac{231}{308}$; $\frac{5}{7}=\frac{220}{308}$; and $\frac{3}{11}=\frac{84}{308}$.

Otherwiſe, $4\times7\times11=308$, common denominator.

$$\begin{array}{rrr}4)308 & 7)308 & 11)308\\ \hline 77 & 44 & 28\\ 3 & 5 & 3\\ \hline 231\text{ num.} & 220\text{ num.} & 84\text{ num.}\end{array}$$

Hence the new fractions are $\frac{231}{308}$, $\frac{220}{308}$ and $\frac{84}{308}$, as above.

(40.) Reduce $\frac{1}{5}$, $\frac{4}{9}$, $\frac{5}{8}$, and $\frac{11}{16}$, to a common denominator.

(41.) Reduce $\frac{3}{7}$ of $\frac{2}{3}$ of $\frac{5}{8}$ and $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{3}{5}$ to a common denominator *.

---

* In page 26 of the Key, read $\frac{3}{7}\times\frac{2}{3}\times\frac{5}{8}=\frac{5}{28}$.

(42.) Reduce $5\frac{1}{8}$, $3\frac{5}{7}$, $4\frac{1}{9}$, and $6\frac{5}{8}$, to improper fractions, having a common denominator.

(43.) Reduce $\frac{\frac{4}{7}}{15}$, $\frac{34\frac{1}{3}}{94}$, $\frac{64}{89\frac{5}{8}}$, and $\frac{5\frac{1}{3}}{3\frac{7}{8}}$, to ſimple fractions, having a common denominator.

(44.) Reduce $\frac{3}{5}$, $\frac{2}{3}$, $\frac{1}{7}$, and $\frac{1}{2}$, to a common denominator.

(45.) Reduce $\frac{4}{7}$, $\frac{5}{9}$, $\frac{3}{8}$, $\frac{7}{8}$, and 19, to a common denominator.

Examples to Prop. 9.

(46.) Required the value of $\frac{5}{8}$ of a £.; or, which is the ſame thing, $\frac{1}{8}$ of £5.

$$\begin{array}{r}
5 \\
20 \\
\hline
8)100 \\
\hline
\text{s. } 12 - 4 \\
12 \\
\hline
8)48 \\
\hline
6\text{d.}
\end{array}$$

Anſwer 12s. 6d.

(47.) Required the value of $\frac{5}{8}$ of a *cwt.*; or, which is the ſame thing, $\frac{1}{8}$ of 5 *cwt*,

$$\begin{array}{r}
5 \\
4 \\
\hline
8)20 \\
\hline
\text{qrs. } 2 - 4 \\
28 \\
\hline
8)112 \\
\hline
14\text{lb.}
\end{array}$$

Anſw. 2qr. 14lb.

(48.) What is the value of $\frac{5}{9}$ of a ſhilling?

(49.) Reduce $\frac{4}{9}$ of a ℔ avoirdupois to its proper quantity.

(50.) What is the value of $\frac{1}{2}$ of $\frac{6}{5}$ of a ℔ Troy?

(51.) Reduce $\frac{3}{5}$ of a league to its proper quantity.

(52.) Reduce $\frac{5}{8}$ of $\frac{3}{5}$ of an acre to its proper quantity.

(53.) What is the value of $\frac{5}{9}$ of 15 yards of cloth?

(54.) What is the value of $\frac{7}{9}$ of a tun of wine?

(55.) What is the value of $\frac{3}{11}$ of a butt of beer?

(56.) What is the value of $\frac{7}{26}$ of a year?

(57.) What is the value of $\frac{5}{9}$ of a chaldron of coals?

(58.) What is the value of $\frac{2}{5}$ of 13s. 4d.?

(59.) What is the value of $\frac{3}{7}$ of 15cwt. 3qr. 14℔?

(60.) What is the value of $\frac{3}{8}$ of a *ſolid* yard?

(61.) What quantity of ale is contained in $\frac{5}{9}$ of 15228 cubic inches?

Examples

Examples to Prop. 10.

(62) Reduce 7s. $6\frac{3}{4}$d. to the fraction of a pound.

7s. $6\frac{3}{4}$d.	20s.
12	12
90	240
4	4
363 farth. numerator;	960 farth. denominator.

Hence $\frac{363}{960}=\frac{121}{320}$£, the fraction required.

(63.) Reduce 15s. 11d. to the fraction of a pound.

(64.) Reduce $5\frac{1}{9}$d, to the fraction of a ſhilling.

(65.) Reduce 1cwt. 2qr. 6℔ 3oz. $8\frac{8}{9}$dr. to the fraction of a Cwt.

(66.) Reduce 5oz. $3\frac{1}{5}$dr. to the fraction of a ℔ Troy.

(67.) Reduce 3qr. $3\frac{1}{9}$n. to the fraction of an Engliſh ell.

(68.) Reduce 147 days 15 hrs to the fraction of a year.

(69.) What part of a pound is 15s. $9\frac{1}{2}$d.?

(70.) What part of a groat is $\frac{2}{3}$ of three half-pence?

(71.) What part of 10cwt. 1qr. 12℔ is 8cwt. 1qr. 25℔ 1oz. $7\frac{3}{13}$drs.?

(72.) Reduce 4 buſh. $2\frac{2}{7}$ pecks of corn to the fraction of a quarter.

(73.) Reduce 1qr. 3n. to the fraction of a yard.

(74.) Reduce 2 roods 15 per. to the fraction of an acre.

Examples to Prop. 11.

(75.) Reduce $\frac{2880}{7}$ of a farthing to the fraction of a pound.

Here a *ſmall* name is brought into a *great*.

Therefore $\frac{2880}{7}\times\frac{1}{4}\times\frac{1}{12}\times\frac{1}{20}=\frac{2880}{6720}$ of a £ $=\frac{3}{7}$ of a £.

(76.) Reduce $\frac{3}{7}$ of a pound to the fraction of a farthing.

Here a *great* name is to be brought into a *ſmall*.

Hence $\frac{3}{7}\times\frac{20}{1}\times\frac{12}{1}\times\frac{4}{1}=\frac{2880}{7}$ of a farthing.

(77.) Reduce $\frac{5}{7}$ of a penny to the fraction of a pound.

(78.) Reduce $\frac{7}{960}$ of a pound to the fraction of a penny.

(79.) Reduce $\frac{5}{8}$ of a *dwt.* to the fraction of a pound Troy.

(80.) Reduce $\frac{1}{300}$ of a ℔ Troy to the fraction of a *dwt.*

(81.) Reduce $\frac{4}{5}$ of a ℔ Avoirdupois to the fraction of a *cwt.*

(82.) Reduce $\frac{1}{160}$ of a *cwt.* to the fraction of a *lb.*

(83.) Reduce $\frac{7}{11}$ of a week to the fraction of a ſecond.

(84.) Reduce $\frac{11}{13}$ of a gallon of wine to the fraction of a hhd.

ADDITION OF VULGAR FRACTIONS.

Rule.

Reduce mixed numbers to improper fractions; complex and compound fractions to ſimple ones; and fractions of different denominators to a common denominator. Then the ſum of the numerators, written over the common denominator, will be the ſum of the fractions required.

Note 1. If the fractions are of different denominations, reduce them to their proper quantities, (by Prop. 9th in Reduction, or reduce them to the ſame denomination by Prop. 11th) and then add them together.

2. When ſeveral fractions are to be added together, it is commonly the beſt to add thoſe two together which may moſt eaſily be reduced to a common denominator, then their ſum, and a third, &c.

3. When ſeveral mixed numbers, as $4\frac{3}{4}$, &c. are to be collected into one ſum, firſt add the fractions to the fractions, and, to the left hand of the ſum, join the ſum of the whole numbers.

Examples.

(1.) Add $3\frac{5}{7}$, $4\frac{5}{8}$, and $\frac{5}{11}$, together.

Firſt $3\frac{5}{7}=\frac{26}{7}$, $4\frac{5}{8}=\frac{37}{8}$. Then the fractions become $\frac{26}{7}$, $\frac{37}{8}$, and $\frac{5}{11}$.

$$\left.\begin{aligned} 26\times8\times11&=2288\\ 37\times7\times11&=2849\\ 5\times8\times\ 7&=\ \ 280 \end{aligned}\right\}\text{ numerators.}$$

$$\frac{5417}{7\times8\times11=616}=8\tfrac{489}{616}\text{ anſwer.}$$

Or thus,

The ſum of $\frac{5}{7}$, $\frac{5}{8}$, and $\frac{5}{11}$, when reduced to a common denominator, is $\frac{1105}{616}=1\frac{489}{616}$. Then $3+4+1\frac{489}{616}=8\frac{489}{616}$, as before. (See the third note.)

(2.) Add $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{3}{7}$, together.

(3.) Add $\frac{1}{5}$ of $\frac{3}{11}$ and $5\frac{3}{8}$ together.

(4.) Add $\frac{1}{9}$, $7\frac{5}{8}$, $\dfrac{45}{94\frac{7}{11}}$, and $\dfrac{47\frac{5}{9}}{314\frac{3}{5}}$, together.

(5.) Add $\frac{1}{9}$ of $\frac{3}{5}$, $\frac{3}{4}$ of 19, and $\frac{5}{8}$ of 12, together.

(6.) Add $\frac{3}{5}$ and $\frac{9}{10}$ of $\frac{5}{11}$ of $15\frac{3}{8}$, together.

(7.) Add $\frac{5}{9}$ of a pound, $\frac{3}{8}$ of a ſhilling, and $\frac{5}{7}$ of a penny, together.

(8.) What is the ſum of $\frac{3}{4}$ of 1l. 10s. $\frac{1}{4}$ of 3l. 10s. and $\frac{1}{50}$ of a hundred guineas?

(9.) Add $\frac{1}{5}$ of a lb. troy to $\frac{1}{8}$ of an ounce.

(10.) Add $\frac{4}{9}$ of a ton to $\frac{5}{12}$ of a cwt.

(11.) Add $\frac{5}{6}$ of 3 ells Engliſh to $\frac{5}{12}$ of a yard.

(12.) Add $\frac{2}{5}$ of a yard, $\frac{3}{7}$ of a foot, and $\frac{5}{11}$ of a mile, together.

(13.) Add $\frac{5}{8}$ of an acre, $\frac{3}{5}$ of nineteen ſquare feet, and $\frac{3}{7}$ of a ſquare inch, together.

(14.) What is the ſum of $\frac{1}{2}$ of a tun of wine and $\frac{3}{5}$ of a hhd?

(15.) Add $\frac{5}{9}$ of a chaldron to $\frac{3}{7}$ of a buſhel.

(16.) Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, and $\frac{1}{5}$ of an hour, together.

(17.) Add $\frac{1}{5}$ of $\frac{3}{4}$ of a year, $\frac{3}{8}$ of $\frac{5}{9}$ of a day, and $\frac{7}{8}$ of $\frac{3}{3}$ of $19\frac{1}{2}$ hours, together.

SUBTRACTION OF VULGAR FRACTIONS.

Rule.

Reduce mixed numbers to improper fractions; complex and compound fractions to ſimple ones; and fractions of different denominators to a common denominator. Then the difference of the numerators, written above the common denominator, will give the difference of the fractions required.

Note 1. If the fractions are of different denominations, reduce them to their proper quantities, &c. as in addition, and then take their difference.

2. In ſubtracting mixed numbers, when the lower fraction is greater than the upper, ſubtract the numerator of the lower fraction from the denominator of the upper, and to their difference add the numerator of the upper fraction, carrying one to the unit's place of the lower whole number.

3. When a fraction is to be ſubtracted from an unit, ſubtract the numerator from the denominator; the remainder will be the numerator to be placed over the denominator.

4. When a proper fraction is to be ſubtracted from any whole number, ſubtract the numerator from the denominator for the numerator of the *remainder*, which muſt be annexed to the whole number, made leſs by 1.

Examples.

(1.) From $\frac{3}{4}$ take $\frac{5}{11}$.

$$\left.\begin{array}{r} 3\times 11=33 \\ 5\times 4=20 \end{array}\right\}\text{numerators.}$$

$$\begin{array}{r} \overline{13} \\ 4\times 11=\overline{44} \end{array}\text{difference.} \qquad \text{Anſw. } \frac{13}{44}.$$

(2.) What is the difference between $\frac{3}{5}$ and $\frac{9}{16}$?

(3.) What is the difference between $3\frac{5}{8}$ and $\frac{2}{3}$ of $\frac{5}{8}$?

(4.) What is the difference between $\frac{49\frac{5}{8}}{97}$ and $\frac{34\frac{3}{5}}{145\frac{3}{11}}$?

(5.) From $115\frac{5}{8}$ take $39\frac{7}{8}$.

(6.) Subtract $\frac{57}{713}$ from an unit.

(7.) Subtract $\frac{11}{13}$ from 365.

(8.) What is the difference between $\frac{2}{3}$ of 15 and $\frac{4}{5}$ of 72?

(9.) To what fraction muſt I add $\frac{3}{5}$ that the ſum may be $\frac{5}{8}$?

(10.) What number is that to which if $7\frac{2}{3}$ be added the ſum will be $17\frac{3}{5}$?

(11.) What number is that from which if you ſubtract $\frac{1}{11}$ of $\frac{5}{9}$ of an unit, and to the remainder add $\frac{3}{5}$ of $\frac{7}{8}$ of an unit, the ſum will be 9?

(12.) What is the difference between $\frac{1}{4}$ of a £ and $\frac{5}{7}$ of a ſhilling?

(13.) From $\frac{5}{6}$ of a lb. Troy take $\frac{5}{8}$ of an ounce.

(14.) From $\frac{3}{8}$ of a ton take $\frac{2}{7}$ of $\frac{3}{4}$ of a lb.

(15.) From $\frac{2}{3}$ of $\frac{3}{5}$ of a hhd of wine take $\frac{3}{5}$ of $\frac{1}{2}$ of a pint.

(16.) From $\frac{3}{5}$ of a league take $\frac{5}{8}$ of a mile.

(17.) From $\frac{5}{9}$ of $365\frac{1}{4}$ days take $\frac{3}{7}$ of $\frac{9}{16}$ of an hour.

MULTIPLICATION OF VULGAR FRACTIONS.

Rule.

Reduce mixed numbers to improper fractions, and complex fractions to ſimple ones. Then multiply all the numerators together for a new numerator, and all the denominators together for a common denominator.

Note 1. The work may be abbreviated by ſtriking out ſuch multipliers as are found both in the numerators and denominators.

2. To multiply a fraction by an integer, divide the denominator of the fraction by the integer, (if poſſible;) but, if that cannot be done, multiply the numerator of the fraction by it.

3. If a proper fraction be multiplied by a proper fraction, the product will be leſs than either the multiplier, or multiplicand. And, if any number, either whole or mixed, be multiplied by a proper fraction, the product will always be leſs than the multiplicand. It ſeems rather paradoxical that the name *multiplication* ſhould be applied to a work which really diminiſhes; when the word, ſtrictly ſpeaking, ſignifies the increaſing of a number by repetition. But, when we conſider, that, whether the multiplier be a whole number or a fraction, the product will bear the ſame proportion to the multiplicand as the multiplier does to an unit; and, from the ſimilarity of fractions to whole numbers in this effect, the taking as much of the multiplicand as is expreſſed by the multiplier is, perhaps, not improperly called multiplication.

Examples.

(1.) Multiply $3\frac{5}{8}$, $\frac{3\frac{7}{8}}{11}$, and $\frac{3}{5}$ of $\frac{9}{10}$, together.

First $3\frac{5}{8}=\frac{29}{8}$, $\frac{3\frac{7}{8}}{11}=\frac{31}{88}$

Then $\frac{29}{8} \times \frac{31}{88} \times \frac{3}{5} \times \frac{9}{10} = \frac{24273}{35200}$, product.

(2.) Required the product of $\frac{3}{7}$ and $\frac{11}{15}$.

(3.) What is the product of 574 by $\frac{37}{84}$?

(4.) Required the product of $\frac{375}{726}$ by 27.

(5.) Required the product of $7\frac{2}{3}$ by 25.

(6.) What is the product of $\frac{5}{8}$ of $\frac{3}{7}$, $\frac{2}{3}$ of $15\frac{1}{2}$, and $\frac{3}{11}$ of 2?

(7.) What is the continual product of $\frac{4}{5\frac{1}{3}}$, $\frac{7\frac{1}{5}}{15}$, and $\frac{35\frac{3}{8}}{129\frac{4}{15}}$?

(8.) What is the product of $\frac{3}{5}$ of $\frac{7}{11}$ of 15, and $\frac{14}{15}$ of $11\frac{5}{8}$?

(9.) Multiply 7ft. 9in. by 3ft. 11in. and that product by 5ft. 3in.

(10.) If a board be 12ft. 9in. long and 5ft. 7in. broad, how many ſquare feet does it contain?

(11.) If a room be 17ft. $9\frac{1}{3}$in. round, and 9ft. 9in. high, how many ſquare feet does it contain?

DIVISION OF VULGAR FRACTIONS.

Rule.

Reduce mixed numbers to improper fractions, complex and compound fractions to ſimple ones. Then invert the diviſor, and proceed exactly as in multiplication.

Note 1. When it can be done, divide the numerator of the dividend by the numerator of the diviſor, and the denominator by the denominator for the quotient.

2. To divide a fraction by an integer, divide the numerator of the fraction by the integer, if poſſible; but, if that cannot be done, multiply the denominator of the fraction by it.

3. If the denominators are equal, place the numerator of the dividend over the numerator of the diviſor for the quotient.

4. If a proper fraction be divided by a proper fraction, the quotient will be greater than either the diviſor or dividend. And, if any whole, or mixed, number be divided by a proper fraction, the quotient will be greater than the dividend; but, if a proper fraction be divided by a whole, or mixed, number, the quotient will be leſs than the dividend. *See the third note in multiplication.*

5. If any whole number, greater than 2, be divided by itſelf, leſs 1, the quotient will be a mixed fraction; and, if this mixed fraction be added to and multiplied by the whole number, the ſum and product will be equal

Examples.

(1.) Divide $\frac{3}{8}$ of $5\frac{1}{3}$ by $\frac{54}{119\frac{3}{7}}$.

Firſt $\frac{3}{8}$ of $5\frac{1}{3}=\frac{3}{8}$ of $\frac{16}{3}=\frac{2}{1}$ dividend, and $\frac{54}{119\frac{3}{7}}=\frac{378}{836}=\frac{189}{418}$ diviſor.

Then $\frac{418}{189}\times\frac{2}{1}=\frac{836}{189}=4\frac{80}{189}$ anſwer.

(2.) Divide $\frac{16}{19}$ by $\frac{3}{7}$.
(3.) Divide $\frac{18}{97}$ by 6.
(4.) Divide $\frac{25}{37}$ by 7.
(5.) Divide $\frac{36}{75}$ by $\frac{9}{25}$.
(6.) Divide $\frac{2}{11}$ by $\frac{7}{11}$.
(7.) Divide $\frac{3}{7}$ of $\frac{5}{9}$ by $\frac{3}{8}$ of $\frac{15}{16}$.
(8.) Divide $15\frac{5}{9}$ by $\frac{7}{8}$ of $\frac{3}{11}$.
(9.) Divide $34\frac{1}{7}$ by $\frac{54\frac{3}{8}}{93\frac{5}{11}}$.
(10.) Divide $\frac{51\frac{1}{11}}{95}$ by $\frac{71}{149\frac{3}{8}}$.
(11.) Divide $\frac{7}{8}$ of $\frac{3}{5}$ of $5\frac{1}{9}$ by $\frac{1}{2}$ of $\frac{1}{5}$ of 19.
(12.) What number, multiplied by $\frac{3}{4}$, will give $15\frac{3}{4}$ for the product?
(13.) What part of 108 is $\frac{5}{12}$ of an unit?
(14.) What number is that which, if multiplied by $\frac{5}{8}$ of $\frac{3}{7}$ of $15\frac{1}{3}$, will produce only $\frac{5}{8}$ of an unit?

THE RULE OF THREE DIRECT IN VULGAR FRACTIONS.

Rule.

State the queſtion as in the Rule of Three in whole numbers. Reduce mixed numbers to improper fractions, complex and compound fractions to ſimple ones, and the firſt and third terms to the ſame denomination. Then invert the firſt term of the ſtating, and multiply the three terms together, and the product will be the anſwer.

Examples.

(1.) If $\frac{5}{8}$ of a yard coſt $\frac{2}{5}$ of a £. what will $\frac{7}{11}$ of an ell Engliſh coſt?

First $\frac{5}{8}$ of a yard $=\frac{5}{8}$ of $\frac{4}{5}=\frac{1}{2}$ of an ell.

Then, if $\frac{1}{2}$ ell : $\frac{2}{5}$£. :: $\frac{7}{11}$ ell.

$\frac{2}{1}\times\frac{2}{5}\times\frac{7}{11}=\frac{28}{55}$£.$=$10s. $2\frac{2}{11}$d. anſw.

(2.) If $\frac{7}{11}$ of an Engliſh ell coſt 10s. $2\frac{2}{11}$d. what will $\frac{5}{8}$ of a yard coſt? *Anſw.* 8s.

(3.) If $\frac{3}{5}$ of a lb. coſt 7s. 9d. what will $54\frac{5}{8}$ lb. coſt?

(4.) If $\frac{3}{11}$ of $\frac{5}{8}$ of 15 ells of holland coſt $2\frac{1}{11}$l. what will $\frac{3}{5}$ of 175 yards coſt at that rate?

(5.) Bought $5\frac{1}{2}$ pieces of ſilk, each containing $35\frac{9}{11}$ ells Engliſh, at 5s. $3\frac{3}{8}$d. per ell, what is the value of the whole quantity?

(6.) Bought $14\frac{5}{11}$ tuns of wine at 3s. $3\frac{5}{8}$d. per quart, how much did I pay for the whole?

(7.) If $\frac{3}{5}$ of $\frac{7}{9}$ of a yard of cloth coſt $\frac{2}{3}$ of $\frac{5}{8}$ of a £. what will 179 Engliſh ells coſt?

(8.) At $7\frac{3}{4}$d. per lb. what will 11 hhds of ſugar amount to, each hhd weighing 4cwt. 3qr. $15\frac{5}{8}$lb.?

THE RULE OF THREE INVERSE IN VULGAR FRACTIONS.

Rule.

State the queſtion as in whole numbers. Reduce mixed numbers to improper fractions, complex and compound fractions to ſimple ones, and the firſt and third terms to the ſame denomination. Then invert the third term of the ſtating, and multiply the three terms together.

Examples.

(1.) If $24\frac{2}{5}$ ſhillings will pay for the carriage of a *cwt.* $137\frac{3}{8}$ miles, how far may $5\frac{3}{8}$ cwt. be carried for the ſame money?

First $137\frac{3}{8}$m.$=\frac{1099}{8}$m. and $5\frac{3}{8}$cwt.$=\frac{43}{8}$cwt.

Then, if $\frac{1}{1}$ cwt. : $\frac{1099}{8}$m. :: $\frac{43}{8}$cwt.

$\frac{1}{1}\times\frac{1099}{8}\times\frac{8}{43}=\frac{1099}{43}$m.$=25\frac{24}{43}$m anſw.

(2.) How many yards of matting, $\frac{3}{4}$ of a foot wide, will be ſufficient to cover a floor that is $15\frac{1}{2}$ feet broad and $27\frac{3}{4}$ feet long?

(3.) How many yards of cloth, at 5s. 8d. per yard, may I give for $57\frac{3}{8}$ yards of cloth at 4s. 3d. per yard, that I may loſe nothing?

(4.) What quantity of ſhalloon, $\frac{3}{8}$ of a yard wide, will line $11\frac{3}{4}$ yards of cloth $1\frac{1}{4}$ yard wide?

(5.) If I have $3\frac{3}{4}$ cwt. carried $15\frac{1}{2}$ miles for 4 guineas, how far ought $9\frac{1}{2}$ cwt. to be carried for the ſame money?

THE RULE OF FIVE IN VULGAR FRACTIONS.

Rule.

State the queſtion as in whole numbers. Reduce mixed numbers to improper fractions, complex and compound fractions to ſimple ones, and the terms in the ſecond line to the ſame denomination as thoſe above them. Then invert the terms, which are to be multiplied together for a diviſor, and take the continual product of all the terms for the anſwer.

Note. This rule will be general, whether you make uſe of the rule of Five, or the univerſal rule of Proportion, for ſtating the queſtion.

Examples.

(1.) If £$3\frac{1}{2}$ be the wages of 13 men for $7\frac{1}{2}$ days, what will be the wages of 20 men for $15\frac{1}{3}$ days?

First $3\frac{1}{2}£ = \frac{7}{2}£$, $7\frac{1}{2}\text{d.} = \frac{15}{2}\text{d.}$ and $15\frac{1}{3} = \frac{46}{3}\text{d.}$

$$\text{Then, if } \begin{array}{l} \frac{13}{1}\text{m.} : \frac{15}{2}\text{d.} \\ \frac{20}{1}\text{m.} : \frac{40}{3}\text{d.} \end{array} \overset{\text{earn}}{\underset{\text{earn}}{\bowtie}} \begin{array}{l} \frac{7}{2}£. \\ * \end{array}$$

$$\frac{1}{13} \times \frac{2}{15} \times \frac{20}{1} \times \frac{46}{3} \times \frac{7}{2}£ = \frac{1288}{117}£ = 11\frac{1}{117}£.$$

(2.) What is the intereſt of 490l. 15s. for $7\frac{3}{4}$ years, at $4\frac{1}{2}$ per cent. per annum?

(3.) If a footman travel 294 miles in $7\frac{2}{3}$ days of $12\frac{1}{2}$ hours long, in how many days, of $10\frac{2}{3}$ hours each, will he travel $147\frac{1}{8}$ miles?

(4.) Bought 5000 deals, of 15 feet long and $2\frac{1}{2}$ inches thick, how many deals are they equivalent to of $12\frac{1}{2}$ feet long and $1\frac{1}{4}$ inch thick?

(5.) If $13\frac{1}{2}$ ells of cloth, $\frac{3}{5}$ yard wide, coſt $5\frac{1}{2}$ guineas, what will $33\frac{1}{4}$ yards, $\frac{2}{3}$ of an ell Engliſh wide, and of the ſame goodneſs, come to?

(6.) *If 248 men, in $5\frac{1}{2}$ days of 11 hours each, dig a trench of 7 degrees of hardneſs, $232\frac{1}{2}$ yards long, $3\frac{2}{3}$ wide, and $2\frac{1}{3}$ deep; in how many days, of nine hours long, will 24 men dig a trench of 4 degrees of hardneſs, $337\frac{1}{2}$ yards long, $5\frac{3}{5}$ wide, and $3\frac{1}{2}$ deep?

A PROMISCUOUS COLLECTION OF QUESTIONS, EXERCISING ALL THE PRECEDING RULES IN VULGAR FRACTIONS.

(1.) What part of 3d. is $\frac{4}{9}$ of 6d.?

(2.) A gentleman bought 3 ſuits of clothes, containing $7\frac{2}{3}$ yards each; the firſt ſuit coſt 17s. per yard, the ſecond $\frac{5}{6}$ of 17s. and the third $\frac{3}{4}$ of 17s. what did the whole coſt him?

(3.) What number is that from which if $14\frac{5}{8}$ be deducted, the remainder will be $47\frac{9}{16}$?

(4.) If $\frac{3}{8}$ of a ſhip be worth 4000 guineas, what is the whole worth?

(5.) Suppoſe I have $\frac{5}{16}$ of a ſhip, worth £16000, what *part* of her ſhall I have left if I diſpoſe of $\frac{3}{8}$ of $\frac{5}{11}$ of $\frac{1}{2}$ my ſhare; and what money is that *part* worth?

(6.) What number is that from which if you deduct $\frac{5}{8}$, of $\frac{3}{7}$, and to the remainder add $\frac{7}{12}$ of $\frac{3}{19}$, the ſum will be 45?

(7.) Suppoſe A can do a piece of work in $6\frac{1}{2}$ days, B can do the ſame in $4\frac{1}{4}$ days, and C in $3\frac{1}{3}$ days; if you ſet them all at work together, in what time will they finiſh it?

---

* This queſtion is taken from Mr. *Emerſon*'s Arithmetic, where a wrong method of ſolution may be ſeen; but *that ſolution* is corrected (and ſeveral other miſtakes in his works) in an appendix annexed to his Surveying.— This queſtion, and its *falſe* ſolution, Mr. *Taylor* has copied into his Arithmetic, publiſhed at *Birmingham*, 1783.

(8.) I have

(8.) I have employed 5 people, A, B, C, D, and E, upon a piece of work. Now I am told that A, B, C, and D, can finiſh it in 13 days; A, B, C, and E, in 15 days; A, B, D, and E, in 12 days; A, C, D, and E, in 19 days; and B, C, D, and E, in 14 days: pray in what time may I reaſonably expect to have my work done by their all working together; and, ſuppoſe I ſhould wiſh to diſcharge 4 of them, which of them would finiſh the work ſooneſt when left to himſelf?

(9.) A *reſervoir* has three cocks, A, B, and C, to let in water, and three others, D, E, and F, to diſcharge it:—now, if A be opened by *itſelf*, the reſervoir, *when empty*, will be filled in 6 hours; if B be opened by *itſelf*, it will be filled in 8 hours; and, if C be opened by *itſelf*, it will be filled in 10 hours. Again, if D be opened by *itſelf*, when the *reſervoir is full*, it will be emptied in 9 hours; if E be opened by *itſelf*, it will be emptied in 11 hours; and, if F be opened by *itſelf*, it will empty the reſervoir in 13 hours:—in what time will the empty reſervoir be *filled* if all the cocks, A, B, C, D, E, F, are ſet open together, ſuppoſing the weight of the column of water in the reſervoir, and the preſſure of the atmoſphere to be uniform during the influx and efflux of the water?

(10.) What is the difference between $\frac{3}{8}$ of $\frac{5}{6}$ of a crown, and $\frac{3}{5}$ of $\frac{9}{10}$ of a guinea?

(11.) Multiply $\frac{1}{2}$ of $\frac{3}{5}$ of $5\frac{3}{8}$, $\frac{17\frac{1}{5}}{94}$, $\frac{14}{95\frac{3}{8}}$, and $\frac{5}{8}$ of 17, together, for the numerator of a fraction; and $\frac{14\frac{5}{6}}{17\frac{3}{5}}$, $\frac{4}{\frac{7}{9}}$, $\frac{\frac{7}{11}}{15\frac{1}{2}}$, and $51\frac{5}{8}$, together for a denominator, and reduce the new fraction to its proper terms.

(12.) Five boys, A, B, C, D, and E, put a number of marbles into a ring in order to play; but, a diſpute happening amongſt them, A ſnatched $\frac{2}{5}$ of the marbles out of the ring; B ſnatched $\frac{3}{8}$ of *thoſe* out of his hand before he got off, and C, who was near, got $\frac{3}{5}$ of the remainder; D ran off with all A had left in the ring, except $\frac{1}{12}$ part, which E got.—A and C, not ſatisfied with what they got, jointly ſet upon D, and ſnatched $\frac{7}{12}$ of what he had got from him, of which number B, in the ſcuffle, got $\frac{1}{5}$, and

E the reſt; C ſnatched from E $\frac{1}{3}$ of the number he had then in hand, and A got $\frac{1}{11}$ of what B had left. Here D obſerved that he had got juſt as many marbles as he put into the ring; and, if E would but give A $\frac{1}{11}$ of what *he* had got, he would try to prevail upon C to give him $\frac{3}{13}$ of what *he then had*, and then they would *all* have equal ſhares. Pray how many marbles were firſt put into the ring, ſuppoſing each boy put in an equal number, and none were loſt in the ſcuffle?

(13.) A father had two ſons; to the elder he left $\frac{35}{79}$ of his eſtate, and $\frac{35}{79}$ of the remainder to the younger ſon; the reſidue was allotted to the widow: now if the elder ſon had £500 more than the younger, pray what was left for the widow, and what was the gentleman's whole eſtate worth?

(14.) If a wall of $57\frac{3}{8}$ yards long, $12\frac{7}{11}$ feet high, and $1\frac{1}{2}$ brick thick, coſt 342l. 15s. building, what will a wall of $34\frac{5}{8}$ yards long, $11\frac{1}{4}$ feet high, and $2\frac{1}{2}$ bricks thick, coſt at the ſame rate per rod?

DECIMAL FRACTIONS.

Definition 1. *Decimal Fractions* are ſuch as have 10, 100, 1000, &c. for their denominator; thus, $\frac{1}{10}$, $\frac{25}{100}$, $\frac{225}{1000}$, &c. are decimal fractions; and theſe are expreſſed by writing the numerator only, with a point before it on the left hand; thus, ·1, ·25, ·225, &c.

2. *When the numerator* of a decimal fraction is written without its denominator, it muſt always conſiſt of as many figures as there are ciphers in the denominator, thus, $\frac{5}{10}$, $= \cdot 5$, $\frac{5}{100}$, $= \cdot 05$, $\frac{5}{1000} = \cdot 005$, &c. Hence the denominator of a decimal fraction is an unit with as many ciphers as there are figures in the decimal.

3. *Ciphers on the right hand* of decimals make no alteration in their value, thus, ·5, ·500, ·5000, &c. are decimals of the ſame value, for $\frac{5}{10}$, $= \frac{500}{1000} = \frac{5000}{10000} = \frac{1}{2}$ by the nature of vulgar fractions.

4. *Ciphers on the left hand* of decimals decreaſe their value; thus, ·5, ·005, ·0005, &c.$=\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$, &c.

Note 1. Decimals, as well as whole numbers, *decreaſe* in a ten-fold proportion towards the right hand; therefore, decimals have the ſame properties as whole numbers, and are ſubject to the ſame rules.

5. *A mixed number* is compoſed of a whole number and a decimal, which are ſeparated from each other by a point, thus, 115·5 ſignifies $115\frac{5}{10}$.

2. A mixed number, as 115·5, may be expreſſed thus, $\frac{1155}{10}$: alſo

$$115{\cdot}005=\frac{115{\cdot}005}{1}=\frac{1150{\cdot}05}{10}=\frac{11500{\cdot}5}{100}=\frac{115005}{1000}, \text{\&c.}$$

ADDITION OF DECIMALS.

Rule.

Place all the decimal points directly under each other, ſo that tenths may ſtand under tenths, and hundredth parts under hundredth parts, &c. in the decimals; and tens under tens, hundreds under hundreds, &c. in the whole numbers. Then add them together as in whole numbers, and from the right hand of the ſum point off as many figures, for decimals, as are equal to the greateſt number of decimals in any of the given numbers.

Examples.

(1.) Add 5·74+3·75+94·375+·745+·005495 together.

$$\begin{array}{r@{}l}
5 & {\cdot}74 \\
3 & {\cdot}75 \\
94 & {\cdot}375 \\
 & {\cdot}745 \\
 & {\cdot}005495 \\
\hline
104 & {\cdot}615495 \text{ ſum.} \\
\hline
\end{array}$$

(2.) Add 5·714+3·456+·543+17·4957 together.
(3.) Add 3·754+47·5+·00857+37·5 together.
(4.) Add 54·34+·375+14·795+1·5 together.
(5.) Add 71·25+1·749+1759·5+3·1 together.
(6.) Add 375·94+5·732+14·375+1·5 together.
(7.) Add ·005+·0057+31·008+·00594 together.

SUBTRACTION

SUBTRACTION OF DECIMALS.

Rule.

Place the leſs number under the greater, the points under the points, tenths under tenths, hundredth parts under hundredth parts, &c. in the decimals; and the whole numbers under thoſe of the ſame denomination. Then ſubtract as in whole numbers, placing the ſeparating point, in the remainder, directly under thoſe above it.

Examples.

(1.) From 57·439 take 5·93754.

57·439
5·93754

51·50146 difference.

(2.) Required the difference between 57·49 and 5·768.

(3.) What is the difference between ·3054 and 3·075?

(4.) Required the difference between 1745·3 and 173·45.

(5.) What is the difference between ſeven-tenths of an unit and the 54 ten thouſandth part of an unit?

(6.) What is the difference between ·105 and 1·00075?

(7.) What is the difference between 150·43 and 754·355?

(8.) From 1754·754 take 375·49478.

(9.) Take 75·304 from 175·01.

(10.) Required the difference between 17·541 and 35·49.

MULTIPLICATION OF DECIMALS.

Rule.

Multiply the decimals as if they were whole numbers, and from the product cut off ſo many decimal places as there are both in the multiplier and multiplicand. If there are not ſo many places in the product, ſupply the defect by prefixing ciphers to the left hand.

Note 1. When any decimal is to be multiplied by 10, 100, 1000, &c. remove the ſeparating point ſo many places to the right hand as there are ciphers; thus, ·543×10=5·43; alſo ·7156×1000=715·6, &c.

2. What was obſerved, in the third note, in multiplication of vulgar fractions, reſpecting a proper fraction, or mixed number, is equally applicable to a pure, or mixed, decimal.

Contracted Multiplication of Decimals.

Rule.

Put the unit's place of the multiplier under that place of the multiplicand which you intend to keep in the product, and *invert* the order of all the other figures, that is, write the decimals on the left hand, and the integers, if any, on the right. In multiplying, always begin with that figure of the multiplicand which ſtands directly over the multiplying digit, and ſet the firſt figure in every product in a right line under each other to the right hand, obſerving to increaſe the firſt figure of every line with what would ariſe, by carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. from the product of the two figures (in the multiplicand) on the right hand of the multiplying digit.

Examples.

Ex. 1. Multiply 4·735	Ex. 2. Multiply ·004735
By ·374	By ·0374
18940	18940
33145	33145
14205	14205
1·770890 prod.	·0001770890 prod.

(3.) Mult. 473·54 by ·057.
(4.) Mult. 137·549 by 75·437.
(5.) Mult. 3·7495 by ·73487.
(6.) Mult. ·04375 by ·47134.
(7.) Mult. ·371343 by ·75493.
(8.) Mult. 49·0754 by 3·5714.
(9.) Mult. ·573005 by ·000754.
(10.) Mult. ·375494 by 574·375.

Examples under the contracted rule.

(1.) Multiply 2·38645 by 8·2175, and let there be only four places of decimals retained in the product.

Contracted way.	*Common way.*	
2·38645	2·38645	
5712·8	8·2175	
190916	11	93225
4773	167	0515
239	238	645
167	4772	90
12	190916	0
19·6107	19·6106	52875

(2.) Let 54·7494367 be multiplied by 4·724753, reſerving only five places of decimals in the product.

(3.) Multiply 475·710564 by ·3416494, retaining three decimals in the product.

(4.) Multiply 3754·4078 by ·734576, retaining five decimals in the product.

(5.) Let 4745·679 be multiplied by 751·4549, and reſerve only the integers in the product.

DIVISION OF DECIMALS.

Rule.

Divide as in whole numbers, and from the right hand of the quotient point off ſo many figures for decimals as the decimal places in the dividend exceed thoſe in the diviſor; but, if the quotient does not contain ſuch a number of figures as is equal to the exceſs, the defect muſt be ſupplied with ciphers to the left hand. If the number of decimal places in the diviſor ſhould be more than thoſe in the dividend, annex ſo many ciphers to the dividend as will make them equal, and the quotient will be integers till all theſe ciphers are uſed; after which, you may continue the quotient to any aſſigned degree of exactneſs, by ſubjoining a cipher continually to the laſt remainder.

Note 1. To divide by 10, 100, 1000, &c. remove the ſeparating point ſo many places to the left hand as there are ciphers; thus, ·543÷10=·0543, and 715·6÷1000=·7156, &c.

2. What was obſerved in the 4th note in Diviſion of Vulgar Fractions, reſpecting a proper fraction, or mixed number, is equally applicable to a pure, or mixed, decimal; alſo the 5th note holds equally true for a whole number, and a mixed decimal.

Contracted Diviſion of Decimals.

Rule.

In diviſion, the firſt figure in the quotient muſt always poſſeſs the ſame place with that figure of the dividend under which the unit's place of its product ſtands. Having thus determined the value of the quotient figures, make uſe of ſo many figures in the diviſor, reckoning from the left hand towards the right, as you intend to have in the quotient. Let each remainder be a new dividend, and, for every ſuch new dividend, leave out one figure to the right hand of the diviſor, obſerving to carry for the increaſe of the figures cut off, as in contracted multiplication.

Note. When there are not ſo many figures in the diviſor as are required to be in the quotient, begin the diviſion with all the figures as uſual, and continue it till the number of figures in the diviſor is equal to the number of figures remaining to be found in the quotient, after which uſe the contraction.

Examples.

(*Ex.* 1.) Divide ·475321 by 97·453.

97·453)·4753210000(·0048774
855090
754660
724890
427190

37378 rem.

(*Ex.* 2.) Divide 475·321 by ·97453.

·97453)475·3210000(487·74, &c.
855090
754660
724890
427190

37378, &c.

(3.) Divide 17·543275 by 125·7.
(4.) Divide 143754·35 by ·7493.
(5.) Divide ·000177089 by ·0374.
(6.) Divide 16 by 960.
(7.) Divide 12 by 1728.
(8.) Divide 47·5493 by 34·75.
(9.) Divide 74·3571 by ·00573.
(10.) Divide ·3754 by 75·714.

Examples under the contracted Rule.

(1.) Divide 754·347385 by 61·34775, and let the quotient contain only three places of decimals.

Contracted way.	*Common way.*	
$61\cdot347\overline{75})754\cdot347385(12\cdot296$	$61\cdot34775)\ 754\cdot347$	$38500\ (12\cdot296$
14086	14086	988
1817	1817	4385
590	590	48350
38	38	353750
1	1	545100

(2.) Divide 59 by ·74571345, and let the quotient contain four places of decimals.

(3.) Divide 17493·407704962 by 495·783269, and let the quotient contain four places of decimals.

(4.) Divide 98·187437 by 8·4765618, and let the quotient contain ten places of decimals.

(5.) Divide 47194·379457 by 14·73495, and let the quotient contain as many decimal places as there will be integers in it.

REDUCTION OF DECIMALS.

Propoſition 1. *To reduce a vulgar fraction to a decimal of equal value.*

Rule.

Annex ciphers to the numerator till it be equal to, or greater than, the denominator; then divide by the denominator as in diviſion of decimals, and the quotient will be the anſwer.

Note. Mr. Colſon, at page 162 of Sir Iſaac Newton's Fluxions, gives the following method for throwing a fraction, having a prime number for its denominator, into a decimal. Let $\frac{1}{29}$ be propoſed: then, by dividing in the common way, till the remainder becomes a ſingle figure, we ſhall have $\frac{1}{29} = \cdot03448\frac{8}{29}$, for the complete quotient; and this equation, multiplied by the numerator 8, will give $\frac{8}{29} = \cdot27586\frac{6}{29}$, and, if this be ſubſtituted in the firſt equation for $\frac{8}{29}$, we ſhall have $\frac{1}{29} = \cdot0344827586\frac{6}{29}$. Again, multiply the equation by 6, and it will give $\frac{6}{29} = \cdot20689655172\frac{7}{29}$; then by ſubſtituting, as before, $\frac{1}{29} = \cdot034482758620689655172\frac{7}{29}$, &c. as far as you pleaſe.

Prop. 2. *To reduce numbers of different denominations, as coins, weights, meaſures, &c. into decimals.*

Rule I.

Reduce the given money, weight, &c. into the loweſt denomination mentioned for a dividend; then reduce the integer into the ſame denomination for a diviſor: the quotient produced by this diviſion will be the decimal required.

Rule II.

Write the given denominations, or parts, regularly under each other, proceeding from the loweſt denomination to the higheſt; let theſe be the dividends. Oppoſite to each dividend, on the left hand, place ſuch a number for a diviſor as will reduce it to the next ſuperior name, and draw a line between them. Begin to divide with the uppermoſt numbers, and write the quotients of each, as decimal parts, on the right hand of the dividend next below it. Divide this mixed number by its diviſor, and ſo on till they are all uſed, the laſt quotient will be the decimal required.

Prop. 3. *To find the value of any decimal fraction in the known parts of an integer.*

Rule.

Multiply the given decimal by the number of parts contained in the next inferior denomination; and, from the right hand of the product, point off ſo many figures as the given decimal conſiſts of. Multiply the remaining decimals by the parts in the next inferior denomination, and from what reſults cut off as before. Proceed thus till you have brought out the leaſt known parts of the integer, and then the ſeveral denominations, on the left hand of the decimal points, will expreſs the value of the decimal,

Examples to Propoſition 1.

(1.) Reduce $\frac{7}{8}$ to a decimal.

$$\begin{array}{l} 8\,)\,7{\cdot}000\,(\,{\cdot}875 \text{ anſwer.} \\ \quad\;\; 60 \\ \quad\;\; \underline{40} \end{array}$$

(2.) Reduce $\frac{4}{7}$ to a decimal.

(3.) Reduce $\frac{1}{243}$ to a decimal.

(4.) Reduce $\frac{3}{4}$ to a decimal.
(5.) Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{5}{8}$ to a decimal.
(6.) Reduce $15\frac{5}{13}$ to a mixed decimal.
(7.) Reduce $\frac{5}{29}$ to a decimal.
(8.) Reduce $\frac{5}{191}$ to a decimal.

Examples to Prop. 2.

(9.) Reduce 18s. 9d$\frac{1}{4}$d. to the decimal of a pound.

By rule 1.

s.	s.	d.
20	18	$9\frac{1}{4}$
12	12	
240	225	
4	4	

960) 903·000000 (·940625

By rule 2.

4	3 · farthings.
12	9 ·75 pence.
20	18·8125 shillings.
	·940625£.

(10.) Reduce 7s. $5\frac{1}{2}$d to the decimal of a pound.

(11.) What decimal part of a pound is three halfpence?

(12.) Reduce 4s. $7\frac{9}{11}$d. to the decimal of a pound.

(13.) Reduce 1 oz. 11dwt. 3gr. to the decimal of a pound Troy.

(14.) Reduce 24 grains to the decimal of an ounce Troy.

(15.) Reduce 5 oz. 4dr. Avoirdupois to the decimal of a pound Troy.

(16.) Reduce 3cwt. 1qr. 14lb. to the decimal of a ton.

(17.) Reduce 2qr. 15lb. to the decimal of a hundredweight.

(18.) Reduce 5lb. 10 oz. 3dwt. 13gr. Troy to the decimal of a hundred weight Avoirdupois.

(19.) Reduce 1qr. 1 nail to the decimal of a yard.

(20.) Reduce 2qr. 3n. to the decimal of an English ell.

(21.) Reduce 14yds 2ft $6\frac{1}{2}$in. to the decimal of a mile.

(22.) What decimal part of an acre is 1r. 37 poles?

(23.) What decimal part of a hogshead of wine is 2qts 1 pint?

(24.) Reduce 3 bush. 3 pecks to the decimal of a chaldron of 32 bushels.

(25.) What decimal part of a year is 3w. 4d. 55hrs. reckoning 365d. 6hrs a year.

(26.) Reduce

(26.) Reduce 2·45 ſhillings to the decimal of a £.
(27.) Reduce 1·074 roods to the decimal of an acre.
(28.) Reduce 176·9 yards to the decimal of a mile.

Examples to Prop. 3.

(29.) Required the value of ·03125 of a pound ſterling.

·03125
20
s. 0·62500
12
d. 7·50000
4
qrs. 2·00000

Anſwer 7½d.

(30.) What is the value of ·7575 of a pound ſterling?
(31.) Required the value of ·75435 of a ſhilling.
(32.) What is the value of ·375 of a guinea?
(33.) What is the value of ·4575 of a hundred-weight?
(34.) What is the value of ·175 of a ton Avoirdupois?
(35.) What is the value of ·05875 of a pound Avoirdupois?
(36.) Required the value of ·02575 of a pound Troy.
(37.) Required the value of ·075 of a yard.
(38.) Required the value of ·475 of an Engliſh ell.
(39.) What is the value of ·04535 of a mile?
(40.) What is the value of ·6375 of an acre?
(41.) What is the value of ·574 of a hogſhead of beer?
(42.) What is the value of ·4285 of a year?
(43.) Required the ſum of ·475 of a pound and ·375 of a ſhilling.
(44.) Required the ſum of ·573 of an inch and ·751 of a yard.
(45.) Required the difference between ·5 of a mile and ·375 of a furlong.
(46.) Required the difference between ·625 of a cwt. and ·20835 of a ton.
(47.) Required the ſum of ·175 ton, ·195 cwt. ·145qr. and ·15lb.
(48.) Required the ſum of ·575lb. Troy and ·845oz.

CIRCU-

CIRCULATING, *or Repeating* DECIMALS.

Definition 1. When the denominator of a vulgar fraction, in its loweſt terms, is not compounded of 2 or 5, or both, the decimal produced from ſuch a vulgar fraction will be infinite; it is called a *repetend*, or *circulating decimal*, from a continual repetition of the ſame figures.

2. *A ſingle repetend* is a decimal where only one figure repeats, as ·222, &c. or ·3333, &c. and theſe may be expreſſed by putting a mark over the firſt figure. Thus ·222, &c. may be denoted by ·2′, and ·3333, &c. by ·3′.

3. *A compound repetend* has the ſame figures circulating alternately, as ·575757, &c. or ·57235723, &c. and theſe may be diſtinguiſhed by marking the firſt and laſt repeating figure. Thus ·5757, &c. may be written ·5′7′, and ·57235723, &c. ·5′723′.

4. *Pure repetends* are ſuch as have no figures in them but what belong to the repetend, as .3′, ·5′, ·4′73′, &c.

5. *Mixed repetends* are ſuch as have ciphers, or ſignificant figures, between the repetend and the decimal point, or ſuch as have whole numbers to the left hand of the decimal point, as ·04′, ·07′53, ·473′, ·357′3′, 6·5′, 4·3′75′, &c.

6. *Diſſimilar repetends* are ſuch as begin at different places from the decimal points, as ·2′53′, ·475′2′, &c.

7. *Similar repetends* are ſuch as begin at an equal diſtance from the decimal points, as ·35′4′, 2·75′34′, &c.

8. *Conterminous repetends* are ſuch as end at the ſame diſtance from the decimal points, as ·125′, ·3′54′, &c.

9. *Similar and conterminous repetends* are ſuch as begin and end at the ſame place after the decimal points, as 53·27′53′, 4·63′25′, and ·46′32′, &c.

REDUCTION of *circulating* DECIMALS.

Propoſition 1. *To reduce a pure repetend to its equivalent vulgar fraction.*

Rule.

Make the given decimal the numerator, and let the denominator be a number conſiſting of ſo many nines as there are figures in the repetend. The terms of this fraction, divided by their greateſt common meaſure, will give the leaſt equivalent vulgar fraction required.

Prop. 2. *To reduce a mixed repetend to its equivalent vulgar fraction.*

Rule.

Rule.

From the given mixed repetend ſubtract the finite numbers for a numerator, and to the right hand of ſo many nines as there are pure repetends annex ſo many ciphers as there are finite decimals for a denominator. Then reduce this fraction to its loweſt terms.

Note 1. Any finite decimal may be conſidered as infinite by making ciphers to recur; thus $\cdot35 = \cdot35000\acute{0}$, &c.

2. If any circulating decimal has a repetend of any number of figures, it may be conſidered as having a repetend of twice or thrice that number of figures, or any multiple thereof. The number $2{\cdot}3\acute{5}\acute{7}$, having two repetends, may be conſidered as having a repetend of 4, 6, 8, 10, &c. places. Thus, $2{\cdot}3\acute{5}\acute{7} = 2{\cdot}3\acute{5}75\acute{7} = 2{\cdot}3\acute{5}757\acute{5} = 2{\cdot}3\acute{5}757575\acute{7}$, &c. Hence any number of diſſimilar repetends may be made ſimilar and conterminous.

3 If any circulating decimal has a repetend of more than one figure, it may be transformed into another decimal, having a repetend of the *ſame* number of figures; thus, $\cdot\acute{5}\acute{7} = \cdot5\acute{7}\acute{5} = \cdot57\acute{5}\acute{7}$, and $3{\cdot}4\acute{7}8\acute{5} = 3{\cdot}47\acute{8}5\acute{7} = 3{\cdot}478\acute{5}7\acute{8} = 3{\cdot}47857\acute{8}\acute{5}$.

4. When any circulating decimal has a repetend of more than one figure, it *cannot* be transformed into another decimal, having a greater or leſs number of figures at pleaſure; but the new repetend muſt always contain either the *ſame* number of places as the original repetend, or ſome *multiple* thereof. Thus, $\cdot\acute{5}\acute{7} = \cdot5\acute{7}\acute{5} = \cdot57\acute{5}\acute{7} = \cdot575\acute{7}\acute{5} = \cdot5757\acute{5}7\acute{5}$: Or, $\cdot\acute{5}7 = \cdot\acute{5}75\acute{7} = \cdot\acute{5}7575\acute{7}$ according to the ſecond note. But $\cdot\acute{5}\acute{7}$ never can be equal to $\cdot\acute{5}7\acute{5}$, as Mr. *Bonnycaſtle* has imagined; for, then $\frac{57}{99}$ would be equal to $\frac{575}{999}$, which evidently is not the caſe. The truth of any of the preceding notes may be examined by turning the given repetends into their equivalent vulgar fractions, and comparing them together by the latter part of Note 2, Prop. 8, Vulgar Fractions.

5. Any ſeries of nines, infinitely continued, is equal to unity, or one, in the next left-hand place. Thus, 999, &c. ad infinitum, $= 1$, $\cdot0999$, &c. $= \cdot1$; alſo, $\cdot00999$, &c. $= \cdot01$, and $5{\cdot}999$, &c. $= 6$.

6. Any number may be multiplied by 9, 99, 999, &c. by annexing ſo many ciphers to the right hand of it as there are nines, and then ſubtracting it from itſelf, thus increaſed. Thus,

$$147 \times 9 = 1470 - 147 = 1323,\quad 147 \times 99 = 14700 - 147 = 14553,$$
$$\text{and } 147 \times 999 = 147000 - 147 = 146853.$$

7. Any number, divided by 9, 99, 999, &c. will be equal to the ſum of the quotients of the ſame number continually divided by 10, 100, 1000, &c. Thus,

$$\frac{425}{10} + \frac{42{\cdot}5}{10} + \frac{4{\cdot}25}{10} + \frac{\cdot425}{10}, \text{ \&c. ad infinitum, } = \frac{425}{9} = 47{\cdot}\acute{2}, \text{ and}$$

$$\frac{425}{1000} + \frac{\cdot425}{1000} + \frac{\cdot000425}{1000}, \text{ \&c. ad infinitum} = \frac{425}{999} = \cdot\acute{4}2\acute{5};$$

hence it appears that every recurring decimal is a geometrical ſeries, decreaſing, ad infinitum, and the equivalent vulgar fraction to every recurring decimal is equal to the ſum of ſuch a ſeries.

S. II

8. If any number be divided by another prime to it, and the diviſion continued on indefinitely, the number of repetends in the quotient will always be leſs than the number of units in the diviſor.

9. If two, or more numbers, that have repetends of equal places be added together, the ſum will have a repetend of the ſame number of places: for, every column of periods will amount to the ſame ſum.

10. If any circulating number be multiplied by any given number, the product will be a circulating number, containing the ſame number of figures in the repetend as before, for every repetend will be equally multiplied, and conſequently muſt produce the ſame product.

Prop. 3. *Having a vulgar fraction given, to find whether its equivalent decimal will be finite or infinite, and how many places the repetend will conſiſt of.*

Rule.

Reduce the given fraction to its loweſt terms, and divide the denominator by 10, 2, or 5, as often as poſſible: then divide 9999, &c. by this reſult till nothing remains, and the number of nines made uſe of will be equal to the number of figures in the repetend. The repetend will always begin after ſo many places of figures as you perform diviſions by 10, 2, or 5; and, if the whole denominator ſhould vanish after theſe diviſions, the decimal will be finite.

Examples to Propoſition 1.

(1.) Required the leaſt equivalent vulgar fraction to ·3′, and ·1′35′.

Firſt, $\cdot 3' = \frac{3}{9} = \frac{1}{3}$, and $\cdot 1'35' = \frac{135}{999} = \frac{5}{37}$.

(2.) Required the leaſt equivalent vulgar fractions to ·6′, ·1′62′, ·7′69230′, ·9′45′, and ·0′9′.

(3.) Required the leaſt equivalent vulgar fractions to ·5′94405′, ·3′6′, and ·1′42857′.

Examples to Prop. 2.

(4.) Required the leaſt equivalent vulgar fractions to 2·41′8′, ·59′25′, ·0084′97133′, and ·53′.

Firſt, $2\cdot41'8' = \frac{2418-24}{990} = \frac{2394}{990} = \frac{133}{55} = 2\frac{23}{55}$; $\cdot59'25' = \frac{5925-5}{9990} = \frac{5920}{9990} = \frac{16}{27}$; $\cdot0084'97133' = \frac{8497133-8}{999999000} = \frac{8497125}{999999000} = \frac{83}{9768}$; and $\cdot5'3' = \frac{53-5}{90} = \frac{48}{90} = \frac{8}{15}$.

(5.) Required the leaſt equivalent vulgar fractions to ·138′, 7·54′3′, ·043′54′, 37·54′, ·67′5′, and ·75′4347′.

(6.) Required the leaſt equivalent vulgar fractions to ·75′, ·43′8′, ·093′, 4·75′43′, ·0098′7′, and ·45′.

Examples to Prop. 3.

(7.) Required to find whether the decimal equivalent to $\frac{249}{29304}$ be finite or infinite; if infinite, how many places the repetend will conſiſt of, whether there will be any finite decimals to the left hand of the repetend, and how many?

Firſt, $\frac{249}{29304}=\frac{83}{9768}$; *then* $\overset{\div 2}{9768}=\overset{\div 2}{4834}=\overset{\div 2}{2442}=1221$, *and* 1221)999999(719: *here are* 6 *nines made uſe of before nothing remains; and the denominator has been divided by* 2 *three times, and cannot be abridged any more; therefore the decimal will be infinite, and will conſiſt of three finite decimals and ſix pure repetends. Thus,* $\frac{83}{9768}$ = ·0084′97133′.

(8.) Whether is the decimal equivalent to $\frac{210}{1120}$ finite or infinite?

(9.) Whether is the decimal equivalent to $\frac{4}{1768}$ finite or infinite?

(10.) Let $\frac{12}{132}$, $\frac{80}{333}$, $\frac{72}{135}$, $\frac{7}{8344}$, and $\frac{255}{792}$, be propoſed.

ADDITION of *circulating* Decimals.

Rule.

Make the repetends ſimilar and conterminous, and to the right hand thereof ſet two or three of the firſt repeating figures, which add together as whole numbers, and carry the tens contained in the left hand row to the right-hand row of the conterminous repetends: collect theſe together into one ſum, like finite decimals, for the anſwer.

Note. The ſum of the repetend, found by the preceding rule, will ſometimes, though very rarely, conſiſt of a number of nines; whenever that is the caſe, reject them, and make the next left-hand figure an unit more. If the decimals to be added contain only ſingle recurring figures, after having made them end together, the ſum of the right hand row may be increaſed by as many units as it contains nines, inſtead of carrying the repetend out.

Examples.

Examples.

(1.) Add ·125′, 4·1′63′, 1·7′143′, and 2·5′4′, together.

Diſſimilar.		*Similar.*		*Similar and conterminous.*		
·125′	=	·125′	=	·125′55555555555′	.....	5555
4·1′63′	=	4·163′16′	=	4·163′16316316316′	......	3163
1·7′143′	=	1·714′371′	=	1·714′37143714371′	......	4371
2·5′4′	=	2·54′54′	=	2·545′45454545454′	......	5454
		The true ſum		8·548′54470131697′		*one to carry.*

(2.) Add 67·34′5′ + 9·6′51′ + ·2′5′ + 17·47′ + ·5′, together.

(3.) Add ·4′75′ + 3·754′3′ + 64·7′5′ + ·5′7′ + ·17′88′, together.

(4.) Add ·5′ + 4·37′ + 49·45′7′ + ·49′54′ + ·7′345′, together.

(5.) Add ·1′75′ + 42·5′7′ + ·37′53′ + ·59′45′ + 3·75′4′, together.

(6.) Add 165·1′64′ + 147.0′4′ + 4·9′5′ + 94·37′ + 4·7′123456′, together.

SUBTRACTION of *circulating* Decimals

Rule.

Make the repetends ſimilar and conterminous, and ſubtract as if they were finite decimals; only obſerve, that if the repetend of the ſubtractor be greater than the repetend of the ſubtrahend, the right-hand figure of the remainder muſt be leſs by unity than it would be if the expreſſions were finite.

Note. If either the ſubtrahend or ſubtractor be finite decimals, they muſt be made ſimilar and conterminous with ciphers.

Examples.

(1.) From 11·47′5′ take 3·457′35′.

Diſſimilar.		*Similar.*		*Similar and conterminous.*		
11·47′5′	=	11·475′7′	=	11·475′75757′	......	575
3·457′35′	=	3·457′35′	=	3·457′35735′	......	735
		The true difference		8·018′40021′		*one to carry.*

(2.) From 47·53′ take 1·7′57′.
(3.) From 17·5′73′ take 14·57′.
(4.) From 17·43′ take 12·345′.
(5.) From 1·127′54′ take ·47′384′.
(6.) From 4·75 take ·375′.
(7.) From 4·794 take ·17′44′.
(8.) From 1·457′ take ·3754.
(9.) From 1·49′37′ take ·1475.

MULTIPLICATION of *circulating* Decimals.

General Rule.

Turn the decimals into their equivalent vulgar fractions, and find the product of theſe fractions: then turn the vulgar fraction, expreſſing the product, into an equivalent decimal fraction, and it will be the product required.

Propoſition 1. *When the right-hand figure of the multiplicand is a ſingle repetend, and the multiplier a finite number.*

Rule. In multiplying, increaſe the right-hand figure of each reſulting line by as many units as there are nines in the product of the firſt figure in that line; and the right-hand figure of each line will be a repetend: make them all end at the ſame place, and then add them together.

Prop. 2. *When the multiplicand is a compound repetend and the multiplier a finite number.*

Rule. Set the repeating figures in the multiplicand twice over, multiply the ſecond period mentally, and carry the tens, contained in the product of the left-hand figure, to the product of the right-hand figure of the firſt period; then multiply the reſt of the figures in the multiplicand as in common multiplication. Proceed thus with each figure in the multiplier, and every product will contain a repetend of the ſame number of places as the repetend in the multiplicand; laſtly, make every product conterminous towards the right-hand before you add them together.

Note. It is poſſible for the product of the repetend to conſiſt of a number of nines; if ever this ſhould happen, increaſe the product of the right-hand figure of the firſt period by an unit.

Prop. 3. *When the multiplicand is a finite number, and the right-hand figure in the multiplier a ſingle repetend.*

Rule. Multiply by the circulating figure as if it were a finite digit; divide this product by 9, and continue the quotient till it becomes a ſingle circulate, if the product does not divide even by 9. Proceed with the remaining figures in the multiplier (if any) as in common multiplication, taking care to ſet the firſt figure of the firſt product exactly under the repeating figure in the multiplier.

Prop. 4. *When the multiplicand is a finite number, and the multiplier a compound repetend.*

Rule. If the multiplier be a mixed repetend, ſubtract the finite decimal from it, and the remainder will be a new multiplier, with which proceed as in common multiplication, and add the ſeveral products together. Then ſet the left-hand figure of this ſum under its third, fourth, fifth, &c. figure towards the right-hand, according as the repetend conſiſts of two, three, four, &c. places, and the reſt in order after it: proceed thus, till the left-hand figure of the ſum falls beyond the right-hand figure. Laſtly, collect theſe numbers into one ſum in the order they are placed, and mark as many figures for a repetend as the repetend of the multiplier conſiſts of.

Note. If the multiplier be a pure repetend, proceed exactly in the ſame manner with it as with the new multiplier above.—The reaſon of the placing the left-hand figure of the ſum under the third, &c. figures, will readily occur to any one who has conſulted the 7th note in Reduction of circulating decimals.

Prop. 5. *When the multiplicand and multiplier are each a ſingle circulate.*

Rule. In multiplying by the repeating digit, increaſe the right-hand figure of the product by as many units as it contains nines; divide this product by 9 till it recurs, and the quotient will be the product of the repetend. Proceed with the finite numbers, if any, as in Prop. 1. taking care to ſet the firſt figure of the firſt product exactly under the repeating figure in the multiplier.

Prop. 6. *When the multiplicand is a compound repetend, and the multiplier a ſingle recurring digit.*

Rule. After having multiplied by the repeating figure, like a finite digit, as directed in Prop. 2, divide the product by 9, till the quotient recurs, or is ſufficiently exact. Proceed with the finite numbers, if any, in the very ſame manner as directed in *that* Prop. taking care to ſet the firſt figure of the firſt product exactly under the repeating figure in the multiplier.

Prop. 7. *When the multiplicand is a ſingle, and the multiplier a compound, repetend.*

Rule. If the multiplier be a pure repetend, proceed as in Prop. 1. If it be a mixed repetend, ſubtract the finite decimal from it, and the remainder will be a new multiplier, with which proceed as above. Then ſet the left-hand figure of the ſum under its third, fourth, fifth, &c. figure towards the right-hand, as directed in Prop. 4.

Prop. 8. *When the multiplicand and multiplier are each a compound repetend.*

Rule. If the multiplier be a pure repetend, proceed as in Prop. 2. If it be a mixed repetend, ſubtract the finite decimal from it, and the remainder will be a new multiplier, with which proceed as above. Then ſet the left-hand figure of the ſum under its third, fourth, fifth, &c. figure, towards the right-hand, as directed in Prop. 4.

Examples.

The general rule needs no example.

Examples to Propoſition 1.

```
(1.) Mult. 4·253′
           by ·257
           ———————
          29773′ ...  33
        21266′6 ...  66
        8506′66 ...  66
        ———————————
Product 1.093106′ one to carry.
```

(2.) Mult. ·3754′ by 14·75.
(3.) Mult. 4·753′ by 7·437.
(4.) Mult. ·30705′ by ·0473.
(5.) Mult. ·14732′ by ·1497.
(6.) Mult. ·37543′ by ·7149.

Examples to Prop. 2.

(6.) Mult. ·42′53′ by 2·57:
2d period.
·42′53′ ... 253
2·57
297′72′ ... 772
212′66′2 ... 662
85′06′50 ... 650
Product 1·0930′86′ *two to carry*

(7.) Mult. ·37′54′ by 17·43.
(8.) Mult. 4·73′5′ by 7·349.
(9.) Mult. 4·1′42857′ by ·1797.
(10.) Mult. 7·14′93′ by 5·43.
(11.) Mult. ·40′705′ by 7·345.

Examples to Prop. 3.

(12.) Mult. ·437
by 3·75′
9)2185
2427′
3059
1311
Product 1·64117′

(13.) Mult. 1·475 by 1·754′.
(14.) Mult. 175·715 by 3·7545′.
(15.) Mult. ·37504 by ·7153′.
(16.) Mult. 5·7534 by ·1735′.
(17.) Mult. ·37493 by ·757′.

Examples to Prop. 4.

(18.) Mult. 4·573 by ·37′5′.
4·573
·37′5′ ·372
3
9146
·372 new mult. 32011
13719
1·701156
17011 ... 56
170 ... 11 &c.
1 ... 70 &c.
Product 1·71833′9′ *one to carry.*

(19.) Mult. 4·573 by ·3′75′.
4·573
·3′75′
22865
32011
13719
1·714875
1714 ... 875
1 ... 71 &c.
Product 1·7165′91′ *one to carry.*

(20.) Mult. 4·7157 by 3·7′543′.
(21.) Mult. 47·1937 by ·007′5′.
(22.) Mult. 4·37595 by 1·7′5435′.
(23.) Mult. 371473 by ·7′5314′.

Examples to Prop. 5.

(24.) Mult. 3·456′
by ·425′
9)17283′33
1920′37′ .. 03 *&c.*
6913′33 .. 33
13826′666 .. 66

Product 1·47100′37′ *one to carry.*

(25.) Mult. 4·57′ by 2·45′.
(26.) Mult. 3·745′ by 1·47′.
(27.) Mult. 5·7195′ by 1·788′.
(28.) Mult. 3·7532′ by ·3425′.
(29.) Mult. 714·32′ by 3·456′.

Examples to Prop. 6.

(30.) Mult. 1·4′56′ by 4·23′.
2d *period.*
1·4′56′ ... 456
4·23′
9)43′69′36, &c.
4 8′54′ ... 854
29′12′9 ... 129
58′25′82 ... 582

Product 6·165′66′ *one to carry.*

(31.) Mult. ·573′4′ by 1·573′.
(32.) Mult. ·41′32′ by 1·432′.
(33.) Mult. 3·7′534′ by ·3757′.
(34.) Mult. 7·00′430′ by 4·7005′.
(35.) Mult. 5·437′15′ by ·37053′.

Examples to Prop. 7.

(36.) Mult. 45·13′ by ·2′45′.
45·13′
·2′45′
2256 6′ ... 66
1805 3′3 ... 33 } one to carry.
9026′6 6 ... 66
11·05766′6666 *&c.*
1105 7666 *&c.*
1 1057 *&c.*
11 *&c.*
11·0′68735402′

This repetend is found by continuing the figures, &c.

(37.) Mult. 3·537′ by 2·4′5′.
2·4′5′
2
2·43 n. mult.
3·537′
2·43
106 1 3′ ... 33
1415 1′1 ... 11 } *
7075′5 5 ... 55
8·59680′0 *&c.*
85968 *&c.*
859 *&c.*
8·6836′3′

This repetend is found as the laſt.

(38.) Mult. ·47053′ by 1·73′5′.
(39.) Mult. 3·4573′ by 54·7′53′.

Examples to Prop. 8.

```
(40.) Mult. 7·7'2' by ·2'97'.       (41.) Mult. ·2'97' by 7·7'2';
           2d period.                          2d period.
     7·7'2' ... 72                     ·2'97' ... 297     7·7'2'
     ·2'97'                             7·65                   7
   ----------                        ----------          -------
    540'9' ... 09 ]                 14'86' ... 486 ]    7·65 n. mult.
   695'4'5 ... 45 }*              17'83'7 ... 837 }one to carry.
  154'5'45 ... 45 ]             20'81'08 ... 108 ]
 -----------                    -------------
 2·29500'0 &c.                   2·274'32'432 &c.
      2295 &c.                          22743 &c.
         2 &c.                            227 &c.
 -------------                              2 &c.
 2·2972'97'                          -------------
 -------------                        2·2972'97'
                                     -------------
```

The ſum of the repetend, in the 2d example, Prop. 7, and in the firſt example in this Prop. conſiſts of an infinite number of nines. See the note in Addition of Circulating Decimals.

(42.) Mult. 4·571'37' by ·14957'3'.
(43.) Mult. 5·7149'3' by 4·75'35'.

DIVISION of *circulating* Decimals.

General Rule.

Turn the decimals into their equivalent vulgar fractions, and find the quotient of theſe fractions: then turn the vulgar fraction, expreſſing the quotient, into an equivalent decimal fraction, and it will be the quotient required.

Propoſition 1. *When the dividend is a ſingle, or compound, repetend, and the diviſor a finite number.*

Rule. Divide as if both the numbers were finite; only, inſtead of bringing down ciphers, bring down the repeating figure, or figures, and continue the quotient till it repeats, or is ſufficiently exact.

Prop. 2. *When the diviſor is a ſingle, or compound, repetend, and the dividend a finite number.*

Rule. If the diviſor be a mixed repetend, annex as many ciphers to the right hand of the dividend as there are pure repetends in the diviſor. Write this dividend and diviſor in the order of diviſion: under theſe write them a ſecond time, each removed ſo many figures towards the right hand as there are pure repetends in the diviſor; ſubtract each lower line from that above it, and the remainders will be a new diviſor and dividend, both finite numbers. If the diviſor be a pure repetend, the dividend *only* muſt be ſubtracted after the ciphers are annexed to it.

Prop. 3. *When both the diviſor and dividend are ſingle, or compound, repetends.*

Rule 1. If they are diſſimilar mixed repetends, make them ſimilar and conterminous, and ſubtract the finite numbers from each of them, the remainders will be a new diviſor and dividend, both finite numbers.

2. If the diviſor and dividend are both pure repetends, make them conterminous, and divide them like finite numbers.

Note 1. If one number is a pure, and the other a mixed, repetend, (without any whole number) by making them ſimilar, the pure repetend will become a mixed one, and conſequently the firſt part of the preceding rule will diſcover the true quotient.

2. If one number is a pure, and the other a mixed, repetend; (compoſed of a pure repetend and a whole number only) make them conterminous, and ſubtract the whole number from the pure repetend belonging to it, for a new diviſor or dividend; then proceed as if both the numbers were finite.

Examples.

The general rule needs no example.

Examples to Propoſition 1.

(1.) Divide 56·6' by 137.

true quotient.

137)56·6'666666(·4'136253'

Rem. 5.

(2.) Divide 24·31'8' by 1·792.

Quotient.

1·792)24·31'8'181818(13·570413, &c.

Rem. 1722.

(3.) Divide 7·54'03' by 14·25.

(4.) Divide 4·3173' by ·075.

(5.) Divide 14·937'5' by 1·788.

(6.) Divide 43·5'75' by ·00456.

Examples to Prop. 2.

(7.) Divide 8·5968 by ·24'5'.

True quotient.

·24'5')8·596800(35·024
2   85968

·243)8·510832 *new dividend.*

; · · ·

(8.) Divide 2·295 by ·2'97'.

Quotient.

·2'97')2·295000(7·7195'4'
2295

·297)2·292705 *new divid.*

Rem. 162.

(9.) Divide 47·345 by 1·7′59′.
(10.) Divide ·35178 by 3·75′3′.
(11.) Divide 17·342 by ·4′5678′.
(12.) Divide ·37453 by 1·4′235′.

Examples to Prop. 3.

(13.) Divide 13·51′69533′ by 4·2′97′.

Firſt, 4·2′97′=4·29′72′=4·29′72972′.

4·29′72972′ | 13·51′69533′
42 | 135
New diviſor 4·2972930 | 13·5169398 *new dividend.*

True quotient.
4·2972930) 13·5169398000 (3·14′5′

Rem. 1953315.

(14.) Divide ·4′5′ by ·1′18881′.

Firſt, ·4′5′=45′4545′

Quotient.
·118881) ·454545000000 (3·8235294, &c.

Rem. 13986

(15.) Div. ·4′75′ by ·37′53′.

Here ·4′75′=·47′54′

True Quot.
·37′53′)·47′54′(1·26′
3 4
·3750)·4750 *new divid.*

250 rem.

(16.) Div. 3·7′53′ by ·2′4′.

Here 3·7′53′=3·7′53753′
and ·2′4′=·2′42424′.
·2′42424′)3·7′53753′(15·484, &c.
3
·242424)3·753750 *new divid.*

56784 rem.

(17.) Divide ·357′2′ by 49·5′735′.
(18.) Divide ·1′54′ by ·5′.
(19.) Divide ·3′ by ·57′6′.
(20.) Divide 4·5′732′ by ·7′.

The RULE of THREE DIRECT and INVERSE, the RULE of FIVE, &c. in decimals, are managed the ſame as in whole numbers; a repetition of thoſe rules, in this place, is therefore unneceſſary.—If the learner wiſhes to exerciſe himſelf in decimals before he proceeds farther, he may take any of the examples in vulgar fractions, under the above rules, and work them decimally.

PRACTICE.

PRACTICE.

Definition. *Practice* has its name from its daily uſe amongſt merchants and tradeſmen, being an eaſy and conciſe method of working moſt queſtions that occur in trade and buſineſs, and is only a contraction of the Rule of Three when the firſt term is an unit.

A Table of the aliquot Parts of Money.

Of a Pound.								*Of a Shilling.*		
s.	*d.*		£	*s.*	*d.*		£	*d.*		
10	0	=	$\frac{1}{2}$	1	3	=	$\frac{1}{16}$	6	=	$\frac{1}{2}$
6	8	=	$\frac{1}{3}$	1	0	=	$\frac{1}{20}$	4	=	$\frac{1}{3}$
5	0	=	$\frac{1}{4}$		10	=	$\frac{1}{24}$	3	=	$\frac{1}{4}$
4	0	=	$\frac{1}{5}$		8	=	$\frac{1}{30}$	2	=	$\frac{1}{6}$
3	4	=	$\frac{1}{6}$		$7\frac{1}{2}$	=	$\frac{1}{32}$	$1\frac{1}{2}$	=	$\frac{1}{8}$
2	6	=	$\frac{1}{8}$		6	=	$\frac{1}{40}$	1	=	$\frac{1}{12}$
2	0	=	$\frac{1}{10}$		5	=	$\frac{1}{48}$	$\frac{3}{4}$	=	$\frac{1}{16}$
1	8	=	$\frac{1}{12}$		4	=	$\frac{1}{60}$	$\frac{1}{2}$	=	$\frac{1}{24}$
1	4	=	$\frac{1}{15}$		$3\frac{3}{4}$	=	$\frac{1}{64}$	$\frac{1}{4}$	=	$\frac{1}{48}$

Rule 1. *When the price is leſs than a penny.* Divide the quantity by the aliquot parts in a penny, then divide that quotient by 12 and by 20.

Rule 2. *When the price is an aliquot part of a ſhilling.* Divide the quantity by the aliquot part, and that quotient by 20.

Rule 3. *When the price is pence and farthings, and they no aliquot part of a ſhilling.* Divide the given quantity by ſome aliquot part of a ſhilling, then conſider what part of this aliquot part the reſt is, and divide the quotient thereby; this quotient, added to the former, will be the anſwer in ſhillings, which divide by 20.

Rule 4. *When the price is more than one ſhilling, but leſs than two.* Let the given number ſtand for ſhillings, and work for the pence and farthings by the preceding rules.

Rule 5. *When the price is any number of ſhillings leſs than 20.* If the number of ſhillings be even, multiply the quantity by half the price, double the firſt figure in the product for ſhillings, and the reſt of the product will be pounds. If the number of ſhillings be odd, find the value for the greateſt even number, as before, to which add $\frac{1}{20}$

of the given quantity for the odd ſhilling, and the ſum will be the anſwer.

Rule 6. *When the price is ſhillings and pence.* If they are an aliquot part of a pound, divide the quantity by that aliquot part, and the quotient will be the anſwer. If they are not an aliquot part, multiply the quantity by the ſhillings, and take parts for the reſt.

Rule 7. *When the price is pounds and ſhillings.* Multiply the quantity by the pounds, and proceed with the ſhillings as in Rule 5.

Rule 8. *When the price is pounds, ſhillings, pence, and farthings.* Multiply the quantity by the pounds, and work for the reſt by the preceding rules.

Note. When the given quantity conſiſts only of 1, 2, or 3, figures, proceed by the 1ſt, 2d, 3d, or 4th, Prop. of Compound Multiplication.

Rule 9. *If there be a fraction in the given quantity,* work for the whole number by ſome of the preceding rules, and find the produce of the fraction by multiplying the price by the numerator, and dividing the product by the denominator; then add them together for the anſwer.

Note. If the price be pounds, ſhillings, and pence, or pounds, ſhillings, pence, and farthings, and if the quantity of things does not exceed 1000, proceed by the fifth Prop. in Compound Multiplication.

A Table of the aliquot Parts of Weights and Meaſures.

Avoirdupois Weight.	
Of a Ton.	*Of* $\frac{1}{2}$ *Cwt. or 56lb.*
	lb.
Cwt..	28 = $\frac{1}{2}$
10 = $\frac{1}{2}$	14 = $\frac{1}{4}$
5 = $\frac{1}{4}$	8 = $\frac{1}{7}$
4 = $\frac{1}{5}$	7 = $\frac{1}{8}$
$2\frac{1}{2}$ = $\frac{1}{8}$	*Of a* $\frac{1}{4}$ *Cwt. or 28 lb.*
2 = $\frac{1}{10}$	*lb.*
	14 = $\frac{1}{2}$
Of a Cwt.	7 = $\frac{1}{4}$
	4 = $\frac{1}{7}$
Qr.	$3\frac{1}{2}$ = $\frac{1}{8}$
2 or 56lb. = $\frac{1}{2}$	*Of a Pound.*
1 or 28lb. = $\frac{1}{4}$	*Oz.*
16 = $\frac{1}{7}$	8 = $\frac{1}{2}$
14 = $\frac{1}{8}$	4 = $\frac{1}{4}$
	2 = $\frac{1}{8}$

Table continued.

Troy Weight.

Of an Ounce.

dwt.	*gr.*		
10	0	=	$\frac{1}{2}$
6	16	=	$\frac{1}{3}$
5	0	=	$\frac{1}{4}$
4	0	=	$\frac{1}{5}$
3	8	=	$\frac{1}{6}$
2	12	=	$\frac{1}{8}$
2	0	=	$\frac{1}{10}$
1	16	=	$\frac{1}{12}$

Of a Dwt.

gr.		
12	=	$\frac{1}{2}$
8	=	$\frac{1}{3}$
6	=	$\frac{1}{4}$
4	=	$\frac{1}{6}$
3	=	$\frac{1}{8}$
2	=	$\frac{1}{12}$

Land-Measure.

Of an Acre.

R.	*P.*		
2	0	=	$\frac{1}{2}$
1	0	=	$\frac{1}{4}$
	32	=	$\frac{1}{5}$
	20	=	$\frac{1}{8}$
	16	=	$\frac{1}{10}$
	8	=	$\frac{1}{20}$

Cloth-Measure.

Of a Yard.

Qr.	*N.*		
2	0	=	$\frac{1}{2}$
1	0	=	$\frac{1}{4}$
	2	=	$\frac{1}{8}$
	1	=	$\frac{1}{16}$

Of an English Ell.

Qr.	*N.*		
2	2	=	$\frac{1}{2}$
1	1	=	$\frac{1}{4}$
1	0	=	$\frac{1}{5}$
	2	=	$\frac{1}{10}$
	1	=	$\frac{1}{20}$

Of a Flemish Ell.

Qr.	*N.*		
1	2	=	$\frac{1}{2}$
1	0	=	$\frac{1}{3}$
	3	=	$\frac{1}{4}$
	2	=	$\frac{1}{6}$
	1	=	$\frac{1}{12}$

Of a French Ell.

Qr.	*N.*		
3	0	=	$\frac{1}{2}$
2	0	=	$\frac{1}{3}$
1	2	=	$\frac{1}{4}$
1	0	=	$\frac{1}{6}$
	3	=	$\frac{1}{8}$
	2	=	$\frac{1}{12}$
	1	=	$\frac{1}{24}$

Rule 10. *When the given quantity is of ſeveral denominations.* Multiply the given price by the higheſt denomination, as in Compound Multiplication, and take parts of the given price for the inferior denominations of the given quantity, and the ſum will be the true value.

Examples to Rule I.

(1) What coſt 4715 yards of tape, at ¼d. per yard?

¼|¼|4715

12)1178¾d.

20)98 2

£4 18 2¾

(2) 371 at ¼d,
(3) 425 at ½d.
(4) 5714 at ¾d.

Examples to Rule II.

(5) 425 yards at 1d.

1d.|1/12|425

2|0)3|5 5

£1 15 5

(6) 3749 at 1d.
(7) 496 at 1½d.
(8) 3741 a[illegible]d.
(9) 574 at 3d.
(10) 1749 at 4d.
(11) 1731 at 6d.

Examples to Rule III.

(12) 354 at 1¼d.

1d |1/12|354

¼|¼| 29 6
7 4½

2|0)3|6 10½

£1 16 10½

(13) 5714 at 1¼d.
(14) 142 at 1¾d.
(15) 1749 at 2¼d.
(16) 134 at 2½d.
(17) 5794 at 2¾d.
(18) 1749 at 3¼d.
(19) 574 at 3½d.
(20) 1749 at 3¾d.
(21) 749 at 4¼d.
(22) 1749 at 4½d.
(23) 3749 at 4¾d.
(24) 173 at 5d.
(25) 146 at 5¼d.
(26) 3741 at 5½d.
(27) 1493 at 5¾d.
(28) 749 at 6¼d.
(29) 1741 at 6½d.
(30) 349 at 6¾d.
(31) 547 at 7d.
(32) 374 at 7¼d.
(33) 5491 at 7½d.
(34) 1649 at 7¾d.
(35) 1498 at 8d.
(36) 749 at 8¼d.
(37) 4719 at 8½d.
(38) 1747 at 8¾d.
(39) 4954 at 9d.
(40) 7143 at 9¼d.
(41) 494 at 9½d.
(42) 374 at 9¾d.
(43) 471 at 10d.
(44) 3751 at 10¼d.
(45) 4967 at 10½d.
(46) 4971 at 11d.
(47) 5794 at 11¾d.

Examples to Rule IV.

(48) 4756 at 12¼d.

¼|1/48|4756
99 1

2|0)485|5 1

£242 15 1

(49) 321 at 12½d.
(50) 479 at 12¾d.
(51) 574 at 13d.
(52) 675 at 13¼d.
(53) 4949 at 13½d.

(54) 574 at 13¾d.
(55) 495 at 14d.
(56) 5714 at 14¼d.
(57) 371 at 14½d.
(58) 4714 at 14¾d.
(59) 3719 at 15d.
(60) 174 at 15¼d.
(61) 4749 at 15½d.
(62) 374 at 15¾d.
(63) 498 at 16d.
(64) 3714 at 16¼d.
(65) 5714 at 16½d.
(66) 494 at 16¾d.
(67) 3751 at 17d.
(68) 494 at 17¼d.
(69) 375 at 17½d.
(70) 5794 at 17¾d.
(71) 4954 at 18d.
(72) 371 at 18¼d.
(73) 579 at 18½d.
(74) 3751 at 18¾d.
(75) 479 at 19d.
(76) 371 at 19¼d.
(77) 471 at 19½d.
(78) 579 at 19¾d.
(79) 471 at 20d.
(80) 3741 at 20¼d.
(81) 494 at 20½d.
(82) 379 at 20¾d.
(83) 4981 at 21d.
(84) 375 at 21¼d.
(85) 3741 at 21½d.
(86) 495 at 21¾d.
(87) 5947 at 22d.
(88) 5931 at 22¼d.
(89) 432 at 22½d.
(90) 541 at 22¾d.
(91) 7194 at 23d.
(92) 5497 at 23¼d.
(93) 714 at 23½d.
(94) 4984 at 23¾d.
(95) 4935 at 23½d.
(96) 3714 at 23¾d.

Examples to Rule V.

(97) What coſt 425 pair of buckles at 6s. per pair?

Here ½ the price is 3s. and 425 multiplied by 3 gives 1275 for the product; double the firſt figure (viz. 5) for ſhillings, and let the 127 ſtand for pounds; then the anſwer will be 127l. 10s.—Had the buckles been 7s. per pair, then $\frac{1}{20}$th of 425 (viz. 21l. 5s.) muſt have been added to 127l. 10s. and the anſwer would have been 148l. 15s.

(98) 475 at 2s.
(99) 379 at 3s.
(100) 1754 at 4s.
(101) 1788 at 5s.
(102) 1789 at 6s.
(103) 414 at 7s.
(104) 5413 at 8s.
(105) 7194 at 9s.
(106) 344 at 10s.
(107) 794 at 11s.
(108) 427 at 12s.
(109) 149 at 13s.
(110) 371 at 14s.
(111) 495 at 15s.
(112) 3741 at 16s.
(113) 794 at 17s.
(114) 494 at 18s.
(115) 371 at 19s.

Examples to Rule VI.

(116) 3754 pair of gloves at 2s. 6d. per pair.

2s. 6d. | ⅛ | 3754

£469 5

(118) 660 at 2s. 6d.
(119) 663 at 4s. 10d.
(120) 924 at 13s. 4d.
(121) 712 at 6s. 8d.
(122) 512 at 7s. 6d.

(117) 3520 buſhels at 3s. 6d.

3

6d. | ½ | 10560
1760

2|0)1232|0

£616

(123) 1749 at 5s. 8d.
(124) 3741 at 4s. 6d.
(125) 493 at 3s. 2d.
(126) 741 at 5s. 9d.

Examples to Rule VII.

(127) 7341 at 2l. 6s.

7341
2

14682 value at 2l.
2202 6 value at 6s.

£16884 6 anſwer.

(129) 754 at 4l. 2s.
(130) 371 at 5l. 3s.
(131) 149 at 9l. 4s.
(132) 374 at 10l. 5s.
(133) 191 at 12l. 6s.
(134) 174 at 3l. 7s.
(135) 512 at 5l. 8s.
(136) 140 at 7l. 9s.
(137) 360 at 2l. 10s.

(128) 435 at 2l. 7s.

2

870 value at 2l.
130 10 value at 6s.
21 15 value at 1s.

£1022 5 anſwer.

(138) 344 at 2l. 11s.
(139) 192 at 3l. 12s.
(140) 351 at 4l. 13s.
(141) 412 at 5l. 14s.
(142) 372 at 2l. 15s.
(143) 741 at 1l. 16s.
(144) 314 at 1l. 17s.
(145) 471 at 1l. 18s.
(146) 374 at 19l. 19s.

Examples to Rule VIII.

(147) 4514 at 2l. 17s. 7½d.

2

9028 value at 2l.
3611 4 ditto at 16s.
6 | ½ | 225 14 ditto at 1s.
1½ | ¼ | 112 17 ditto at 6d.
28 4 3 ditto at 1½d.

£13005 19 3 anſwer.

(148) 471 at 5l. 14s. 9¼d.
(149) 3714 at 2l. 13s. 11¾d.
(150) 415 at 4l. 11s. 10½d.
(151) 341 at 5l. 13s. 9½d.
(152) 7494 at 10l. 17s. 10d½.
(153) 34121 at 11l. 14s. 8d¾.
(154) 7251 at 14l. 11s. 5½d.

Examples to Rule IX.

(155) $3749\frac{3}{8}$ at 3l. 15s. 6d.

```
   3l. 15s. 6d.  the price.                 3749
                                               3
   ──────────                             ──────
8)11   6   6  three times ditto.           11247
   ──────────                               2624  6
    1   8  3¾ 3-8ths of ditto.      6d.|½|  187   9
   ──────────                                 93 14  6
                                   ⅛ add      1  8  3¾
                                          ──────────
                                      £14153 17  9¾ anſwer.
```

Note. $\frac{1}{8}$ of 3 times the price is the ſame as 3 times $\frac{1}{8}$ of the price, or $\frac{3}{8}$, by the nature of fractions.

(156) $371\frac{1}{5}$ at 3l. 14s. $7\frac{1}{2}$d. (159) $4759\frac{1}{11}$ at 4l. 15s. $9\frac{3}{4}$d.
(157) $4917\frac{1}{8}$ at 4l. 18s. $10\frac{1}{2}$d. (160) $574\frac{3}{5}$ at 19l. 11s. 6d.
(158) $1375\frac{5}{9}$ at 2l. 19s. $11\frac{3}{4}$d. (161) $1749\frac{1}{12}$ at 4l. 19s. $10\text{d}\frac{1}{4}$.

Examples to Rule X.

(162) What is the value of 18cwt. 1qr. 11lb. of tobacco, at 6l. 19s. 11d. per cwt.?

```
                      £   s.  d.
1 qr. is ¼ cwt.       6  19  11
                              2
                     ───────────
                     13  19  10
                              9
                   ─────────────
                    125  18   6    value of 18 cwt.
  7 lb  |¼|           1  14  11¾   value of ¼ cwt.
  3½lb  |½|              8   8¾·75     ditto of 7lb.
  ½lb.  |7|              4   4¼·875    ditto of 3½lb.
                             7¼·982⅐   ditto of ½lb.
                   ─────────────────
                  £128    7  2½·607⅐  anſwer.
                   ─────────────────
```

(163) 19cwt. 3qr. 11lb. of hops, at 4l. 11s. 9d. per cwt.
(164) 19cwt. 3qr. 19lb. of ſugar, at 2l. 4s. 8d. per cwt.
(165) 11cwt. 1qr. 16lb. of ſoap, at 3l. 7s. per cwt.
(166) 9 cwt. 3qr. 10lb. of treacle, at 1l. 18s. 9d. per cwt.
(167) 9ton 13cwt. 3qr. 19lb. at 14l. 15s. 9d. per ton.
(168) 3qr. 19lb. 10 oz. at 11l. 12s. $5\frac{1}{2}$d. per cwt.
(169) 74 oz. 2dwt. 12gr. of ſilver, at 4s. $11\frac{1}{2}$d. per oz.
(170) A pair of chaſed ſilver ſalts, weight 7 oz. 11dwt. at 8s. $11\frac{1}{4}$d. per oz.

(171) 571 oz. 14dwt. 16½gr. at 3l. 11s. 9¼d. per oz.

(172) What is the rent of 725a. 2r. 19p. of land, at 2l. 11s. 9d. per acre?

(173) 51a. 3r. 15p. at 4l. 10s. per acre.

(174) 97a. 14p. at 3l. 11l. 10d. per acre.

(175) 514 yds 3qrs 2n. at 17s. 9½d. per yard.

(176) 125 ells Eng. 1qr. 1n. at 1l. 11s. 9½d. per ell.

(177) What coſt 17 French ells 1qr. 3n. of Bruſſels lace, at 3l. 19s. 11¼d. per ell?

(178) 349 Flem. ells 1qr. 3n. of holland, at 1l. 11s. 6d. per ell.

(179) 475yds 3qr. 2n. at 1l. 14s. 9¼d. per ell Engliſh.

(180) 375⅜ ells Engliſh, at 18s. 11¼d. per yard.

TARE AND TRET.

Definitions.

1. *Tare and Tret* are practical rules for deducting certain allowances made by merchants and tradeſmen in ſelling their goods by weight.

2. *Tare* is an allowance made to the buyer for the weight of the box, barrel, bag, cheſt, wrappers, &c.

3. *Tret* is an allowance of 4lb. in 104lb. for waſte, duſt, &c.

4. *Cloff*, or *draught*, is an allowance of 2lb. for every 3cwt. made by the ſeller to the buyer, that the weight may hold good when ſold by retail.

5. *Groſs Weight* is the whole weight of any ſort of goods, together with the box, barrel, bag, &c. that contains it.

6. *Suttle* is when part of the allowance is deducted from the groſs.

7. *Neat Weight* is what remains after all allowances are deducted.

Propoſition I.

When the Tare is at ſo much in the whole groſs weight to find the neat weight.

Rule. Subtract the tare from the groſs, and the remainder will be the neat weight.

Prop. 2. *When the tare is at ſo much per box, bag, barrel, &c. to find the neat weight.*

Rule. Multiply the number of boxes, bags, &c. by the tare, and ſubtract the product from the groſs.

Prop. 3. *When the tare is at ſo much per hundred weight, to find the neat weight.*

Rule. If the tare be an aliquot part of a *cwt.* divide the groſs weight by the aliquot part, and the quotient will be the tare to be deducted from the groſs. If the tare be not an aliquot part of a *cwt.* firſt take ſome aliquot part of a *cwt.* and then part of *that* part, &c. according to the nature of the queſtion, the ſum of the quotients belonging to theſe parts will be the whole tare, which deduct from the groſs.

Prop. 4. *The groſs weight of any ſort of merchandiſe given to find the neat weight when the tret is allowed with tare.*

Rule. Find the tare, as before, and ſubtract it from the groſs, the remainder will be the *ſuttle.* Then, divide the ſuttle by 26, and the quotient will be the tret, which deduct from the ſuttle.

Prop. 5. *The groſs weight of any ſort of merchandiſe given to find the neat weight, when tare, tret, and cloff, are allowed.*

Rule. Find the neat weight by the laſt rule, and call that the *ſecond* ſuttle. Then divide the ſecond ſuttle by 168, and the quotient will be the cloff, which deduct from the ſecond ſuttle.

Note 1. The above rule will only give the neat weight when cloff is 2 lb. for every 3 cwt. which is generally the caſe. The 168 is found by dividing 3 cwt. or 336 lb. by 2; hence it will be very eaſy to find a diviſor when any other allowance of cloff is made. At the Cuſtom-houſe, the following allowances are made upon goods imported, viz. 1 lb. upon goods not weighing leſs than 1 cwt.—2 lb. from 1 to 2 cwt.—3 lb. from 2 to 3 cwt.—4 lb. from 3 to 10 cwt.—7 lb. from 10 to 18 cwt.—and 9 lb. from 18 to 30 cwt. and upwards.

2. There are other allowances, ſuch as *break,* which is ſometimes at ſo much per hhd, bag, &c. and *damage,* which is ſo much in the whole for any part of the merchandiſe which may have received injury.

Examples to Propoſition 1.

(1.) What is the neat weight of 6 hhds of tobacco, each weighing 12 cwt. 3qr. 11lb. groſs, tare in the whole 854lb.?

12cwt. 3qr. 11lb.
6

77 0 10 whole groſs weight.
7 2 14 tare.

69 1 24 neat weight.

28)854

4)30 - 14

7 2 14

(2.) Required the neat weight of 27 bales of ſilk, each weighing $349\frac{1}{2}$lb. groſs; tare in the whole 3cwt. 1qr. 15lb.

(3.) Required the neat weight of 29 hhds of tobacco, each weighing 14cwt. 3qr. 17lb. groſs; tare in the whole 1547lb.

(4.) In 43 bags of cotton, each weighing 3cwt. 1qr. $11\frac{1}{4}$lb. groſs, tare in the whole $77\frac{1}{2}$lb. what is the neat weight?

(5.) What is the neat weight of 4 hhds of ſugar, weighing as follows, viz.

No.		Cwt.	qr.	lb.		lb.
1	-	5	3	11	Tare	25
2	-	4	1	10	—	18
3	-	7	2	14	—	37
4	-	9	1	24	—	53

Examples to Prop. 2.

(6.) What is the neat weight of 12 hhds of tobacco, each weighing 5cwt. 3qr. 14lb. groſs; tare per hhd 97lb.?

cwt. qr. lb.
5 3 14
12

70 2 0 whole groſs weight.
10 1 16 tare.

60 0 12 neat weight.

lb.
97
12

28)1164

4)41 - 16

10 16

(7.) Required the neat weight of 19 caſks of indigo, each weighing 4cwt. 1qr. 19lb. groſs; tare per caſk 37lb.

(8.) Required the neat weight of 47 hhds of tobacco, weighing 147cwt. 1qr. 11lb. groſs; tare 75lb. per hhd.

(9.) In 19 bags of pepper, each 84½lb. groſs; tare per bag 4¼lb. how many lbs. neat?

(10.) In 75 bales of ſilk, each weighing 254lb. groſs, tare per bale 14lb. how many lbs. neat?

(11.) What is the neat weight of 354 barrels of figs, each weighing 124lb. groſs; tare 11lb. per barrel?

Examples to Prop. 3.

(12.) What is the neat weight of 7 barrels of figs, each weighing 2cwt. 1qr. 12lb. groſs; tare 21lb. per cwt.?

		cwt.	qr.	lb.	
		2	1	12	
				7	
14lb.	⅛	16	2	0	groſs.
7lb.	½	2	0	7	tare at 14lb. per cwt.
		1	0	3½	ditto at 7lb. per cwt.
		3	0	10½	whole tare.
		13	1	17½	neat weight.

(13.) Required the neat weight of 29 barrels of pot-aſh, each weighing 1cwt. 3qr. 18lb. groſs; tare 12lb. per cwt.

(14.) Required the neat weight of 15 caſks of argol, weighing groſs 97cwt. 2qr. 15lb. tare 15l. per cwt.?

(15.) Required the neat weight of 19 barrels of anchovies, each weighing 35lb. groſs; tare 11½lb. per cwt.

(16.) Required the neat weight of 17 hhds of tobacco, each weighing 4cwt. 3qr. 14lb. groſs; tare 19lb. per cwt.

Examples to Prop. 4.

(17.) In 7 hhds of ſugar, weighing groſs 47cwt. 2qr. 4lb.; tare in the whole 10cwt. 2qr. 14lb.; tret 4lb. per 104, how much neat weight?

	cwt.	qr.	lb.	
	47	2	4	groſs.
	10	2	14	tare.
26)	36	3	18	*ſuttle.*
	1	1	19	tret.
	35	1	27	neat.

(18.) How much neat weight is contained in 12cwt. 3qr. 19lb. groſs; tare in the whole 37lb.; tret 4lb. per 104?

(19.) Required the neat weight of 19 cheſts of ſugar, each weighing 7cwt. 3qr. 19lb. groſs; tare 12lb. per cwt. tret 4lb. per 104.

(20.) Suppoſe $19\frac{1}{4}$lb. per cwt. tare, and 4lb. per 104lb. tret, were allowed on 19 caſks of prunes, each 4cwt. 1qr. 14lb. groſs, what would be the neat weight?

Examples to Prop. 5.

(21.) Required the neat weight of 45 hhds of tobacco, weighing groſs 224cwt. 3qr. 20l.; tare 25cwt. 3qr.; tret 4lb. per 104lb.; cloff 2lb. for every 3 cwt.

	cwt.	qr.	lb.	
	224	3	20	groſs.
	25	3	0	tare.
26)	199	0	20	*ſuttle.*
	7	2	18	tret.
168)	191	2	2	*ſecond* ſuttle.
	1	0	$15\frac{19}{28}$	cloff.
	190	1	$14\frac{9}{28}$	neat.

(22.) In 7 hhds of tobacco, each weighing groſs 5cwt. 3qr. 17lb.; tare 11lb. per cwt.; tret 4lb. per 104; cloff 2lb. for every 3cwt. how much neat weight?

(23.) The neat weight of 5 caſks of currants is required, each weighing 7cwt. 3qr. 11lb. groſs; tare 2qr. 11lb. per caſk; tret 4lb. per 104lb.; and cloff 2lb. per 336lb.

CLASS II. *exerciſing all the Propoſitions.*

(24.) Bought 19cwt. 1qr. 27lb. groſs of tobacco in leaf, at 5l. 0s. 4d. per cwt. neat, and 12cwt. 3qr. 19lb. groſs in rolls, at 5l. 17s. 8d. per cwt.; the tare of the former was 149lb. and the latter $48\frac{1}{2}$lb.; what did the tobacco ſtand me in?

(25.) Bought $17\frac{1}{4}$ hhds of ſugar, each 10cwt. 1qr. 14lb.; tare 7lb. per cwt; tret 4lb. per 104lb.; what is the value at 1l. $12\frac{2}{3}$s. per cwt. neat?

(26.) Bought 7 hogſheads of treacle, each weighing 4cwt. 3qr. 17lb. groſs; tare 17lb. per cwt.; break 8lb. per hhd; and damage in the whole, $99\frac{1}{4}$lb.; what is the value at 1l. 17s. 6d. per cwt. neat?

(27.) In 29 parcels, each weighing 3cwt. 3qr. 14lb. groſs; tare 8lb. per cwt.; tret 4lb. per 104lb; and cloff 2lb. per 3cwt; how much neat weight, and what is the value at a guinea and a half per cwt. neat?

(28.) The neat value of a hhd of Barbadoes ſugar was 4l 14s 6d. the cuſtom and fees 2l 11s 4d. freight 1l 1s 6d. factorage 5s 9d. the groſs weight was 11cwt 1qr 15lb. tare $11\frac{1}{3}$lb. per cwt.; pray what was the ſugar rated at per cwt. neat in the bill of parcels?

(29.) In 7 hhds of oil, each weighing 3cwt 2qr 14lb. groſs; tare 21lb. per cwt. how many gallons neat, and what is the value at 5s. 4d. per gallon?

(30.) I have imported 87 jars of Lucca oil, each containing 57 gallons; what came the freight to at 5s 3d. per cwt. neat, reckoning 1lb. in 11lb. for tare, and $7\frac{1}{2}$lb. of oil to a gallon?

INTEREST.

Definition 1. *Intereſt* is the premium, or money, which one perſon allows to another for the uſe of any ſum of money for a determinate ſpace of time.

2. *The principal* is the money lent.

3. *The rate per cent.* is a certain ſum, agreed on between the borrower and the lender, to be paid for the uſe of every £100 in the principal for a year.

4. *The amount* is the principal and its intereſt added together.

SIMPLE INTEREST.

Definition. *Simple Intereſt* is the money ariſing from the principal only, though ſuch intereſt ſhould remain unpaid for any number of years; thus, if the intereſt of £100 for 1 year be £4, it will be £8 for 2 years, &c. or £2 for half a year, £1 for a quarter of a year, &c.

Propoſition 1. *To find the intereſt of any ſum of money, aving the principal, the time of its continuance in years, and he rate per cent. given.*

Rule. Multiply the principal by the rate *per cent.* that roduct, divided by 100, will give the intereſt for one ear. Then, if the intereſt for one year be multiplied by he number of years given in the queſtion, the product ill be the intereſt for *that* time. Or, multiply the principal by the rate per cent. and that product by the time; he laſt product divided by 100 will give the intereſt required.

Note 1. If there be any parts annexed to the whole years, as $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, &c. after you have found the intereſt for the number of years, add $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, &c. of one year's intereſt to it.

2. If the rate of intereſt have any part or parts, annexed to it, as $\frac{1}{4}$, or $\frac{3}{4}$, &c. after you have multiplied the principal by the whole number, take the reſpective part, or parts, of the principal, which add to the product, and proceed for the given time as above.

3 If the rate per cent. is an aliquot part of 100, or if it can be divided into convenient aliquot parts, take the ſame part or parts, of the principal for the intereſt of one year.

Prop. 2. *To find the intereſt of any ſum of money, having the principal, the time of its continuance in days, and the rate per cent. given.*

Rule. As 365 days are to the intereſt of the given ſum for a year, ſo are the days given to the intereſt required.

Or, reduce the principal into the loweſt denomination contained in it, then multiply it by the number of days, and *that* product by the rate *per cent.* for a dividend: let this dividend be divided by 36500, and the quotient will be the anſwer in the ſame denomination as the principal was reduced to.

Note. If the intereſt of a ſum of money is required for any number of weeks, reduce them into days, and proceed as above; or, as 52 weeks are to the intereſt of the given ſum for a year; ſo are the weeks given to the intereſt required.

Prop. 3. *To find the intereſt of any ſum of money, having the principal; the time of its continuance in years and months, or years, months, and days; and the rate per cent. given.*

Rule. Find the intereſt for the years by the firſt rule, work for the months by the aliquot parts of a year, and for the days by the aliquot parts of a month, reckoning 12 months to a year, and 30 days to a month.

Note. Though the rule to Prop. 3, be not preciſely accurate, yet it will be found not leſs uſeful than the others which are ſo; for, in ſome caſes, it is cuſtomary to conſider the time elapſed different ways. Thus, in the courts of law, intereſt is always calculated in years, quarters, and days; but, in calculating the intereſt on the public bonds of the South-ſea and India Companies, and in the Bank of England, &c. the time is generally taken in calendar-months and days; and on Exchequer-bills in quarters of a year and days.

Prop. 4. *When the amount, time, and rate per cent. are given to find the principal.*

Rule. As the amount of £100, at the rate and for the time given, is to £100, ſo is the amount given to the principal.

Prop. 5. *When the amount, principal, and time, are given to find the rate per cent.*

Rule. As the principal is to its intereſt, for the whole time, ſo is £100 to *its* intereſt for the ſame time; divide this intereſt by the time and the quotient will be the rate *per cent.*

Prop. 6. *When the principal, rate per cent. and amount, are given to find the time.*

Rule. As the intereſt of the principal for one year, at the given rate, is to one year, ſo is the whole intereſt to the time required.

Examples to Propoſition 1.

(1.) What is the intereſt of 357l 10s. for 3 years, at 5 *per cent. per annum?*

357l. 10s. principal.
5 rate per cent.
£17,87 10
20
s. 17,50
12
d. 6,00

£ s. d.
17 17 6
3
56 12 6 *Anſ.*

Or, thus,
5l. is $\frac{1}{20}$ | 357l. 10s.
Intereſt for 1 year £17 17 6
3
Intereſt for 3 years £53 12 6

Or thus,
£357 10 principal.
5 rate per cent.
1787 10
3 time
£ 53,62 10
20
s. 12,50
12
d. 6,00

Anſw. £53 12 6

(2.) Required the intereſt of 349l 10s. for 7 years, at per cent. per annum.

(3.) Required the intereſt of 429l 11s 6d. for 6 years, at per cent. per annum.

(4.) What is the intereſt of 625l 15s. for $3\frac{1}{2}$ years, at per cent. per annum?

(5.) What is the intereſt of 494l 13s 9d. for $5\frac{3}{4}$ years, t 5 per cent. per annum?

(6.) Required the intereſt of 700 guineas, for 9 years, t $4\frac{1}{4}$ per cent. per annum.

(7.) Required the intereſt of 420l. for $7\frac{1}{4}$ years, at $3\frac{1}{2}$ er cent. per annum.

(8.) Required the intereſt of 500l 15s. for $5\frac{1}{2}$ years, at $4\frac{1}{4}$ per cent. per annum.

(9.) Required the intereſt of 97l 18s 6d. for $3\frac{3}{4}$ years, t $4\frac{3}{4}$ per cent. per annum.

Examples to Prop. 2.

(10.) Required the intereſt of 357l 10s. for 65 days, at 5 per cent. per annum.

The intereſt for 1 year, by the firſt example, is 17l. 17s. 6d.
Then, as 365 days : 17l. 17s. 6d. :: 65 days to 3l. 3s. $7\frac{1}{4}$d. $\frac{65}{73}$.

Or thus,

The principal reduced, to the loweſt term mentioned in it, is 7150 ſh. which multiply by 5, the rate per cent. and then by 65, the number of days, and the laſt product will be 2323750 ſh. for a dividend, which divide by 36500, after the manner of compound diviſion, and the quotient will be 63s. $7\frac{1}{4}$d. $\frac{65}{73}$, or 3l. 3s. $7\frac{1}{4}$d. $\frac{65}{73}$ as above.

(11.) Required the intereſt of 194l 11s 6d. for 315 days, at $4\frac{1}{2}$ per cent. per annum.

(12.) What is the intereſt of 700l. for 149 days, at $4\frac{3}{4}$ per cent. per annum?

(13.) Required the intereſt of 494l 12s 10d. for 29 weeks, at 5 per cent. per annum.

(14.) Required the intereſt of 347l 10s. for 18 weeks, at 4 per cent. per annum.

(15.) Required the intereſt of 540l 10s. from Jan. 7, 1799, to Sept. 22, in the ſame year, at 4 per cent. per annum.

(16.) What is the intereſt due on an *Exchequer*-Bill of 400l. value, at $3\frac{1}{2}$ per cent. per annum, for $2\frac{1}{2}$ years, and 59 days?

(17.) Required the intereſt due upon an *Exchequer*-Bill of 100l. value, for 294 days, reckoning the intereſt at 3d. per day.

Examples to Prop. 3.

(18.) Required the intereſt of 342l 10s. for 3 years, 4 months, and 15 days, at 4 per cent. per annum.

342l. 10s.
4

13|70 0
20

14|00

4m.	$\frac{1}{3}$	13l. 14s.		intereſt for 1 year.
		3		
		41 2		intereſt for 3 years.
15d.	$\frac{1}{8}$	4 11	4	intereſt for 4 months.
		11	5	intereſt for 15 days.
		46 4	9	*anſwer.*

(19.) Required the intereſt of 500 guineas for 5 years, 9 months, and 27 days, at $4\frac{3}{4}$ per cent. per annum.

(20.) What is the intereſt due upon an India Bond of 500l. value, at $3\frac{3}{4}$ per cent. per annum, from May 15, 1787, to September 22, 1789?

(21.) Sold an India Bond, of 100l. value, with Intereſt due thereon, for 2 months 17 days, at 4 per cent. per annum, premium 10s. what is its value?

(22.) A gentleman left his daughter, by will, 875l 10s. to be paid her when ſhe is 21 years of age, with intereſt at 5 per cent. per annum. Now ſhe was 18y. 7m. 3d. old at her father's deceaſe, reckoning 12 months to a year, and 30 days to a month. Pray what will be the amount of her fortune when ſhe comes of age?

Examples to Prop. 4.

(23.) What principal, put to intereſt for ſeven years at 5 per cent. per annum, will amount to 465l 8s 3d?

5l. intereſt of 100l. for 1 year.
7 time.

35 intereſt of 100l. for 7 years.
100

135 amount of 100l. at 5 per cent. per annum, for 7 years.
As 135l. : 100l. :: 465l. 8s. 3d. to 344l. 15s. anſwer.

(24.) What principal, put to intereſt for 5 years, will amount to 570l 16s 6d. at 4 per cent. per annum?

(25.) What principal, put to intereſt for $3\frac{3}{4}$ years, at $4\frac{1}{2}$ per cent. per annum, will amount to 205l 11s. $7\frac{3}{4}$d.$\frac{4}{5}$?

(26.) What principal, put to intereſt for $4\frac{1}{2}$ years, will amount to 350l 12s 6d. at $3\frac{3}{4}$ per cent. per annum?

Examples to Prop. 5.

(27.) At what rate per cent. will 475l 13s 9d. amount to 570l 16s 6d. in 5 years time?

570l. 16s. 6d. amount.
475 13 9 principal.

95 2 9 intereſt.

As 475l. 13s. 9. : 95l. 2s. 9d. :: 100l. to 20l.
This 20l. divided by 5, the number of years, gives 4l. the rate per cent.

(28.) At what rate per cent. will 344l 15s. amount to 465l 8s 3d. in 7 years time?

(29.) At what rate per cent. will 175l 18s. amount to 205l 11s $7\frac{3}{4}$d.$\frac{4}{5}$ in $3\frac{3}{4}$ years?

(30.) At what rate per cent. will 300l. amount to 350l 12s 6d. in $4\frac{1}{2}$ years?

Examples to Prop. 6.

(31.) In what time will 344l 15s. amount to 465l 8s 3d. at 5 per cent. per annum?

344l. 15s. principal.
5 rate per cent.

£ 17|23 15
20

s. 4|75
12

d. 9|00

465l. 8s. 3d. amount.
344 15 0 principal.

120 13 3 whole intereſt.

As 17l. 4s. 9d. : 1 year :: 120l. 13s. 3d. to 7 years, anſw.

(32.) In what time will 475l 13s 9d. amount to 570l 16s 6d. at 4 per cent. per annum?

(33.) In what time will 175l 18s. amount to 205l 11s $7\frac{3}{4}$d.$\frac{4}{5}$, at $4\frac{1}{2}$ per cent. per annum?

(34.) In what time will 300l. amount to 350l 12s 6d. at $3\frac{3}{4}$ per cent. per annum?

CLASS II. *Promiſcuous Queſtions.*

(35.) A young gentleman, whoſe father has been dead 12 years, is informed by his guardians that his fortune amounts in caſh to 26416l. and that they have allowed him 5 per cent. ſimple intereſt for the uſe of the money his father left him, except 100l. which was deducted annually for his education; if the gentleman is now 21 years of age, pray what money did his father leave him?

(36.) Lent my friend 20l. October 21, 1782; on the 22d of May, 1784, I borrowed of him 150l. and on July 30, in the ſame year, 150l. more. On July 21, 1785, I paid him 15l 18s.—on Auguſt 21, 40l.—on October 21, 50l.—on February 13, 1786, I paid 9l 12s.—on June 13, 111l.—and on January 13, 1787, 80l. How ſtood our account at that period, allowing 5 per cent. ſimple intereſt for the money?

(37.) Lent 500 guineas at 4½ per cent. per annum, which by the 25th of September, 1788, was raiſed by the intereſt to 700l 15s. Pray on what day and in what year did I lend the money?

(38.) If 100l. in 11 years gain 38l 10s. in what time would any other ſum gain as much intereſt as will make its amount 5 times the principal?

(39.) What difference is there between the intereſt of 500l. for 4¾ years, at 5 per cent. and half that ſum for twice the time, at half the ſame rate per cent.?

(40.) Lent *William Adamſon* per bill, (dated Auguſt 1, 1786) payable 2 months after date, 957l 18s. which I received as follows, *viz.* October 5, 94l 17s. November 27, 47l 19s 6d. December 15, 100 guineas; January 1, 1787, 55l 11s 4d. March 15, 101l 14s. May 12, 105 guineas; Auguſt 19, 140l 2s 6d. September 11, 50l 6d. and on March 15, 1788, I received the balance of the principal. Pray what intereſt ought I to claim at 4 per cent.?

BROKAGE, or *Brokerage*.

Definition. *Brokage* is an allowance of ſo much *per cent.* made to perſons called Brokers; who, from their knowledge of merchants and the different branches of commerce, are generally employed in buying or ſelling goods or others.

Propoſition 1. *To find what allowance muſt be made to a roker for buying or ſelling goods, having the rate per cent. and alue of the goods, &c. given.*

Rule 1. Divide the given ſum by 100, and take parts rom the quotient with the rate *per cent.*

Or, 2. As £100, is to the rate per cent. ſo is the given um, to the brokage.

Note. The allowances made to brokers are generally at 2s. or 2s. 6d. er cent. but, ſhould the brokage ſo far accumulate, from repeated negotiations, as to exceed 20s. per cent. it muſt be calculated by the following rule of commiſſion, or by the ſecond rule given above.

Examples.

(1.) Suppoſe I employ a broker to ſell goods for me to the amount of 715l. 15s. what is his allowance at 3s. 9d. per cent.?

```
     £  s.                    £   s.  d.
   £7|15 15        |2 6|1/3|7   3  1¼  ·2
       20          |   |   |-------------
   ------          |1 3|1/2|   17 10½  ·9
   s. 3|15         |   |   |    8 11¼ ·45
       12                  -------------
     ----                  £1   6 10  ·35 anſwer.
   d. 1|80                 ---------------
        4
     ----
   f. 3|20
```

Or thus,

As 100l. : 3s. 9d. :: 717l. 15s. : 1l. 6s. 10d. $\frac{7}{20}$ anſwer.

(2.) When a broker ſells goods to the amount of 7134l 15s 10. what may he demand for brokerage, if he is allowed 5s 9d. per cent.?

(3.) Suppoſe I employ a broker to ſell goods for me to the amount of 1057l 17s. what may he demand for brokerage, if I allow him 4s 7d. per cent.?

(4.) What is the brokerage of 3759l 17s 6d. at 19s 9¼d. per cent.?

Class II.

(5.) If a broker ſells goods to the value of 750l. 19s. at an allowance of $\frac{5}{8}$l. per cent. how much is due to him?

(6.) Required the brokerage of 2947l. 15s. 6d. at $\frac{3}{8}$l. per cent.

COMMISSION.

Definition. *Commiſſion* is an allowance made by merchants to their factors, or agents, in foreign countries, for buying or ſelling goods; and is generally at a certain rate *per cent.* according to the cuſtom of the country where the factors reſide.

Propoſition. To find what allowance muſt be made to a factor at any rate per cent. having the ſum given, from which his commiſſion is to be taken.

Rule 1. Multiply the ſum by the rate per cent.; the product, divided by 100, will give the commiſſion.

Or, 2. As £100, is to the rate per cent. ſo is the given ſum, to the commiſſion.

Note. If the rate per cent. be leſs than 20s. proceed by the preceding rule for brokerage, or by the ſecond rule given above.

Examples.

(1.) If I empower my factor to purchaſe goods for me to the amount of 500l. 14s. what does his commiſſion come to at $2\frac{1}{2}$ per cent.?

500l. 14s.
$2\frac{1}{2}$

1001 8
250 7

l. 12|51 15
20

s 10|35
12

d. 4|20
Anſwer 12l. 10s. $4\frac{1}{5}$d.

Or thus,
£$2\frac{1}{2}$|$\frac{1}{40}$|500l. 14s.

£12 10 $4\frac{1}{5}$

Or,
If 100l. : 2l. 10s. :: 500l. 14s. to 12l. 10s. $4\frac{1}{5}$d. anſwer.

(2.) My factor informs me that he has bought goods, n my account, to the amount of 757l. 14s. what comes is commiſſion to at $3\frac{1}{4}$l. per cent.?

(3.) My factor informs me that he has ſold goods on my ccount to the amount of 500l. 17s. what comes his comiſſion to at $1\frac{3}{8}$ per cent.?

(4.) Conſigned goods to my factor, as per invoice, to he amount of 1175l. 14s. what does his commiſſion come o at $4\frac{3}{8}$ per cent.?

CLASS II.

(5.) If I allow my factor $7\frac{5}{8}$ per cent. for commiſſion, hat may he demand for purchaſing goods for me to the mount of 977l. 18s.?

(6.) What does the commiſſion of 7497l. 15s. come to t $12\frac{7}{8}$ per cent.?

INSURANCE.

Definition. *Inſurance* is a ſecurity given in conſideration of a premium of ſo much per cent paid down by the proprietors of goods, &c. to the inſurers, whereby they engage to anſwer for the loſs or damage of ſhips, houſes, goods, &c. by ſtorms, fires, or other accidents.

Propoſition. To find what premium muſt be given for an Inſurance of property, to any amount, at any rate per cent.

Rule I. Multiply the value of the property by the rate *per cent.* the product, divided by 100, will give the premium to be paid down. If the rate *per cent.* be leſs than 20s. divide the value of the property by 100, and take parts from the quotient with the rate *per cent.*

Or, II. As £100, is to the rate per cent. ſo is the given ſum, to the premium.

Examples.

(1.) What premium muſt be paid for an inſurance of goods to the amount of 500l. 14s. at $2\frac{1}{2}$ per cent.? *Anſwer* 12l. 10s. $4\frac{1}{5}$d.

This example is the ſame as the firſt in Commiſſion, and muſt be worked in the ſame manner.

(2.) What premium muſt be paid for inſuring goods to the amount of 715l. 15s. at 3s. 9d. per cent.? *Anſw.* 1l. 6s. 10d.

This example is the ſame as the firſt in Brokerage, and muſt be worked in the ſame manner.

(3.) What premium muſt be given as a pledge for the inſurance of an Eaſt-India ſhip and cargo, valued at 47575l. 18s. when the rate of inſurance is 17$\frac{7}{8}$ per cent.?

(4.) Shipped off goods for *Jamaica* to the value of 4794l. 18s. when the rate of inſurance was 11$\frac{5}{8}$ per cent. what premium muſt be paid in London for an inſurance to recover the ſaid value in caſe of failure of the voyage?

CLASS II.

(5.) When the inſurance of goods to a certain port is 15$\frac{3}{4}$ per cent. what premium muſt be given as a pledge for the ſecurity of goods to the amount of 7000 guineas?

(6.) Suppoſe I inſure goods to the amount of 300l. 18s. what premium muſt I pay at the rate of 2s. 6d. per cent.?

(7.) My factor at *Barbadoes* conſigns goods to me, amounting to the value of 579l. 15s. 6d. what premium muſt I pay for an inſurance of thoſe goods at 11$\frac{3}{8}$ per cent.?

PURCHASING OF STOCK.

Definition. *Stock* is a general name for the capitals of our trading companies, and the money borrowed by government, at ſo much per cent. to defray the expences of the nation.

Propoſition. *To aſcertain the value of any quantity of ſtock at any given rate per cent.*

Rule. If the current price of the ſtock to be transferred be under par, viz. leſs than £100, multiply the ſtock by the rate *per cent.* the product, divided by 100, will give the purchaſe. If the price of the ſtock be above par, multiply the quantity to be transferred by ſuch part of the rate *per cent.* as exceeds 100; divide this product by 100 as before, to which add the given ſtock for the whole purchaſe.

Or, As £100 ſtock is to the rate *per cent.* or current price, ſo is the ſtock to be transferred to its current value.

Note. The principal trading companies in England are the Eaſt-India and South-ſea companies. Every capital ſtock, or fund of a company, is raiſed for ſome particular purpoſe, and limited by parliament to a certain ſum; it therefore follows, that, when that ſum is completed, no ſtock can be bought of the company; yet the ſhares already purchaſed may be transferred from one perſon to another. The government annuities, and other ſecurities of money, which has at any time been raiſed by the authority of parliament for the public ſervice, are to be conſidered as national debts, contracted on the credit of ſome certain tax; various intereſts for which debts are annually paid to the public from the produce of the taxes; and muſt continue to be ſo paid till theſe debts are redeemed or paid off, by the ſame authority by which they were contracted. This plan of raiſing money for the exigencies of the ſtate, commenced ſoon after the Revolution in 1688, and is the eaſieſt and beſt method of raiſing money, both for the ſubject and the ſtate, when managed with economy and prudence. We know, from experience, that taxes, laid on ſuch articles as could well ſupport the weight of them, have produced conſiderable ſurpluſſes; that is, they have amounted to more than the abſolute ſecurity engaged for; hence foreigners as well as natives have been induced to advance their money on ſo ſaſe a foundation. Theſe ſurpluſſes, after payment of the intereſt they ſtand charged with, are carried to a ſeparate and diſtinct account, known by the name of the *Sinking Fund.* This fund was to be kept *moſt ſacredly* for the valuable purpoſe of leſſening, or *ſinking*, and paying off gradually the national debt, by an act of George I. anno 1716; and had not this act been rendered ineffectual by ſubſequent acts, the national debt, and conſequently the taxes raiſed to pay off its intereſt, could never have amounted to that enormous height, which we now find them. The prices of ſtocks are continually fluctuating above and below *par*; for, if there are more buyers than ſellers, a perſon, who is indifferent about ſelling, will not part with his ſhare without a conſiderable profit: and, on the contrary, if many are diſpoſed to ſell, and few inclined to buy, the value of ſuch ſhares will naturally fall. So, when a perſon, who is unacquainted with tranſactions of this nature, reads in the papers the price of ſtocks, where bank-ſtock is marked perhaps 140½, India ſtock 155½, South-ſea ſtock 97⅜a⅝, he is to underſtand that £100 of thoſe reſpective ſtocks ſell at ſuch a time for thoſe ſeveral ſums. In comparing the prices of the different ſtocks one with another, it muſt be remembered, that the intereſt due on them from the laſt payment is taken into the current price, and the ſeller never receives any ſeparate conſideration from it, except in the caſe of India bonds, where the intereſt due is calculated to the day of ſale, and paid by the purchaſer over and above the premium agreed for. But, as the intereſt on the different ſtocks is paid at different times, this, if not rightly underſtood, would lead a perſon, not well acquainted with them, into conſiderable miſtakes in his calculation of their value; ſome always having a quarter's intereſt due on them more than others, which makes an appearance of a conſiderable difference in their price, when, in reality, there is none; thus, for inſtance, Old South-ſea Annuities ſell for $85\frac{1}{2}$, or £85l. 10s. while New South-ſea Annuities fetch only £$84\frac{3}{4}$, or 84l. 15s. though each of them produce the ſame annual ſum of 3 per cent. but the Old Annuities have a quarter's more intereſt due on them than the New Annuities, which amounts to 15s. the exact difference. If a perſon purchaſe a *nominal* £100 in the 3 per cents. for £60, he makes 5 per cent. of his money; for

£60 : £3 :: £100 : £5.

Examples.

(1.) What muſt be given for 750l. 16s. in the 3 per cent. annuities, when $64\frac{1}{8}$l. will buy 100l.?

$|\frac{1}{8}|$ 750l. 16s.
8

6006 8
8

48051 4
93 17

£481|45 1
20

s. 9|01

Anſwer. 481l. $9\frac{1}{100}$s.

Or thus,

As 100l. : $64\frac{1}{8}$l. :: 750l. 16s. : 481l. $9\frac{1}{100}$s.

(2.) What is the purchaſe of 540l. 16s. Bank-ſtock, at $112\frac{5}{8}$ per cent.?

Here the rate exceeds 100l. by $12\frac{5}{8}$.

540l. 16s.
$12\frac{5}{8}$

6489 12
$\frac{5}{8}$ = 338 0

£68|27 12
20

s. 5|52
12

d. 6|24 = $\frac{6}{25}$.
£68 5 $6\frac{6}{25}$.
540 16 0 add.

609 1 $6\frac{6}{25}$ anſw.

Or thus,

As 100. : $112\frac{5}{8}$l. :: 540l. 16s. : 609l 1s. $6\frac{6}{25}$d.

(3.) What is the purchaſe of 7575l. 15s. Bank-ſtock, at $125\frac{3}{8}$ per cent.?

(4.) Required the purchaſe of 900l. South-ſea ſtock, at $89\frac{1}{4}$ per cent.

(5.) What muſt be given for 1759l. 18s. 9d. India ſtock, when $196\frac{1}{4}$l. will purchaſe 100l.?

CLASS II.

(6.) Bought 5000l. capital ſtock in the 3 per cent. conſolidated annuities, and paid brokerage $\frac{1}{8}$ per cent. on the capital, what was the purchaſe at $85\frac{3}{8}$ per cent.?

(7.) What is the value of 759l. 10s. South-ſea old annuities, at $64\frac{1}{4}$ per cent. brokerage $\frac{1}{8}$ per cent?

(8.) On February 28, 1784, a gentleman employed a oker to purchaſe 70,000l. 3 per cent. annuities, for the *ſcounters* in May, at 61 per cent.—ſome little time after, e *honeſt* broker informs his employer that the 3 per cents. e fallen 4 per cent. below the price he bought at, and that e houſe are *bulls* for the *reſcounters*; upon which the gen-eman orders him to ſell out, on May 25, at 56$\frac{1}{2}$ per cent. hat did he loſe by this buſineſs, allowing the broker $\frac{1}{8}$ per nt. for buying and ſelling?

DISCOUNT.

Definition. Diſcount, or *Rebate*, is an allowance made r the payment of any ſum of money before it becomes e: and the preſent worth of any ſum, or debt, is ſuch a m as if put to intereſt for the time, and at the rate for hich the diſcount is to be made, would amount to the m, or debt, due.

Propoſition. Any ſum, due ſome time hence, being given to find s preſent value to the creditor, diſcounting at any rate per cent.

Rule. As the amount of £100, for the given rate and ime, is to £100, ſo is the given ſum to its preſent worth. he difference between the given ſum and its preſent value ill give the diſcount.

Or, As the amount of £100, for the given rate and ime, is to the intereſt of £100 for *that* time, ſo is the iven ſum to the diſcount. The difference between the iven ſum and its diſcount will give the preſent value.

. Note. The preceding rule is built upon this baſis, viz. that the *preſent worth* of any ſum of money, due ſome time hence, put to intereſt for the time, for which the diſcount is to be made, ſhould amount to the ſum, or debt, due: and that the *diſcount*, put to intereſt for the ſame time, ſhould amount to the intereſt of the ſum due for that time.

2. Thus, the *preſent worth* of £100, due one year hence, diſcounting at the rate of 5 per cent. is £95 4s. 9$\frac{1}{7}$d. and the *diſcount* of £100 for one year, at the rate of 5 per cent. is 4l. 15s. 6d.$\frac{6}{7}$d. according to the rule. Now, if the creditor ſhould put the preſent money allowed him (viz. £95 4s. 9$\frac{1}{7}$d.) to intereſt, at the rate of 5 per cent. for 1 year, it will amount to £100 exactly, and therefore he is not injured: again, if the debtor puts the diſcount allowed *him* (viz. £4 15s. 2$\frac{6}{7}$d.) to intereſt, at the rate of 5 per cent. for one year, it will amount to £5, the exact ſum which he might have made of the £100, had he kept it in his hands till it became due.

3. It is cuſtomary with bankers, in diſcounting bills, to calculate th intereſt of the ſum drawn for, in the bill, from the time of their diſcounting it to the time it becomes due, including the days of grace.—By thi practice they make the diſcount more than it ought to be.

4. When goods are bought or ſold, on which diſcount is to be made fo preſent payment at any rate per cent. if no time is ſpecified, the intereſt of the value of the goods for a year is the diſcount.

5. When goods are ſold to any amount, payable at different times, at the ſame or different rates per cent. calculate the preſent worth of each payment ſeparately, as a debt independent of the other payments, and the ſum of theſe will be the preſent value of the goods to the ſeller.

Examples.

(1.) What is the preſent worth and diſcount of 550l. 10s. for 9 months, at 5 per cent. per annum?

6m.	½	£5		intereſt of 100l. for 1 year.
3m.	½	2	10	ditto for ½ year.
		1	5	ditto for ¼ year.
		3	15	ditto for ¾ of a year.
		100	0	
		£103	15	amount of 100l. for ¾ of a year.

As 103l. 15s. : 100l. :: 550l. 10s. : 530l. 12s. 0½d. $\frac{26}{83}$, the preſent worth; which, deducted from 550l. 10s. gives 19l. 17s. 11¼d. $\frac{57}{83}$ for the diſcount.

Or thus,

As 103l. 15s. : 3l. 15s. :: 550l. 10s. : 19l. 17s. 11¼d. $\frac{57}{83}$ the diſcount; which, deducted from 550l. 10s. gives 530l. 12s. 0½d. $\frac{26}{83}$ for the preſent worth.

(2.) Required the preſent worth of 594l. 14s. 9d. due 8 months hence, allowing a diſcount of 5¼ per cent. per annum.

(3.) Sold goods to the value of 915l. 17s. payable 7 months hence; what muſt I allow for preſent payment, at 8 per cent. per annum?

(4.) How much ready money ſhould I have for a note of 75l., which would be due 19 months hence, if I allow a diſcount of 5 per cent. per annum?

(5.) If a legacy of 900l. be left me on the 22d of September, to be paid at Chriſtmas, what muſt I receive if I allow 6 per cent. per annum diſcount for preſent payment?

(6.) What is the diſcount of 1500l. for 57 days, at 7⅝ per cent. per annum?

Class II.

(7.) A bond, bearing date the 5th of July, 1788, is yable at Midſummer, 1789, valued 1757l. 10s. what eſent money would have diſcharged it at the time it was ade, allowing a diſcount of $7\frac{3}{8}$ per cent. per annum?

(8.) Sold goods to the value of 747l. 18s. one third of ich is due immediately, $\frac{1}{3}$ at the end of 7 months, and e reſt at 10 months; what ready money ought I to reive if I allow a diſcount of $8\frac{5}{8}$ per cent.? *Anſ.* 719l. 4s. $9\frac{1}{2}$d.

(9.) Sold goods to the value of 800l. 16s. payable as llows, viz. $\frac{1}{4}$ at 2 months, $\frac{1}{6}$ at 3 months, $\frac{1}{7}$ at 9 months, at 11 months, and the reſt at 12 months; what muſt be ſcounted for preſent payment, at 5 per cent. per annum?

(10.) What ready money will diſcharge a debt of 89l. 19s. 10d. due 3 years, 3 quarters, and 41 days nce, diſcount at $5\frac{3}{8}$ per cent. per annum?

(11.) What difference is there between the intereſt of 1000 for 20 years, and the diſcount of the ſame ſum for at time, the rate per cent. in each caſe being 5l.?

(12.) Sold 500l. worth of goods to 5 *different* perſons, z. 100l. worth each; the firſt man pays at one year's end, e 2d at two, the 3d at three, the 4th at four, and the h at five, years' end. Now, if I allow each of them a ſcount of 6 per cent. for preſent payment, what ſhall I ceive for my 500l.—and, ſuppoſing that I am not in imediate want of the money, and have no fear of loſing any art of it, whether will it be better to receive the debts they become due, and put them out to intereſt, at 6 per ent. till the end of 5 years; or to receive their preſent alue, and put it out to intereſt at the rate of 6 per cent. ll the end of 5 years?

EQUATION of PAYMENTS.

Definition. When ſeveral debts are payable at different imes, bearing no intereſt till after the term of payment, he finding a time at which, if they are all paid together, neither the debtor nor the creditor will ſuffer loſs, is called *quating*, or reducing the times of payment to one.

Propoſition. To find the equated time at which ſeveral debts, payable at different times, may be paid at once, without loſs either to the debtor or creditor, allowing ſimple intereſt.

Rule. Multiply each payment by the time at which it becomes due; then divide the ſum of the products by the ſum of the payments, and the quotient will be the time required.

Note 1. As this rule of Equation of Payments has been the occaſion of more diſputes than all the rules of arithmetic put together, the reader will not be diſpleaſed to find here the ſeveral ſuppoſitions on which its principal defenders have founded their demonſtrations.

2. *Mr. Cocker* ſuppoſes the equated time will be true, ‘ When the ſum of the intereſts of the ſeveral debts which are payable before the equated time, from their terms to that time, is equal to the ſum of the intereſts of the debts payable after the equated time, from that time to their terms.’ But the argument by which he attempts to prove the truth of the rule is, according to *Mr. Malcolm*, very erroneous.

3. *Mr. Hatton* ſuppoſes the equated time to be true, ‘ When the intereſt of the ſum of the debts, from the time of the queſtion to the equated time, is equal to the ſum of the intereſts of the ſeveral debts from the time of the queſtion to the ſeveral terms of payment; and then, by an example, ſhews that the rule agrees with this ſuppoſition.

4. *Mr. R. Burrow*, in his Diary for the year 1777, reduces the ſubject ‘ To find in what time the whole ſum of the ſingle payments will produce the ſame amount as that which ariſes from the ſum of all the ſingle payments, together with the *intereſt* of each payment from the time of its becoming due to the time of the laſt payment;’ and then gives an algebraical demonſtration, which ſhews that the rule is true according to this ſuppoſition.

5. That the rule is univerſally true, according to *any* of theſe ſuppoſitions, or that, if it be true according to one of them, it muſt neceſſarily be true according to the whole, is demonſtrated in the appendix to the Complete Practical Arithmetician, annexed to the Key.

6. The following rule is the ſame as thoſe which *Dilworth*, *Walkinghame*, and ſeveral other writers, have given as the true one.——*Kerſey's rule.* Find the preſent worth of each debt, diſcounting from the time at which it is payable, (by the rule of Diſcount) then find (by Prop. 6. of Simple Intereſt) in what time the ſum of theſe preſent-worths will amount to the ſum of the debts, and that is the time ſought. There are other rules given by different authors, as *Sir Samuel Moreland's*, *Ward's*, *&c.* but, upon a cloſe attention to their principles, they will be found exactly the ſame as one or other of the rules already given: indeed, the foundation of *Burrow's* demonſtration ſeems to have been taken from *Moreland's* rule. *Malcolm's* rule will be given in the ſecond part of this treatiſe; its requiring an extraction of the ſquare-root makes it inadmiſſible in this place.

Examples.

Examples.

(1.) A owes B 110l. whereof 50l. is to be paid at two ears' end, 40l. at $3\frac{1}{2}$ years' end, and 20l. at $4\frac{1}{2}$ years' end; t what time may B receive the whole at once, without rejudice to either party?

50	multiplied	by 2	gives	100
40	——	by $3\frac{1}{2}$	—	140
20	——	by $4\frac{1}{2}$	—	90
110	ſum of the payments.			330 ſum of the products.

The 330, divided by 110, gives 3 years, the anſwer.——If the times f payment, or debts, are of different denominations, they muſt always e reduced to the ſame denomination before you attempt to ſolve a queſtion y the rule.

(2.) A debt of 500l. is to be diſcharged, viz. 100l. at 2 months, 200l. at 4 months, and the reſt at 6 months, what is the equated time for the payment of the whole?

(3.) A debt of 700l. is to be diſcharged thus; £150 preſent, £300 at 6 months, £200 at nine months, and the reſt at 12 months; what is the equated time for the payment of the whole?

(4.) A merchant buys goods to the amount of £750, £350 of which is to be paid at 3 months, and the reſt at 9 months;—to prevent farther trouble, it is agreed to pay the whole at once, and to prolong the time of the firſt payment in proportion to the ſhortening the time of the ſecond; at what time muſt the whole be diſcharged without prejudice to either?

(5.) A debt of 500l. 15s. is payable as follows: £150 at 2 months, 147l. 17s. at 74 days, 137l. 18s. at 95 days, and the reſt at 5 months. It is to be diſcharged at one payment; what is the equated time, reckoning 30 days to a month?

CLASS II.

(6.) A owes B a certain ſum, which is to be diſcharged as follows, viz. $\frac{1}{2}$ at 4 months, $\frac{1}{4}$ at 5 months, $\frac{1}{8}$ at 7 months, and the reſt at 10 months. Now, if both parties ſhould agree to have the whole diſcharged at once, what is the equated time?

(7.) A debt is to be diſcharged thus, $\frac{1}{5}$ preſent, $\frac{1}{6}$ at 25 days, $\frac{1}{9}$ at 3 months, and the reſt at 4m. 17d. what time may the whole be paid at once?

(8.) Three legacies are left by a gentleman, in his will, payable by his executors to *one* perſon, or his heirs. The firſt legacy of 500l. 18s. is payable in $\frac{1}{2}$ a year, the 2d of 900l. 17s. 6d. is payable in 1 year 114 days, and the 3d of 1700l. 18s. 4$\frac{1}{4}$d. is payable in 2$\frac{1}{2}$ years. The legatee and executors have agreed that the payment of theſe ſums ſhall be made at once; at what time muſt that be, that neither party may be injured, allowing ſimple intereſt?

COMPOUND INTEREST.

Definition. Compound Intereſt is that which is produced not only from the ſum of money lent as the principal, but alſo from the intereſt, which, (when unpaid,) as it becomes due, is added to the principal.

Propoſition. To find the intereſt of any ſum of money, unpaid, for any equal number of payments at any rate per cent.

Rule. Find the amount of the given principal for the time of the firſt payment by Simple Intereſt; then conſider this amount as the principal for the ſecond payment, and find *its* amount as before. Proceed thus through all the payments, always conſidering the laſt amount as the principal of the next payment; then, if the given principal, or money lent, be deducted from the laſt amount, the remainder will be the intereſt required.

Note. The above rule will be true, whether the payments are made yearly, half-yearly, quarterly, monthly, or by any other aliquot part of a year: thus, for half-yearly payments, take half the rate per cent. and twice the number of years;—for quarterly payments, take $\frac{1}{4}$ of the rate per cent. and four times the number of years, &c. But the given time muſt be complete years, half-years, or quarters; thus, you cannot find the intereſt of a given ſum payable yearly, for 4$\frac{1}{2}$ years, 4$\frac{3}{4}$ years, &c. by the above rule, as directed by *Mr. Vyſe*, in the note page 101, 6th edition, of his Tutor's Guide.—The 2d, 4th, 5th, 6th, and 7th queſtions in *Walkinghame*'s arithmeti[illegible] are calculated according to *Vyſe*'s note, and ſeveral other writers have calculated ſome of their examples in a ſimilar manner. Thoſe who are acquainted with logarithmical arithmetic, and the involution of numbers to fractional powers, will, however, readily perceive that the method of calculation is erroneous.

Examples.

(1.) What is the compound intereſt of 357l. 10s. for years, at 5 per cent. per annum?

5l. is $\frac{1}{20}$)	357l.	10s.		principal.
	17	17	6	intereſt for the firſt year.
$\frac{1}{20}$)	375	7	6	amount for ditto.
	18	15	4$\frac{1}{2}$	intereſt for the 2d year.
$\frac{1}{20}$)	394	2	10$\frac{1}{2}$	amount for ditto.
	19	14	1$\frac{1}{2}$-$\frac{9}{10}$	intereſt for the 3d year.
	413	17	0 -$\frac{9}{10}$	amount for ditto.
	357	10	0	*principal.*
Anſwer	£56	7	0 -$\frac{9}{10}$	whole intereſt, which is £2 14 6-$\frac{9}{10}$

more than the ſimple intereſt of the ſame ſum. *See Example 1, Simple Intereſt.*

(2.) What is the compound intereſt of 700l. 18s. for 4 years, at 5 per cent. per annum?

(3.) What is the compound intereſt of 1057l. 17s. 6d. for 6 years, at 4 per cent. per annum?

(4.) Required the amount of 500l. 17s. for 5 years, at 4¼ per cent. compound intereſt?

(5.) What will 700l. amount to in 7 years, at 4¾ per cent. per annum, compound intereſt?

CLASS II.

(6.) Find the ſeveral amounts of 500l. payable yearly, ½-yearly, and quarterly, being forborne 4 years, at 5 per cent. per annum. *Anſwer,* £607 15s. 0½d. *for yearly,* £609 4s. 0¼d. *for half-yearly, and* £609 18s. 10½d. *for quarterly payments.*

(7.) What is the amount of 715l. for 6 years, the intereſt payable ½-yearly, at 4¼ per cent. per annum?

(8.) What is the compound intereſt of 740l. 18s. for 9¼ years, by quarterly payments, at 4 per cent. per annum?

FELLOWSHIP.

Definition. *Fellowſhip* is a general rule by which the accounts of merchants, &c. trading in company, with a joint ſtock, are adjuſted; ſo that every partner may have his due ſhare of the gain, or ſuſtain a proportional part of the loſs, according to the money he has advanced in the ſtock and the time of its continuance therein.

SINGLE FELLOWSHIP.

Definition. *Single Fellowſhip* is when different ſtocks are employed for any certain equal time. The effects of bankrupts are by this rule properly divided among their creditors, legacies adjuſted in deficiencies of aſſets, &c.—It likewiſe teaches us to divide any given number into unequal parts, proportional to certain other given numbers.

Propoſition. Having each man's particular ſtock and the whole gain or loſs given to find each man's part of the gain or loſs.

Rule. As the whole ſtock is to the whole gain or loſs, ſo is each man's particular ſtock to his particular ſhare of the gain or loſs.

Method of proof. Add all the ſhares together, and the ſum will be equal to the given gain or loſs when the work is right.

Note 1. When there are many partners concerned, the following rule, which is beſt performed by decimals, will be found uſeful.—Divide the whole gain, or loſs, by the whole ſtock, and the quotient will be a common multiplier, by which multiply every man's particular ſtock, and the ſeveral products will give each man's ſhare of the gain or loſs.

2. *Propoſition. To divide any given number into any number of unequal parts proportional to certain other given numbers.*

Rule. Make the ſum of the numbers, to which the required parts muſt be proportional, the firſt term; the number to be parted, or divided, the ſecond; and each of the given numbers, to which the required ones muſt be proportional, the ſeveral third terms of ſo many ſtatings in the Rule of Three, the fourth terms of which will be the reſpective parts required.

Examples.

(1.) Three merchants, A, B, and C, enter upon a joint adventure; A puts into the common ſtock 250l. 10s. B 300l. 15s. and C 410l. 18s. After all expences were paid, a clear gain of 327l. 11s. 6d. was to be divided amongſt them; what was each merchant's ſhare?

250l. 10s. A's ſtock.
300 15 B's ſtock.
410 18 C's ſtock.

£962 3 *ſum*, or the whole ſtock.

	l.	s.		l.	s.	d.		l.	s.		l.	s.	d.	rem.
As	962	3	:	327	11	6	::	250	10	:	85	5	8½	— 3338, A's ſhare.
	962	3	:	327	11	6	::	300	15	:	102	7	10½	— 666, B's ſhare.
	962	3	:	327	11	6	::	410	18	:	139	17	10¾	—15239, C's ſhare.
											327	11	6	proof.

(2.) Two merchants trade together; A put into the ſtock 500l. 17s. 10d. and B 700 guineas; they gained 300l. 15s. what is each perſon's ſhare thereof?

(3.) Four merchants, A, B, C, and D, entered into partnerſhip with a ſtock of 5075l. 18s. of which A contributed 574l. 16s. B 947l. 18s. 6d. C. 3044l. 17s. and D the reſt; they gained 1358l. 18s. what was each merchant's ſhare thereof in proportion to his ſtock?

(4.) The money and effects of a bankrupt, after every unavoidable expence is deducted, amount to 7174l. 14s. At this time he is indebted to A 540l. 14s. to B 770l. 18s. to C 4005l. 14s. to D 975l. 18s. 9d. and to E 3000 guineas, how muſt it be divided amongſt them, and what will they receive in the pound?

(5.) Six merchants, A, B, C, D, E, and F, ſuſtained a loſs of £79750 by ſhipwreck on a foreign coaſt;—A put on-board, as part of the cargo, to the value of 7754l. 17s. B 15749l. 14s. C 3497l. 16s. D 5754l. 18s. 10d. E 3775l. 19s. and F 37497l. 19s. 8d. whereof there was a *ſalvage* in the cargo of 18750l. which was ſold in the country for 7347l. clear gain; what was each merchant's loſs?

(6.) Three merchants, A, B, and C, freight a ſhip with wine; A put on board 500 tuns, B 340, and C 94; by a ſtorm at ſea they were obliged to caſt 150 tuns overboard; what loſs does each ſuſtain?

(7.) Let the number 1680 be divided into 6 ſuch parts as ſhall be to each other as 1, 2, 3, 4, 5, and 6, reſpectively?

(8.) Three merchants enter into partnerſhip with a ſtock of 1789l. 4s. their ſeveral ſtocks are in proportion as 7, 8, and 9; they gained £500: required each perſon's ſtock and gain?

CLASS II.

(9.) There was a mixture of 3 different kinds of wine, in which, for every 3 gallons of one kind, there was 4 of another, and 7 of a third; what quantity of each kind is in a mixture of 292 gallons?

(10.) A father left his eſtate of £19090 among 3 ſons, in ſuch manner, that, for every £2 that A gets, B ſhall have 3, and C 5; how is the eſtate divided?

(11.) An old lady left 229l. 13s. 4d. to be divided amongſt three of her nieces, A, B, and C, thus; as often as A had £$5\frac{5}{9}$, B had £$4\frac{3}{7}$; and, as often as B had £$4\frac{1}{9}$, C had £$3\frac{2}{3}$; pray what money did the old lady leave to each of them?

(12.) Divide £500 amongſt 4 people, thus; give A $\frac{1}{2}$, B $\frac{1}{3}$, C $\frac{1}{4}$, and D $\frac{1}{5}$.

(13.) Two perſons traded together; the difference of their ſtocks was 51l. 11s. 6d. A's gain was 57l. 18s. and B's 29l. 14s. required each perſon's ſtock?

(14.) Three merchants, A, B, and C, freight ſhips to *Liſbon*, with ſugar, to the value of 15778l. 2s. 6d. A bought 250cwt. 1qr. 22lb. at 2l. 16s. per cwt. B paid 2l. 6s. 8d. per cwt. for his; but, meeting with a ſtorm at ſea, the ſailors were under the neceſſity of caſting out part of the ſhip's lading.——A's proportional part caſt overboard was equal to the $\frac{1}{100}$ part of their whole *cargo*, and $3\frac{3}{4}$ times the *whole* quantity caſt over-board was equal to $3\frac{1}{3}$ times the *whole freight* of A and B. When they came to land, A ſold his remaining part for 4 guineas per cwt. and found himſelf a loſer 10 per cent. beſides charges. B advanced the remaining part of his commodity 20 per cent. and C gained 4s. 8d. per cwt. by the quantity he ſaved.—What did each merchant loſe by this voyage, the charge thereof amounting to 500 guineas?

DOUBLE FELLOWSHIP.

Definition. *Double Fellowſhip* is that which ſuppoſes the ſeveral ſtocks, advanced for the purpoſes of trade, to be continued for unequal times, or to be increaſed or diminiſhed at pleaſure, with the conſent of the ſeveral partners, any time during the continuance of ſuch partnerſhip.

Propoſition. Given each man's ſtock, the time of its conti-uance, and the whole gain or loſs, to find each man's part of e gain or loſs.

Rule. Multiply each man's ſtock by the time of its ontinuance. Then, as the ſum of all the products is to e whole gain or loſs, ſo is each man's product to his rt of the gain or loſs.

Method of proof as in ſingle fellowſhip.

Note. The truth of this rule may be ſhewn thus; when the times are ual, the ſhares of the gain or loſs are evidently as the ſtocks, as in ingle Fellowſhip; and, when the ſtocks are equal, the ſhares are as the mes; wherefore, when neither are equal, the ſhares muſt be as their roducts.

2. When there are many partners concerned, divide the whole gain or ſs by the ſum of the products of each man's ſtock and time, and the uotient will be a common multiplier, by which multiply the product of ach man's ſtock and time ſeparately, to obtain his ſhare of the gain or oſs.—This rule is beſt adapted to decimals.

3. The following rules and obſervations will be found very uſeful in ſolving difficult queſtions in Compound Fellowſhip; and, on this account, will doubtleſs be acceptable to the generality of readers.

4. *Prop.* 1. *Given each man's ſtock and time, and one man's gain (or loſs) to find each man's particular gain (or loſs) and conſequently the whole.*

Rule. As the product of that man's ſtock and time, whoſe gain or loſs is given, is to his gain or loſs, ſo is the product of any other man's ſtock and time to his gain or loſs.

5. *Prop.* 2. *Given each man's gain (or loſs) and time, and the whole ſtock, to find each man's particular ſtock.*

Rule. Multiply each man's gain or loſs into all the times except his own. Then, as the ſum of the products is to the whole ſtock, ſo is each man's product to his ſtock.

6. *Prop.* 3. *Given each man's gain (or loſs) and time, and one man's ſtock, to find each man's particular ſtock.*

Rule. As that man's gain (or loſs) whoſe ſtock is given, is to the product of his ſtock and time, ſo is any other man's gain (or loſs) to the product of *his* ſtock and time. Theſe products, divided by their reſpective times, will give their ſeparate ſtocks.

7. *Prop.* 4. *Given each man's ſtock and gain (or loſs) and the ſum of their times, to find their particular times.*

Rule. Reduce each man's ſtock and gain (or loſs) into one denomination, and multiply each man's gain (or loſs) into all the ſtocks except his own. Then, as the ſum of the products is to the ſum of the times, ſo is each man's product to his time.

8. *Prop.* 5. *Given each man's ſtock and gain (or loſs) and one man's time, to find the particular times of all the reſt.*

Rule. Reduce each man's ſtock and gain (or loſs) into one denomination, and multiply each man's gain (or loſs) into all the ſtocks except his own. Then, as that man's product, whoſe time is given, is to *that* time, ſo is any other man's product to his time.

Examples.

(1.) Three merchants, A, B, and C, enter into partnerſhip; A puts in 89l. 5s. for 5 months, B 92l. 15s. for 7 months, and C 38l. 10s. for 11 months: with this ſtock they traffic, and gain 86l. 16s. Required each perſon's ſhare of the gain in proportion to his ſtock, and the time of its continuance.

89l.	5s.	multiplied	by 5	gives	446l.	5s.	A's product.
92	15	——	by 7	—	649	5	B's product.
38	10	——	by 11	—	423	10	C's product.
		Sum of the	products		1519	0	

		l.		l.	s.		l.	s.		l.	s.	
As	1519	:	86	16	::	446	5	:	25	10	A's gain.
	1519	:	86	16	::	649	5	:	37	2	B's gain.
	1519	:	86	16	::	423	10	:	24	4	C's gain.
									£86	16	proof.

(2.) Three merchants, A, B, and C, engage in partnerſhip; A puts in 547l. 19s. 6d. for 7 months, B 475l. 18s. for 9 months, and C 1747l, 14s. for 4 months; they trade, and gain 225l. Required each perſon's ſhare thereof?

(3.) Four farmers, A, B, C, and D, jointly hired a paſture of a neighbour for 20 guineas, into which A turned 7 oxen for 13 days, B 9 oxen for 14 days, C 11 oxen for 25 days, and D 15 oxen for 37 days; how much muſt each farmer pay for his ſhare of the paſture?

(4.) A family of 10 perſons took a large houſe for ½ a year, for which they were to pay 26l. 2s. 6d. for that time. Now, at the end of 14 weeks, they took in 4 lodgers, and 3 weeks after 4 more; and ſo on for every 3 weeks (during the term) they took in 4 more lodgers. What muſt one of each claſs pay per week of the rent?

(5.) Three merchants enter into partnerſhip, and trade as follows; A put in £150, and at the end of 7 months took out £50; 5 months after that he put in £170:—B put

£205, and at the end of 5 months £110 more, but took £150 four months after:—C put in 300 guineas, and, en 6 months had elapſed, he drew out £150, but 9 nths after he put in £500:—their partnerſhip continued months, at the end of which time they gained £450. quired each perſon's ſhare thereof?

CLASS II. *exercifing the notes, &c.*

(6.) Three merchants traded together as follows: A put £500 for 3 months, B £350 for 5 months, and C £400 r 2 months, by which he received 29l. 12s. 7$\frac{1}{9}$d. profit. hat muſt A and B receive for *their* reſpective ſtocks, and hat did they gain in the whole?

(7.) Three merchants traded together in this manner: 's money continued 8 months, for which he received l. 4s. gain; B's continued 6 months, for which he had l. 16s. 9$\frac{1}{3}$d. and C's 12 months, by which he was entled to receive 79l. 11s. 2$\frac{2}{3}$d.—Their whole ſtock was 227, hence is required each perſon's particular ſtock?

(8.) A, B, and C, are in company, and put in together 3822; A's money was in 3 months, B's money was in 5 onths, and C's money was in 7 months; they gained £234, hich was ſo divided, that $\frac{1}{2}$ of A's gain was equal to $\frac{1}{3}$ of 's gain, and $\frac{1}{3}$ of B's gain was equal to $\frac{1}{4}$ of C's gain; hat did each merchant gain and put in?

Note. This queſtion is taken from *Hill*'s Arithmetic, where a falſe method of ſolution may be ſeen; which queſtion, and its falſe ſolution, are copied into *Meſſrs. Birks*' Arithmetic, vide 2d edit. page 367. After each man's gain is found, (by note 2, Single Fellowſhip) their reſpective ſtocks muſt be found by note 5, prop. 2, Double Fellowſhip.

(9.) X, Y, Z, in company, make one common ſtock of £4262; X's money was in 4 months, Y's 6 months, and Z's 9 months. They gained £420, which was to be divided in the following manner, viz. $\frac{1}{2}$ of X's gain to be equal to $\frac{1}{3}$ of Y's, and $\frac{1}{3}$ of Y's gain to be equal to $\frac{1}{4}$ of Z's. Quere what each perſon gained and put in?

Note. This queſtion, which is extracted from *Vyſe*'s Tutor's Guide, is ſimilar to the preceding, and muſt be ſolved in a ſimilar manner.—Mr. *Vyſe* uſes the ſame method of ſolution as *Meſſrs. Birks*'.

(10.) Four merchants, A, B, C, and D, trade together; A clears 76l. 4s. in 6 months, B 57l. 10s. in 5 months, C 100 guineas in 12 months, and D (with a ſtock of 200 guineas) 78l. 15s. in 9 months. Required each man's particular ſtock?

(11.) Three perſons, A, B, and C, traded together; A's ſtock was 89l. 5s. B's 92l. 15s. and C's 38l. 10s. Their reſpective gains were £25 10s. £37 2s. and £24 4s. alſo, if the times that each perſon's ſtock was employed in trade be added together, the ſum will be 23 months, pray how long was each man's ſtock in trade?

(12.) A merchant (B) in trade, with a capital of £5000, after a certain time, agreed to take a friend (C) into partnerſhip, who was to have a ſhare in the profits according to the money he advanced and the time of its continuance. Now C put £1400 into the ſtock, and they traded together in this manner, till, willing to enlarge their ſphere of trade, they admitted another perſon (D) as a partner with a ſtock of £1800.—At the end of 3 years (or 36 months) reckoning from the time that B commenced buſineſs, B's gain was found to be £1125, C's £210, and D's £202 10s. quere, how long was C's and D's money employed in trade, and what did each merchant gain per cent. for his money?

(13. Two merchants, A and B, traded together with a ſtock of 315; A's money was employed 12 months, and B's only 8; when they came to divide the profits of their traffic, they had equal ſhares.—Pray what money did each perſon put into the ſtock?

(14.) A certain village is poſſeſſed by three proprietors, who are deſirous of having it incloſed for their mutual benefit. A's property, upon a ſurvey of the quantity and quality, is 394a. 3r. 34p. at 18s. per acre; B has 417a. 1r. 14p. at an average of 19s. 6d. per acre; and C has 714a. 3r. at a guinea an acre. Out of theſe an allowance of 5s. 6d. in the pound is to be made for the tithes. What quantity of land muſt be allotted for theſe tithes, at an average quality of 19s. 9½d. per acre? *Anſw.* 420*a.* 1*r.* 26*p.*

LOSS AND GAIN.

Definition. *Loſs and Gain* is a rule that diſcovers what is gained or loſt in the buying or ſelling of goods; and inſtructs the merchant, or trader, to raiſe or lower the price his goods ſo as to gain or loſe ſo much *per cent.* &c.

Note. By the prime coſt, or ſelling price of an integer, in the fol- wing propoſitions and rules, is meant the prime coſt, or ſelling price, *r yard, pair, dozen, pound, cwt. gallon, tun, &c.* of any quantity of ods, or it may ſignify the whole value in any of the propoſitions, ex- pt the firſt, fifth, and ſixth.

Propoſition 1. *Given the prime coſt and ſelling price of an teger of any quantity of goods to find the whole gain or loſs.*

Rule. Calculate the value of the goods at the prime oſt and ſelling price of an integer, by the Rule of Three, r Practice, and the difference of theſe values will be the ain or loſs.

Prop. 2. *Given the prime coſt and ſelling price of an in- eger of any quantity of goods to find the gain or loſs per cent.*

Rule. As the prime coſt of an integer is to £100, ſo is he advanced or reduced price of ſuch integer to a fourth umber; which, if greater than £100, the exceſs will be he gain; but, if leſs than £100, the defect will be the oſs, *per cent.*

Prop. 3. *Given the prime coſt of an integer, and the propoſed ain or loſs per cent. to find the ſelling price of ſuch integer.*

Rule. As £100 is to £100, with the gain added to, or he loſs ſubtracted from, it, ſo is the prime coſt of an in- teger to the required price per integer.

Prop. 4. *Given the price of an integer, with the gain or loſs per cent. by ſuch a price, to find the gain or loſs at any other price.*

Rule. As the given price of an integer is to £100 with the gain per cent. added to, or loſs ſubtracted from, it, ſo is the propoſed price to a fourth number. If this fourth number be greater than £100, the exceſs will be the gain; but, if it be leſs, take it from £100, and the remainder will be the loſs *per cent.*

Prop. 5. *Given the price at which an integer of any quantity of goods is ſold, and the gain or loſs per cent. by ſuch ſale, to find the whole gain or loſs.*

Rule. Find the whole value of the goods at the ſelling price per integer. Then, as £100, with the gain *per cent.* added to, or loſs ſubtracted from, it, is to £100, ſo is the whole value at which the goods were ſold to the whole prime coſt. The difference between the whole value at which the goods were ſold and the whole prime coſt will give the whole gain or loſs.

Prop. 6. *Given the prime coſt of an integer of any quantity of goods, and the gain or loſs per cent. by the whole quantity, to find the whole gain or loſs.*

Rule. Find the whole value of the goods at the prime coſt per integer. Then, as £100 is to £100, with the gain added to, or loſs ſubtracted from, it, ſo is the whole value of the goods, at the price they coſt, to the whole value at the gain or loſs *per cent.* propoſed. The difference between theſe values will give the whole gain or loſs.

Note. More propoſitions and rules might, with propriety, have been given here; but, if the ſcholar thoroughly underſtand the rules already given, and their application, I am perſuaded he will not meet with any embarraſſment in any queſtion in *Loſs and Gain*, however complicated it may be.

Examples to Propoſition 1.

(1.) Bought 119¾cwt. of ſugar at 1l. 15s. per cwt. whether ſhall I gain or loſe if I ſell it by retail for 6d. per lb.?

If 1cwt. : 1l. 15s. :: 119¾cwt. : 209l. 11s. 3d. prime coſt.
If 1lb. : 6d. :: 119¾cwt. : 335l. 6s. ſold for.
Then 335l. 6s.—209l. 11s. 3d.=125l. 14s. 9d. gain.

(2.) Bought 15cwt of cheeſe at 1l. 11s. 6d per cwt. which I ſell by retail at 4½d. per lb. what ſhall I gain or loſe by ſo doing?

(3.) If I buy 240 eggs, *viz.* ½ at 3 for a penny, and the other ½ at 2 for a penny, whether ſhall I gain or loſe if I mix them together, and ſell them 5 for 2 pence?

(4.) A merchant bought 12 tuns of wine at £75 12s. per tun, which he ſold at 7s. per gallon; but, by misfortune, a pipe was ſtaved, and rendered unſaleable. Whether did the merchant gain or loſe, and how much by ſuch ſale?

(5.) Bought 340 yds of cloth at 5s. 4d. a yard, and ſold it again at 7s. 6d. per yard; what did I gain in the whole?

Examples to Prop. 2.

(6.) If wine is bought at 7s. 6d per gallon, and ſold for 10s. what is gained *per cent.* by ſuch ſale?

If 7s. 6d. : 100l. :: 10s. : 133l. 6s. 8d.

Then 133l. 6s. 8d.—100l.=33l. 6s. 8d. the gain per cent.

r, 10s.—7s. 6d.=2s. 6d. and 2s. 6d.=⅓ of 7s. 6d. therefore 100÷3 =33l. 6s. 8d. anſwer.

(7.) A merchant has a quantity of damaged tobacco, hich, including all expences, ſtands him in 17¼d. per *lb.* hat will he loſe per cent. by a ſale at 13½d. per *lb.*?

(8.) Bought 27 yards of cloth for 17 guineas, and ſold hem again at 9s. 10d. per yard; what was the gain or loſs er cent.?

(9.) *Bought a quantity of goods for 60l. and ſold, hem again for 75l. what was the gain per cent.?

(10.) Bought a quantity of cloth at 7s. 6d. per yard, hich, upon examination, I find not ſo good as I expected. Now, if I ſell it at 6s. 2½d. per yard, what ſhall I loſe *per ent.* by it?

Examples to Prop. 3.

(11.) Bought muſlin at 4s. 8d. per yard; at what price muſt I ſell it per yard to gain 12½ per *cent.*?

If 100l. : 112l. 10s. :: 4s. 8d. : 5s. 3d. anſwer.

Or, 12l. 10s.=⅛ of 100l. and ⅛ of 4s. 8d.=7d. Hence 4s. 8d.+ 7d.=5s. 3d. as before.

(12.) If I buy cloth at 11s. 6d. per yard, how muſt I ſell it to gain 20l. per *cent.*?

(13.) A Mancheſter man bought a quantity of yarn at 6s. per bundle, which not proving ſo good as he expected, he ſold it ſo as to loſe 6l. per cent. by it; what was the ſelling price?

(14.) If I buy tobacco at 12 guineas per cwt. at what rate muſt I ſell it per cwt. to gain 15l. per cent.?

(15.) Bought a quantity of cloth at 7s. 6d. per yard, which not proving ſo good as I expected, I have reſolved to loſe £17½ per *cent.* by it; how muſt I ſell it per yard?

Examples to Prop. 4.

(16.) A ſtationer ſold quills at 11s. per thouſand, by which he cleared 60l. per cent. but, they growing ſcarce, he raiſed them to 13s. 6d. per thouſand; what was his gain per cent. by the latter price?

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* This queſtion is taken from *Mr. Malcolm*, and is that alluded to by *Dr. Hutton*, which *Lowe* copied into his arithmetic with the falſe anſwer.

If 11s. : 160l. :: 13s. 6d. : 196l. 7s. $3\frac{3}{11}$d.
Then 196l. 7s. $3\frac{3}{11}$d.—100l.=96l. 7s. $3\frac{3}{11}$d. anſwer.

(17.) If, when I ſell cloth at 8s. 9d. per yard, I gain 12l. per cent. what will be the gain per cent. when it is ſold for 10s. 6d. per yard?

(18.) A woollen-draper in London had a quantity of black cloth by him, and, being afraid of its being damaged, he ſold it at 15s. per yard, and by ſo doing loſt 14l. *per cent.* but, a general mourning coming unexpectedly, he was enabled to advance his cloth to a guinea per yard; what did he gain or loſe *per cent.* by the latter ſale?

(19.) If a plumber gains 12l. 10s. *per cent.* when lead is ſold at 20l. 9s. 6d. a fother, what would he gain or loſe *per cent.* when it is ſold only at 17l. 1s. 3d. the fother?

Examples to Prop. 5.

(20.) A merchant ſold 5t. 3hhds $53\frac{1}{2}$gall. of wine at 6s. 8d. *per gallon*, and by ſo doing gained $6\frac{1}{2}$l. *per cent.* What was the prime coſt of his wine, and what did he gain in the whole?

If 1 gall. : 6s. 8d. :: 5t. 3hhd. $53\frac{1}{2}$g. : 500l. 16s. 8d. ſold for.
Again, If 106l. 10s. : 100l. :: 500l. 16s. 8d. : 470l. 5s. $3\frac{181}{213}$d. prime coſt.
Then 500l. 16s. 8d.—470l. 5s. $3\frac{181}{213}$=30l. 11s. $4\frac{32}{213}$d. whole gain.

(21.) A merchant ſold 15cwt. 3qr. 18lb. of ſugar at $7\frac{1}{2}$. per lb. and his profit *per cent.* was 25l. what did he gain in the whole?

(22.) *If I ſell 500 deals at 15d. a piece, and 9l. *per cent.* loſs, what do I loſe in the whole quantity?

(23.) A had 15 pipes of Malaga wine, which he parted with to B at $4\frac{1}{3}$l. *per cent.* profit, who ſold them to C for

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* This queſtion is the 9th in *Dilworth*'s Arithmetic, the anſwer to which, as *Mr. Vyſe* obſerves in his Tutor's Guide, is obtained upon falſe principles. This gentleman has given a correct ſolution to it in his Key, (vide 4th edit. page 170,) from *Dr. Hutton*. The 23d queſtion is taken from *Vyſe*, and in his Key, (page 172) may be ſeen a ſolution to it, exactly upon the ſame *falſe* principles that he has been juſt before correcting in *Dilworth*. The ſame queſtion is likewiſe ſolved by *Meſſrs. Birks*, and ſome other authors, in a ſimilar manner. Part of the true ſolution is contained in the 20th queſtion; for, C's prime coſt and profit will be the ſame as the prime coſt and profit found in that queſtion, and not 468l. 5s. 7d. and 32l. 11s. 1d. as theſe gentlemen have determined it.

38l. 11s. 6d. advantage; C made them over to D for 500l. 16s. 8d. and cleared thereby 6½ *per cent.* what did this wine coſt A *per gallon?*

Examples to Prop. 6.

(24.) Bought 60 reams of paper at 15s. per ream, by the ſale of which I loſt 4l. per cent. what did I loſe in the whole?

If 1r. : 15s. :: 60r. : 45l. prime coſt.
If 100 : 96 :: 45l. : 43l. 4s. ſelling price.
Then 45l.—43l. 4s.=1l. 16s. whole loſs.

(25.) Sold 7 pieces of cloth, each containing 35½ yards, on account of damage, at a loſs of 10l. *per cent.* what did I loſe in the whole, the prime coſt being 15s. per yard?

(26.) Bought 475 yards of cloth at 10s. 6d. per yard, by which I gained 30l. *per cent.* what did I gain in the whole?

Class II. *Promiſcuous Examples.*

(27.) Bought 127 hhds of ſugar, each containing 4½ *cwt.* at 3l. 0s. 8d. *per cwt.* how muſt I ſell the ſugar per lb. to gain 50 guineas by the whole?

(28.) A merchant bought 1400 caſks of tallow, at 2l. 5s. per caſk, and ſold one half of it at 2l. 15s. per caſk; but the reſt being worſe than he expected, he is willing to ſell it at ſuch a price per caſk that he may exactly make his purchaſe-money of the whole. At what rate muſt he ſell it?

(29.) A merchant bought 100 yards of velvet for 112l. at what rate muſt he ſell it per yard to gain as much by the whole quantity as four yards are ſold for?

(30.) Sold a quantity of Virginia ſnakeroot for 20l. and by ſo doing loſt 20l. *per cent.* whereas I ought to have gained as much *per cent.* as the ſnakeroot coſt. Quere my loſs in point of trade?

(31.) A tea-dealer purchaſed 120lb. of tea; ⅔ of which he ſold at 10s. 6d. per lb. but the reſt, being damaged, he ſold at a loſs of 3l. 12s. after which he found he had neither gained nor loſt. What did the tea coſt him per lb. and what was the damaged tea ſold for?

(32.) My factor at Leghorn returned me 800 barrels of anchovies, each weighing 14lb. neat, worth 12½d. per lb.

in

in lieu of 7490lb. of Virginia tobacco; by which conſignment I find that I have gained 17l. *per cent*. Pray what was the prime coſt of a lb. of my tobacco to the factor?

(33.) A merchant ſent goods to *Boulogne* to the value of 3475l. 15s. by the ſale of which he gained 40l. ſterling per cent.—The value of the goods he ſent over and the gain were returned in commodities, by the ſale of which in England he loſt 15l. per cent. What was his gain at the laſt?

(34.) Sold a piece of cloth, containing 5000 ells Flemiſh, for 4250 guineas, and gained upon every yard $\frac{1}{8}$ of the prime coſt of an Engliſh ell. What did the whole piece ſtand me in?

BARTER.

Definition. When merchants or tradeſmen exchange one commodity for another, it is called *Bartering*; and, by the rule of proportion, the price and quantity of the goods ſo exchanged are determined, ſo that neither party may ſuſtain a loſs by ſuch traffic.

Propoſition 1. *Given the price of an integer of any quantity of goods, to find the correſponding quantity of any other ſort of goods, at any given price per integer.*

Rule. Find the value of that commodity, whereof the quantity is given, by the Rule of Three, or Practice. Then, as the price of an integer of the *required* quantity of goods is to *that* integer, ſo is the value of the *given* quantity, found before, to the required quantity.

Note. Several queſtions that fall under this propoſition may be ſolved, with the leaſt trouble and attention, by the univerſal rule of proportion.

Prop. 2. *Given the price of an integer of any quantity of goods, to find the quantity of any other kind of goods, (at any given price per integer) when part of the value is paid in money, or any other kind of merchandiſe.*

Rule. Find the whole value of that commodity, whereof the quantity is given, by the Rule of Three, or Practice; from which ſubtract the ſum of money to be paid down, or the value of the given quantity of goods in exchange. Then, en, as the price of an integer, of the *required* quantity goods, is to *that* integer, ſo is the remaining value to accounted for, to the required quantity.

Note. Several other propoſitions and rules in Barter are omitted as ſu- rfluous, the queſtions they are intended to ſolve being of no real uſe; ſuch to find in what proportion one perſon ought to advance his goods to other who has raiſed *his* goods above their real value, &c. For, in the l exchange of goods, when both parties have mentioned their ready- oney prices, if one perſon's goods are advanced to a bartering price, the her perſon's muſt be advanced in the ſame proportion, and conſequently e balance between them will remain exactly the ſame as if the ready- oney price *only* had been uſed.

Examples to Propoſition 1.

(1.) A and B barter; A has 3½lb. of pepper at 13½d. er lb. B has ginger at 15¼d. per lb. How much ginger uſt B give for A's pepper?

| ½ | 1s. 1½d. value of 1lb. of A's pepper.
3

3 4½ value of 3lb.
6¾ ditto of ½lb.

3 11¼ ditto of 3½lb.

Then, as 15¼d. : 1lb. :: 3s. 11¼d. : 3lb. $1\frac{35}{61}$oz. anſwer.

By the univerſal rule of proportion.

equal
If 3½lb. at 13½d. 1 value in pence.
*lb. at 15¼d 1 (the ſame) value.
equal

13½ × 3½ = 47¼ dividend, which, divided by 15¼, the diviſor, gives $3\frac{6}{61}$lb. = 3lb. $1\frac{35}{61}$oz. as before.

(2.) A would exchange 400 gallons of Jamaica rum, worth 7s. 9d. per gallon, with B for London porter, at 9d. a gallon; how many gallons of porter muſt A receive of B, in exchange for his rum?

(3.) A hop-factor, A, exchanged 5cwt. 1qr. 10lb. of hops, at 2s. 4½d. per lb. for wheat at 5s. 9d. per buſhel, with a farmer B; what quantity of wheat did B give A for his hops?

(4.) How many yards of cloth, at 18s. 6d. per yard, muſt I give for 5000 yards of baize, at 13½d. per yard?

(5.) A delivered 6 hogſheads of brandy, at 6s. 8d. per gallon, to B for 252 yards of cloth; what ought the cloth to be worth per yard?

(6.) A has 288 ells of cloth, worth 1s. 3d. per ell, which he would barter with B for cheeſe at 19s. per cwt. what weight of cheeſe ought B to give for the cloth?

(7.) A and B bartered; A had 14cwt. 3qrs. of ſugar, worth 1l. 17s. per cwt. which he bartered for wine worth 3s. 9d. per gallon; how much wine did A receive?

(8.) A chandler and a butcher trade as follows; the butcher has 3cwt. 2qr. 16lb. of tallow at 1l. 17s. 4d. per cwt. and the chandler rates his candles at 5s. 2d. per dozen. How many lbs of candles muſt the chandler give the butcher for his tallow?

Examples to Prop. 2.

(9.) A and B barter as follows; A has 41 cwt. of hops at 30s. per cwt. for which B gives him 20l. in ready money, and the reſt in ſugar at 6d. per lb. What quantity of ſugar muſt B give A?

30s. | $\frac{1}{2}$ | 41l.
20 10 As 6d. : 1lb. :: 41l. 10s. : 14cwt. 3qr. 8lb. anſwer.

61 10 value of A's hops.
20 0 paid down.

41 10 to account for.

(10.) A and B barter; A has 750 yards of canvas, worth 10d. per yard, for which B gives him 475 yards of ſerge at 11$\frac{1}{2}$d. per yard, and the balance in cotton at 3s. per yard; how many yards of cotton muſt A receive?

$\frac{1}{2}$d. | $\frac{1}{24}$ | 475
19 9$\frac{1}{2}$

20)455 2$\frac{1}{2}$

£22 15 2$\frac{1}{2}$

| 10d. | $\frac{1}{24}$ | 750

£ 31 5 value of A's canvas.
22 15 2$\frac{1}{2}$ value of B's ſerge.

£ 8 9 9$\frac{1}{2}$ to account for.

As 3s. : 1yd. :: 8l. 9s. 9$\frac{1}{2}$d. : 56yds. 2$\frac{7}{18}$qrs. anſwer.

(11.) A has 700 gallons of rum at 4s. 6d. per gallon, for which B gives him 27 guineas in money, and the reſt in cotton at 11$\frac{1}{2}$d. per lb. how much cotton muſt A receive?

(12.) A has 57qrs. 6buſh. of corn, worth 1l. 11s. 6d. r quarter, for which B will give 14cwt. 3qr. 18lb. of gar at 4l. 14s. per cwt. and the balance in raiſins at 7d. r lb. Should theſe perſons barter, what quantity of iſins ought B to give A?

(13.) A has 27 cwt. of cheeſe, worth 1l. 11s. 4d. per wt. and B has 25 pieces of cloth, worth 1l. 19s. 10½d. r piece; ſhould theſe perſons barter together, to whom ill the balance, if any, be due?

CLASS II.

(14.) A gave B 120 yards of Kerſey, 3½ yards of which oſt 15s. 9d. for ſtockings at 7s. per pair, and hats at 6s. 6d. ach; B gave A as many hats as pairs of ſtockings; how any of each did he give?

(15.) Two merchants have various kinds of goods to arter; A has 735 yards of India ſilk, worth 8s. 6d. per yard, 532 canes worth 3s. each, and 16 pieces of muſlin worth 4l. each; B has ſcarlet cloth worth 1l. per yard, glaſs manufacture at 1s. 8d. per lb. and a finer kind at 2s. 4d. per lb. How many yards of cloth and pounds of each ſort of glaſs muſt B give A, admitting that he gives as many pounds of each ſort of glaſs as he gives yards of cloth?

(16.) A merchant, A, *of London*, ſent 8752 yards of cloth, worth 1l. 11s. 6d. per yard, to B in *Jamaica*; and deſired him to return him ¼ of the value in ſugar at 1l. 15s. 6d. per cwt. ⅛ of the value in pepper at 7l. 3s. 9d. per cwt. and the reſt in rum at 5s. 6d. per gallon. Each merchant ran the riſk, and paid the charges of the commodity he ſent over; pray what quantity of ſugar, pepper, and rum, did A receive?

(17.) A and B barter; A has 24 puncheons of rum, worth 4s. 9d. per gallon; for which B gives him 150 guineas in caſh, and 714 yards of cloth. What ought B's cloth to be worth per yard?

(18.) A bartered tobacco, worth 3s. 4d. per lb. at 3s. 9d. per lb. with B for tea at 6s. 3d. per lb. When A ſold the tea, he found himſelf a gainer of 17l. 6s. 8d. per cent. and in the whole 8l. 10s. 8d. What did A ſell the tea for per lb. and what quantity of tobacco and tea were bartered?

EXCHANGE.

Definition 1. *By Exchange* is meant the bartering, or exchanging, the money of one place for that of another, by means of an inſtrument in writing, called a *Bill of Exchange*; and conſiſts in finding what quantity of the money of one city or country will be equal to any given ſum of another, according to a given *courſe* of exchange.

2. *The Courſe of Exchange* is the value agreed upon by merchants, or their factors; and is continually fluctuating above or below the *Par of Exchange*, according as the demand for bills is greater or leſs.

3. *The Par of Exchange* is that quantity of the money of one country which is intrinſically equal to a certain quantity of the money of another, whether *real* or *imaginary*.

4. *The real money* of every empire, kingdom, ſtate, &c. ſignifies one piece, or more, of any kind of metal, coined by the authority of that empire, kingdom, ſtate, &c. and current at a certain value by virtue of ſuch authority.

5. *The imaginary Money* is uſed in keeping accounts, and includes all the denominations which are uſed to expreſs any ſum of money, of which no ſuch ſpecies are current in any empire, kingdom, ſtate, &c.

6. *The Agio* denotes the difference in foreign countries between the *current*, or *caſh-money*, and the *exchange*, or *bank-money*, the latter being compounded of a finer, or purer, metal than the former.

Note. When current, or caſh-money is taken in payment, the merchants have an allowance of ſo much per cent. according to what exchange-money is worth more than the current-money.

7. *Bank-notes*, in the buſineſs of exchange, are ſuch as are obtained from foreign bankers for money lodged in their bank.—Theſe are called *bank-money*.

8. *Uſance* is a certain ſpace of time allowed, by one country to another, for the payment of bills of exchange. It varies according to the cuſtom of countries, and frequently in proportion to the diſtance of places from each other.—Bills are either payable at *ſight*, or ſo many days after ſight; at *uſance*, double *uſance*, or half *uſance*.

9. *The Days of grace* are a certain number of days al-
ved for the payment of bills of exchange, after the ex-
ation of the term ſpecified in ſuch bills, and are variable
different countries. In ſome countries no days of grace
e allowed.

The neceſſary TABLES of EXCHANGE.

Table I. DENMARK, ZEALAND, NORWAY, &c.

At *Copenhagen*, *Elſinore*, *Bergen*, *Drontheim*, *&c.*

The loweſt piece of money is a Skilling, value $\frac{9}{16}$d. ſterling.

			s.	d.				s.	d.
6 Skillings	=	1 Duggen	0	3⅜	4 Marcs	=	1 Crown	3	0
6 Skillings	=	1 *Marc	0	9	6 Marcs	=	1 Rix-dollar	4	6
0 Skillings	=	1 Rix-marc	0	11¼	11 Marcs	=	1 Ducat	8	3
4 Skillings	=	1 Rix-ort	1	1½	14 Marcs	=	1 Hatt-Ducat	10	6

The *Danes* exchange by the Rix-dollar, value 4s. 6d. ſterling; but the
ourſe of exchange varies from 45d. to 58d. ſterling per Rix-dollar.

Table II. SWEDEN, LAPLAND, &c.

At *Stockholm*, *Upſal*, *Tornea*, *&c.*

The loweſt piece of money is a **Runſtic*, value $\frac{7}{36}$d. ſterling.

			s.	d.
2 Runſtics	=	1 Stiver	0	0 $\frac{7}{18}$
8 Runſtics	=	1 copper Marc	0	1 $\frac{5}{6}$
3 copper Marcs	=	1 ſilver Marc	0	4 $\frac{2}{3}$
4 copper Marcs	=	1 copper Dollar	0	6 $\frac{2}{9}$
9 copper Marcs	=	1 Caroline	1	2
3 copper Dollars	=	1 ſilver Dollar	1	6⅔
3 ſilver Dollars	=	1 Rix-dollar	4	8
2 Rix-dollars	=	1 Ducat	9	4

The chief medium of commerce in *Sweden* is copper, which is exceedingly inconvenient, ſome of the pieces being nearly as large as tiles: theſe and ſmall bank-notes are almoſt their only circulating money. They have gold ducats and eight marc pieces, valued each 5s. 2d. but theſe are very ſcarce.

The *Swedes* exchange by the copper dollar, value 6$\frac{2}{9}$d. ſterling, but the courſe of exchange is ſubject to great variations.

Table III. MUSCOVY, or RUSSIA.

At *Peterſburg*, *Archangel*, *Moſcow*, *&c.*

The loweſt piece of money is a *Poluſca*, value $\frac{27}{100}$d. ſterling.

2 Poluſcas

			s.	d.
2 Poluſcas	=	1 Denuſca	0	$0\frac{27}{100}$
2 Denuſcas	=	1 *Copec	0	$0\frac{27}{50}$
3 Copecs	=	1 Altin	0	$1\frac{31}{50}$
10 Copecs	=	1 Grivener	0	$5\frac{2}{5}$
25 Copecs	=	1 Polpotin	1	$1\frac{1}{2}$
2 Polpotins	=	1 Poltin	2	3
10 Griveners, or 2 Poltins	=	1 Ruble	4	6
2 Rubles	=	1 Xervonitz	9	0

The *Ruſſians* exchange by the Ruble, value 4s. 6d. the Rubles a converted into Florins current of *Amſterdam*, and the current into ban money, and thence into ſterling, according to the courſe of exchange be tween *England* and *Amſterdam*. Sometimes they exchange directly wi *London*, from 4 to 5s. per Ruble.

Table IV. LIVONIA. (A Ruſſian Province.)

At *Riga*, *Revel*, *Narva*, *&c.*

The loweſt piece of money made uſe of is a *Blacken*, value $\frac{7}{90}$d. ſterling.

			s.	d.
6 Blackens	=	1 Groſh	0	$0\frac{7}{15}$
9 Blackens	=	1 Vording	0	$0\frac{7}{10}$
2 Groſhen	=	1 Whiten	0	$0\frac{14}{15}$
6 Groſhen	=	1 Marc	0	$2\frac{4}{5}$
30 Groſhen	=	1 Florin	1	2
3 Florins	=	1 *Rix-dollar	3	6
18 Marcs	=	1 Albertus	4	$2\frac{2}{5}$
64 Whiten	=	1 copper-plate Dollar	4	$11\frac{11}{15}$

Table V. POLAND and PRUSSIA.

At *Cracow*, *Warſaw*, *&c. Dantzic*, *Koningſberg*, *Elbing*, *Thorn*, *&c.*

The loweſt piece of money made uſe of is a *Shelon*, value $\frac{7}{45}$d. ſterling.

			s.	d.
3 Shelons	=	1 Groſh	0	$0\frac{7}{15}$
5 Groſhen	=	1 Couſtic	0	$2\frac{1}{3}$
3 Couſtics	=	1 Tinſe	0	7
18 Groſhen	=	1 Ort	0	$8\frac{2}{5}$
30 Groſhen	=	1 Florin	1	2
3 Florins	=	1 *Rix-dollar	3	6
8 Florins	=	1 Ducat	9	4
5 Rix-dollars	=	1 Frederic d'Or	17	6

Dantzic, &c. exchange by the Groſh, value $\frac{7}{15}$d. ſterling, with *London* by the way of *Amſterdam*; 270 Groſh. being equal to 1 Pound Flemiſh; and 110 Groſh. make 1 Rix-dollar, bank of *Hamburgh*.

Table VI. GERMANY.

At *Hamburgh, Altena, Lubec, Bremen, &c.*

e loweſt piece of money made uſe of is a **Tryling*, value $\frac{3}{128}$d.
ng.

			s.	d.
2 Trylings	=	1 *Sexling	0	0 $\frac{3}{64}$
2 Sexlings	=	1 Fening	0	0 $\frac{3}{32}$
12 Fenings	=	1 Shilling Lub	0	1 $\frac{1}{8}$
16 Shill Lub	=	1 *Marc	1	6
2 Marcs	=	1 Slet Dollar	3	0
3 Marcs	=	1 Rix-Dollar	4	6
6¼ Marcs	=	1 Ducat	9	4½
7½ Marks, or 120 Shill Lub	=	1 *Pound Flemiſh	11	3
6 Fenings	=	1 Denier Groſs, or Penny Flemiſh.		
2 DeniersGroſs, or 2 Pence Flem.	=	1 Sol Lub		
12 DeniersGroſs, or 6 Sols Lub	=	1 Sol Groſs, or Shilling Flemiſh.		
20 Sols Groſs, or 2½ Rix-dollars	=	1 Pound Flemiſh.		
8 Sols Groſs	=	1 Rix-Dollar.		

Lubs is a term derived from the city of Lubec, where the ſhillings, or ls Lubs, are coined.

Theſe places exchange either by the Rix-dollar, Slet-dollar, or Marc.

At *Franckfort, Nuremburg, Wurtſburg, &c.*

The loweſt piece of money in uſe is a *Fening*, value $\frac{7}{60}$d. ſterling.

			s.	d.
4 Fenings	=	1 Creuitzer	0	0 $\frac{7}{15}$
3 Creuitzers	=	1 Keyſer Groſs	0	1 $\frac{2}{5}$
4 Creuitzers	=	1 Batzen	0	1 $\frac{13}{15}$
15 Creuitzers	=	1 Ort Gold	0	7
60 Creuitzers	=	1 Gould	2	4
90 Creuitzers	=	1 *Rix-dollar	3	6
2 Goulds	=	1 hard Dollar	4	8
4 Goulds	=	1 Ducat	9	4

Theſe places exchange by the Gould, Guilder, or Florin, and Rix-dollar;

Table VII. The SEVEN UNITED PROVINCES.

At *Amſterdam, Rotterdam, Leyden, Middleburg, Fluſhing, &c.*

The loweſt piece of money in uſe is a **Pening*, value $\frac{21}{320}$d. ſterling.

			s.	d.
8 Penings	=	1 *Grot	0	0 $\frac{21}{40}$
2 Grots	=	1 Stiver	0	1 $\frac{1}{20}$
6 Stivers	=	1 Scalin	0	6 $\frac{3}{10}$

20 Stivers

			s.	d.
20 Stivers	=	1 Guilder	1	9
$2\frac{1}{2}$ Guilders	=	1 Rix-dollar	4	$4\frac{1}{2}$
6 Guilders	=	1 *Pound	10	6

As this is one of the moſt uſeful tables in exchange, I ſhall give it in another form for the encouragement of the learner.

*Penings
8 = 1 *Grot, or Penny,
16 = 2 = 1 Stiver,
96 = 12 = 6 = 1 Scalin,
320 = 40 = 20 = $3\frac{1}{3}$ = 1 Guilder, or Florin,
800 = 100 = 50 = $8\frac{1}{3}$ = $2\frac{1}{2}$ = 1 Rix-dollar,
1920 = 240 = 120 = 20 = 6 = $2\frac{2}{5}$ = 1 *Pound Flemiſh.

If the ſcholar be taught to reduce all the tables to this form, before he enter upon any of the examples, it will be a great advantage to him.

Table VIII. The NETHERLANDS.

At *Ghent*, *Oſtend*, *&c.* *Antwerp*, *Bruſſels*, *&c.*

The loweſt piece of money in uſe is a **Pening*, value $\frac{9}{160}$ ſterling.

			s.	d.
4 Penings	=	1 Urche	0	$0\frac{9}{40}$
8 Penings	=	1 *Grot	0	$0\frac{9}{20}$
2 Grots	=	1 Petard	0	$0\frac{9}{10}$
6 Petards	=	1 *Scalin	0	$5\frac{2}{5}$
7 Petards	=	1 Scalin	0	$6\frac{3}{10}$
40 Grots	=	1 *Florin	1	6
$17\frac{1}{2}$ Scalins	=	1 Ducat	9	3
240 Grots	=	1 *Pound Flem.	9	0

Table IX. The NETHERLANDS, *continued.*

At *Dunkirk*, *St. Omer*, *Boulogne*, *&c.*

The loweſt piece of money in uſe is a *Denier*, value $\frac{1}{24}$d. ſterling.

			£	s.	d.
12 Deniers	=	1 Sol	0	0	$0\frac{1}{2}$
15 Deniers	=	1 *Petard	0	0	$0\frac{5}{8}$
15 Sous, or Sols	=	1 *Piette	0	0	$7\frac{1}{2}$
20 Sols	=	1 *Livre Tournois	0	0	10
3 Livres	=	1 Ecu of Exchange	0	2	6
24 Livres	=	1 Louis d'Or	1	0	0
24 Livres	=	1 Guinea	1	1	0
$30\frac{3}{8}$ Livres	=	1 Moeda	1	7	0

At *Liſle*, *Cambray*, *Valenciennes*, *&c.* a *Piette = 15 *Petards, and is worth $9\frac{3}{8}$d. ſterling; and 20 Petards = a *Florin, value 1s. $0\frac{1}{2}$d. alſo $10\frac{1}{2}$ Livres Tournois = a Ducat, value 9s. 3d.

Table X. FRANCE.

At *Paris*, *Rouen*, *Bourdeaux*, *Bayonne*, *Marſeilles*, *Lyons*, &c. The loweſt piece of money in uſe is a *Denier*, value $\frac{1}{24}$d. ſterling.

					£	s.	d.
3	Deniers	=	1	Liard	0	0	0$\frac{1}{8}$
2	Liards	=	1	Dardene	0	0	0$\frac{1}{4}$
3	Livres	=	1	Ecu of Exchange	0	2	6
6	Livres	=	1	Ecu	0	5	0
12	Deniers	=	1	Sol	0	0	0$\frac{1}{2}$
20	Sols	=	1	*Livre Tournois	0	0	10
10	Livres	=	1	*Piſtole	0	8	4
24	Livres	=	1	Louis d'Or	1	0	0

Tournois is a term of the ſame import in France, as ſterling in England.

Table XI. SPAIN.

At *Madrid*, *Cadiz*, *Seville*, &c. they reckon by the *New Plate*; and the loweſt piece of money, which is a *Maravedie*, is worth $\frac{43}{272}$d. ſterling.

					s.	d.
2	Maravedies	=	1	Quartil	0	0$\frac{43}{136}$
34	Maravedies	=	1	Rial	0	5$\frac{3}{8}$
2	Rials	=	1	Piſtarine	0	10$\frac{3}{4}$
8	Rials	=	1	*Piaſtre of Exchange	3	7
10	Rials	=	1	Dollar	4	6
375	Maravedies	=	1	*Ducat of Exchange	4	11$\frac{1}{2}$
32	Rials	=	1	*Piſtole of Exchange	14	4
36	Rials	=	1	Piſtole	16	9

At *Gibraltar*, *Malaga*, *Denia*, &c. they reckon by the money of *Veillon*, a *Maravedie* of which is worth $\frac{23}{272}$d. ſterling.

					s.	d.
2	Maravedies	=	1	Ochavo	0	0$\frac{23}{136}$
4	Maravedies	=	1	Quartil	0	0$\frac{23}{68}$
34	Maravedies	=	1	*Rial Veillon	0	2$\frac{7}{8}$
15	Rials	=	1	*Piaſtre of Exchange	3	7
512	Maravedies	=	1	Piaſtre	3	7
60	Rials	=	1	*Piſtole of Exchange	14	4
2048	Maravedies	=	1	Piſtole of Exchange	14	4
70	Rials	=	1	Piſtole	16	9

At *Barcelona*, *Saragoſſa*, *Valencia*, &c. they reckon by the *Old Plate*; a *Maravedie* of which is worth $\frac{27}{128}$d. ſterling.

					s.	d.
16	Maravedies	=	1	Soldo	0	3$\frac{3}{8}$
2	Soldos	=	1	Rial	0	6$\frac{3}{4}$
16	Soldos	=	1	*Dollar	4	6
60	Soldos	=	1	Piſtole	16	9

Table XII. PORTUGAL.

At *Liſbon*, *Oporto*, &c.

The loweſt piece of money made uſe of is a **Re*, value $\frac{27}{400}$d. ſterling.

			£	s.	d.
10 Rez	=	$\frac{1}{2}$ Vintin	0	0	0$\frac{27}{40}$
20 Rez	=	1 Vintin	0	0	1$\frac{7}{20}$
5 Vintins	=	1 Teſtoon	0	0	6$\frac{3}{4}$
4 Teſtoons	=	1 Cruſade of Exchange	0	2	3
24 Vintins	=	1 New Cruſade	0	2	8$\frac{2}{5}$
1000 Rez	=	1 *Milre	0	5	7$\frac{1}{2}$
48 Teſtoons	=	1 Moidore	1	7	0
64 Teſtoons	=	1 Joaneſe	1	16	0

Table XIII. ITALY.

1. The Republic of Genoa and the Iſland of Corſica, containing the towns of *Genoa*, *&c.* *Baſtia*, *&c.*

The loweſt piece of money made uſe of in theſe places is a *Denari*, value $\frac{43}{1200}$d. ſterling.

			s.	d.
12 Denari	=	1 Solidi	0	0$\frac{43}{100}$
20 Solidi	=	1 *Lire	0	8$\frac{3}{5}$
5 Lires	=	1 Croiſade	3	7
5$\frac{3}{4}$ Lires, or 115 Solidi	=	1 *Pezzo of Exchange	4	1$\frac{9}{20}$

Some merchants in *Genoa* keep their accounts in Pezzos, Solidi, and Denari; others in Lires, Solidi, and Denari.

At *Florence*, *Leghorn*, *&c.* in Tuſcany.

The loweſt piece of money in uſe is a *Denari*, value $\frac{5}{144}$d. ſterling.

			s.	d.
4 Denari	=	1 Quatrini	0	0$\frac{5}{36}$
12 Denari	=	1 Solidi	0	0$\frac{5}{12}$
20 Solidi	=	1 *Lire	0	8$\frac{1}{3}$
6 Lires	=	1 Piaſtre of Exchange	4	2
7$\frac{1}{2}$ Lires	=	1 Ducat	5	2$\frac{1}{2}$
22 Lires	=	1 Piſtole	15	6

3. At *Venice*, *Bergamo*, &c. in the republic of *Venice*.

The loweſt piece of money in uſe is a *Picoli*, value $\frac{1}{36}$d. ſterling.

			s.	d.
12 Picoli	=	1 Solidi	0	0$\frac{1}{3}$
6$\frac{1}{2}$ Solidi	=	1 *Groſs	0	2$\frac{1}{6}$
20 Solidi	=	1 *Lire	0	6$\frac{2}{3}$
124 Solidi	=	1 Ducat current	3	5$\frac{1}{3}$
24 Groſs	=	1 *Ducat of Exchange	4	4
54 Solidi	=	1 Teſtoon	1	6

Table XIV. TURKEY and the LEVANT.

Viz. the *Morea*, the *Iſlands of Candia*, *Cyprus*, &c.

The loweſt piece of money uſed in theſe places is a *Mangar*, value $\frac{3}{20}$d. ſterling.

			s.	d.
4 Mangars	=	1 *Aſper	0	0$\frac{3}{5}$
3 Aſpers	=	1 Parac	0	1$\frac{4}{5}$
5 Aſpers	=	1 Beſtic	0	3
10 Aſpers	=	1 Oſtic	0	6
20 Aſpers	=	1 Solota	1	0
80 Aſpers	=	1 *Piaſtre	4	0
100 Aſpers	=	1 Caragrouch	5	0
10 Solotas	=	1 Xeriff	10	0

The above tables of the real and imaginary money of all the principal trading places on the *Continent of Europe*, along the ſea-coaſt, from the *White Sea* to the *Levant*, have, for the ſake of regularity, been given in a geographical order.

Table XV. IRELAND.

Dublin, *Cork*, *Londonderry*, &c.

The Iriſh have no coin of their own, but are ſupplied by the different nations with which they traffic.

A farthing Iriſh, is worth only $\frac{3}{13}$d. ſterling.

			£	s.	d.
2 Farthings	=	1 Halfpenny	0	0	0$\frac{6}{13}$
2 Halfpence	=	1 *Penny	0	0	0$\frac{12}{13}$
6$\frac{1}{2}$ Pence	=	$\frac{1}{2}$ a Shilling	0	0	6
12 Pence	=	1 *Shilling Iriſh	0	0	11$\frac{1}{13}$
13 Pence	=	1 Shilling	0	1	0
65 Pence	=	1 Crown	0	5	0
20 Shillings	=	1 *Pound Iriſh	0	18	5$\frac{7}{13}$
22$\frac{3}{4}$ Shillings	=	1 Guinea	1	1	0

Table XVI. *The Continent of* AMERICA.

The Colonies of *Nova Scotia*, *New England*, *Virginia*, &c.

In the American colonies accounts are kept in pounds, ſhillings, and pence, as in England, and their money is called *currency*. The ſcarcity of caſh obliges them to ſubſtitute a paper-currency, and the courſe of exchange varies in proportion to the ſcarcity of money.

Table XVII. The WEST-INDIA ISLANDS.

Viz. Jamaica, Barbadoes, &c.

A *halfpenny, in *Jamaica*, &c. is worth only $\frac{57}{160}$d. ſterling.

2 Halfpence

			£	s.	d.
2 Halfpence	=	1 *Penny	0	0	$0\frac{57}{80}$
$7\frac{1}{2}$ Pence	=	1 Bit	0	0	$5\frac{11}{32}$
12 Pence	=	1 *Shilling	0	0	$8\frac{11}{20}$
75 Pence	=	1 Dollar	0	4	6
7 Shillings	=	1 Crown	0	5	0
20 Shillings	=	1 *Pound	0	14	3
24 Shillings	=	1 Piſtole	0	16	9
30 Shillings	=	1 Guinea	1	1	0

Note. In the above tables the imaginary money is marked with an aſteriſk *, and the figure, ſtanding at the right hand of the denomination of each foreign piece, is the intrinſic value thereof, according to the lateſt and beſt aſſays made at the mint of the tower of London.—Hereafter follow proper rules for finding what quantity of the money of one country, &c. will be equal to a given quantity of another, according to a given courſe of exchange; and in order to avoid prolixity and unneceſſary repetition, the places are reduced to three claſſes.

CLASS I.

Places which give the uncertain price of exchange for the pound ſterling.

Theſe are HAMBURGH, LUBEC, &c. in GERMANY; the SEVEN UNITED PROVINCES, the NETHERLANDS, &c.

Propoſition 1. *To reduce the currency of any ſtate into bank or exchange money, and the contrary.*

Rule. As 100, with the agio added to it, is to 100, ſo is any given ſum current to its value in bank-money.

And, As 100 is to 100 with the agio added to it, ſo is any given ſum of bank-money to its value current.

Note. The exchange is always ſuppoſed to be made in bank-money, and therefore the currency of any ſtate, or kingdom, which uſes this denomination of money, muſt always be reduced to bank-money before exchange can be made.

Prop. 2. *Given the courſe of exchange between Great-Britain and any foreign country, city, &c. which exchanges for the pound ſterling, to change any given quantity of ſterling money into the money of that country, &c.*

Rule. As £1 ſterling is to the given courſe of exchange, ſo is the given ſum, in ſterling money, to its correſponding value in foreign money.

Note. Whenever the firſt term of a ſtating is 1, as in this propoſition, the work may be performed by *Practice.*

Prop. 3. *Given the courſe of exchange to or from any foreign country, city, &c. which exchanges with Great-Britain for the pound ſterling, to change any given quantity of ſuch foreign money into ſterling money.*

Rule. As the courſe of exchange is to £1 ſterling, ſo is the given ſum, in foreign money, to its correſponding value in ſterling money.

CLASS II.

Places which give the certain ſpecies of their money for the uncertain number of pence ſterling.

Theſe are DENMARK, RUSSIA, POLAND, FRANCE, SPAIN, PORTUGAL, ITALY, &c. and almoſt every other place in the world with which exchanges are made, except thoſe already mentioned in Claſs I. and thoſe which belong to Claſs III.

Prop. 4. *Given the courſe of exchange between Great-Britain and any foreign country, city, &c. which exchanges for any number of pence ſterling, to change any quantity of ſterling money into the money of that country, &c.*

Rule. As the number of pence ſterling, contained in the courſe of exchange, is to the integer of foreign money, ſo is the given ſum, in ſterling money, to its correſponding value in foreign money.

Prop. 5. *Given the courſe of exchange between Great-Britain and any foreign country, city, &c. which exchanges for any number of pence ſterling, to change any quantity of ſuch foreign money into ſterling.*

Rule. As the integer of foreign money is to the number of pence contained in the courſe of exchange, ſo is the given quantity of foreign money to its correſponding value in ſterling money.

CLASS III.

Places which exchange with GREAT-BRITAIN at an advanced rate per cent.

Theſe are the ISLES OF MAN and IRELAND, the WEST-INDIA ISLANDS, and the Continent of AMERICA.

Prop. 6. *Given the courſe of exchange between Great-Britain and any place which gives a variable ſum of money, more than*

than £100 for £100 ſterling, to change any quantity of ſterling money into the currency of that place.

Rule. As £100 ſterling is to £100 with the courſe of exchange per cent. added to it, ſo is the given ſterling money to the currency required.

Note. In this and the following rule, by the courſe of exchange muſt be underſtood the exceſs of the currency above £100. Thus, if £100 ſterling be worth £110 currency, the exchange is at 10 per cent.

This exceſs, were it authoriſed by cuſtom, might be called the *agio*.

Prop. 7. Given the courſe of exchange between Great-Britain and any place which gives a variable ſum of money, more than £100 for £100 ſterling, to change any quantity of the currency of that place into ſterling money.

Rule. As £100, with the courſe of exchange per cent. added to it, is to £100, ſo is the given currency to the ſterling required.

OF THE GAIN OR LOSS PER CENT. BY THE RISING OR FALLING OF THE COURSE OF EXCHANGE.

Prop. 8. To determine the gain or loſs per cent. by the different courſes of exchange with places that exchange by the pound ſterling, or with places that exchange for a variable number of pence ſterling.

Rule. If the gain or loſs *per cent.* be conſidered with reſpect to the *par* of exchange, ſay, As the par of exchange is to £100, ſo is the given *courſe* of exchange to a fourth number; which, if greater than £100, the exceſs will be the gain; but, if leſs than £100, the defect will be the loſs *per cent.*—But, if the gain or loſs *per cent.* be conſidered with reſpect to any other *courſe* of exchange, ſay, As the given *courſe* of exchange is to £100, ſo is the propoſed *courſe* of exchange to a fourth number; which, if greater than £100, the exceſs will be the gain; but, if leſs than £100, the defect will be the loſs *per cent.*

See *Prop.* 2. in *Loſs and Gain.*

Examples to Propoſition 1. (Tables VII. and XIII.)

(1.) A merchant at *Amſterdam* is poſſeſſed of 3750 guilders 10 ſtivers currency, which he wiſhes to turn into bank money, the agio at 4⅛ per cent. what will be the value in guilders bank?

As 104⅛ : 100 :: 3750g. 10s. : 3593g. 5s. $13\frac{149}{167}$pen. anſwer.

(2.) If the agio between the current and bank-money of the United Provinces be $4\frac{3}{8}$ per cent. how many guilders current will be equal to 3593 guilders 5 ſtivers $13\frac{149}{167}$ penings bank?

As 100 : $104\frac{3}{8}$ :: 3593g. 5s. $13\frac{149}{167}$p. : 3750g. 10s. anſwer.

(3.) Change 577 guilders 14 ſtivers current money into florins bank, agio $5\frac{4}{5}$ per cent.?

(4.) Change 765 guilders 9 ſtivers, bank, into current, agio $5\frac{5}{8}$ per cent.?

(5.) In 7570 guilders 15 ſtivers current, how many rix-dollars bank, agio $4\frac{7}{8}$ per cent.?

(6.) If the agio between the current and bank money of the Netherlands be 25 per cent. how many pounds Flemiſh bank will be equal to 797l. Flemiſh current?

(7.) The agio of Venice is 20 per cent. how much current money of Venice will be equal to 790 ducats bank?

Examples to Prop. 2. (Table VI.)

(8.) In 127l. 3s. 4d. ſterling, how many *Hamburgh* marcs, &c. exchange at $32\frac{1}{3}$ ſols gros, or 2328 fennings, per £ ſterling?

As £1 : 2328 fen. :: £127 3 4 : 296044 fen. = 1541 marcs 14 ſols lub. 4 fen.

(9.) How many *Hamburgh* marcs are contained in 4751l. 15s. ſterling, exchange at 2450 fennings per £ ſterling?

(10.) In 475l. 18s. ſterling, how many marcs, &c. exchange at 36s. 6d. Flemiſh per £ ſterling?

(11.) In 749l. 14s. ſterling, how many marcs bank, exchange at 35 ſols gr. 1 den. per £ ſterling, 6 fennings being equal to 1 denier, and 12 deniers to 1 ſol gros?

(12.) In 754l. 18s. 9d. ſterling, how many rix-dollars current, exchange at 34 ſols gr. $9\frac{1}{4}$ den. per £ ſterling, agio $108\frac{1}{4}$?

(Table VII.)

(13.) If I pay 757l. 18s. 7d. in *London*, what muſt I draw my bill for on *Amſterdam*, exchange at 1l. 15s. 9d. Flemiſh per £ ſterling?

(14.) If I pay in *London* 754l. 11s. 9d. ſterling, how many guilders, &c. may I draw for at *Amſterdam*, exchange at 34 ſcalins $4\frac{1}{2}$ grots per £ ſterling?

(15.) In 479l. 14s. ſterling, how many rix-dollars current, agio $4\frac{5}{8}$, and exchange at 34s. $7\frac{1}{2}$d. per £ ſterling?

(Table II.)

(16.) In 547l. 19s. 10d. ſterling, how many copper dollars of *Sweden*, exchange at $47\frac{1}{2}$ copper dollars per £ ſterling?

(17.) In 3749l. 14s. $10\frac{1}{2}$d. how many dollars, &c. exchange at 48 copper dollars per £ ſterling?

Examples to Prop. 3. (Table VI.)

(18.) Reduce 1541 marcs 14 ſols lub. 4 fen. bank-money of *Hamburgh*, into ſterling, exchange at $32\frac{1}{3}$ ſols gros per £ ſterling?—72 fennings being equal to 1 ſol gros.

As $32\frac{1}{3}$ ſols gr. : 1l. :: 1541m. 14ſ. l. 4f. : 127l. 3s. 4d. anſwer.

(19.) In 1788 rix-dollars 21 ſols lub. how many pounds ſterling, exchange at $34\frac{1}{2}$ ſols gros per £ ſterling?

(20.) In 747 rix-dol. 2 marcs 14 ſols lub. how many £ ſterling, exchange at 32s. 6d. Flemiſh per £ ſterling?

(21.) In 743 rix dol. 4 ſols gr. *ſlet* money, agio $4\frac{5}{8}$ per cent. exchange 33s. 9d. Flemiſh per £ ſterling, how many £ ſterling?

(22.) In 1749 marcs 13 ſols lub. current, agio $9\frac{3}{8}$ per cent. and 474 *ſlet* dollars 2 ſols gr. agio $4\frac{3}{8}$ per cent. exchange at 35s. 8d. *Flemiſh*, per £ ſterling, how many £ ſterling?

(Table VII. and VIII.)

(23.) Remitted from *Amſterdam* to *London* a bill of 1747l. 14s. 7d. Flemiſh; how many pounds ſterling is the ſum, exchange at 34s. 7d. Flemiſh per £ ſterling?

(24.) What muſt I draw for at *London* if I pay at *Rotterdam* 7495 guild. 14 ſtiv. current, agio $5\frac{3}{8}$ per cent. exchange at 34 ſcalins 4 grots per £ ſterling?

(25.) A merchant remits a bill of exchange from *Antwerp* to *England*, when the courſe is 34s. 3d. Required the value of 774l. 18s. *Flemiſh* at that rate in *London*?

(Table II.)

(26.) In 7123 copper dollars 14 runſtics, how many pounds ſterling, exchange at $48\frac{1}{2}$ copper dollars per £ ſterling?

(27.) In 5749 ſilver doll. 1 copper doll. 2 copper marcs 3 runſtics, how many £ ſterling?—Exchange at 49 copper dollars per £ ſterling.

Examples to Prop. 4. (Table I.)

(28.) In 747l. 18s. 10d. ſterling, how many rix-dollars of *Denmark*, exchange at 47d. ſterling per rix-dollar?

As 47d. : 1 rix-dol. :: 747l. 18s. 10d. : 3819 rix-dol. 1 marc $10\frac{26}{47}$ ſkillings.

(29.) In 749l. 16s. ſterling, how many rix-dollars, &c. exchange at $49\frac{1}{2}$d. ſterling per rix-dollar?

(Table III.)

(30.) In 7574l. 19s. ſterling, how many *Ruſſian* rubles, &c. exchange at 4s. 7d. ſterling per ruble?

(31.) In 574l. 18s. ſterling, &c. how many rubles, exchange at 4s. $9\frac{1}{2}$d. per ruble?

Examples to Prop. 5. (Table I.)

(32.) In 3819 rix-dollars 1 marc $10\frac{26}{47}$ ſkillings of *Denmark*, how much ſterling money, exchange at 47d. ſterling per rix-dollar?

As 1 rix-dol. : 47d. :: 3819 rix-dol. 1 marc $10\frac{26}{47}$ ſkill. : £747 18 10.

(33.) In 9751 rix-dol. 4 m. 3 ſkill. how much ſterling, exchange at $48\frac{3}{4}$d. ſterling per rix-dollar?

(Table III.)

(34.) In 7454 rub. 4 griv. 6 cop. how many £ ſterling, exchange at 4s. 9d. per ruble?

(35.) In 7479 rubles, how much ſterling, exchange at 4s. $7\frac{1}{2}$d. per ruble?

CLASS II. (Tables III. and VII.)

In the following examples, the rules belonging to the propoſitions hitherto made uſe of are to be uſed occaſionally.

(36.) In 4759 rub. 44 cop. exchange at 124 copecs p rix-dollar current at *Amſterdam*, agio $3\frac{1}{2}$ per cent. ho much ſterling money?—the exchange between *Amſterda* and *London* being 34s. 6d. *Flemiſh* per £ ſterling?

(37.) Remitted from *London* to *Peterſburgh*, by the w of *Amſterdam*, 495l. 17s. 6d. ſterling, the exchange b tween *London* and *Amſterdam* being 34s. 8d. per £ ſterlin and between *Amſterdam* and *Peterſburgh* 52 ſtivers p ruble; what is the value of this remittance in rubles, &c.

(38.) Received from *Archangel*, per bill of exchang 7437 rub. 5 griv. 24 cop. exchange at 121 copecs per ri dollar current of *Amſterdam*, agio $3\frac{1}{8}$ per cent. and 34s. 7 Flemiſh per £ ſterling; what is the value of this bill?

(Tables IV. V. and VII.)

(39.) In 7947 florins of *Dantzic*, exchange at 2 groſhen per £ Flemiſh, and 33s. 5d. Flemiſh per £ ſterlin how much ſterling money?

(40.) In 749l. 17s. 6d. ſterling, how many rix-dolla &c. exchange at 274 groſhen per £ Flemiſh, and 34s. 8 per £ ſterling?

(41.) In 4795 flor. 14 groſhen 2 couſtics, how ma pounds ſterling, exchange at 273 groſhen per £ Flemi current, agio $3\frac{3}{8}$ per hundred guilders, and 33s. 7d. Flemi per £ ſterling?

(Tables IX. and X.)

(42.) In 636 livres Tournois 3 ſols $9\frac{1}{2}$ deniers, how man £ ſterling, exchange at $31\frac{1}{2}$d. per ecu of 3 livres?

(43.) Bought wine of a merchant at *Bourdeaux* to th amount of 57475 livres 6 ſols; for what ſterling mone muſt the merchant draw his bill, exchange at 53d. per ec

(44.) A bill of 759l. 18s. 9d. is remitted to *Paris* by merchant in *London*; what is the value in *French* crown exchange at 54d. per ecu?

(45.) A gentleman (on his travels) received at *Par* 3749 crowns 2 livres 10 ſols for a bill of exchange, th value whereof in *England* was 843l. 14s. 3d. What w the courſe of exchange between *England* and *France*?

(Tab

(Table XI.)

(46.) In 749l. 18s. ſterling, how many piaſtres, or pieces eight, at *Madrid*, exchange at 45⅜d. ſterling per ſtre?

(47.) In 1347 piaſtres 2 rials 24 maravedies of *Madrid*, w much ſterling, exchange at 47½d. per piaſtre?

(48.) In 9749 rials of plate, how many £ ſterling, exnge at 43⅛d. per piaſtre?

(49.) Bought raiſins of a merchant at *Malaga* to the ount of 7549 rials Veillon; for what ſterling money ſt the merchant draw his bill, exchange at 41½d. per ſtre?

(Table XII.)

(50.) In 7434 cruſades 347 rez, how many £ ſterling, change at 65d. per milre?

(51.) A merchant at *Liſbon* remits to *London* 4756 milrez eſtoons 4 vintins 10 rez, exchange at 64½d. per milre; w much ſterling muſt be paid in *London* for this remitce?

(52.) If a bill of 1788l. 17s. ſterling be drawn upon *don*, what is the value at *Oporto*, in milrez, exchange at ½d. per milre?

(53.) If 2000 milrez were paid at *Liſbon* for a bill upon *don* of 666l. 13s. 4d. what was the courſe of exchange?

(Table XIII.)

(54.) How much ſterling money may a perſon receive *London* if he pays in *Genoa* 947 dollars, exchange at ½d. per dollar?

(55.) *London* is indebted to *Genoa* 1749l. 17s. 6d. for w many pezzos may *Genoa* draw on *London*, the exchange 47¾d. per pezzo?

(56.) In 747l. 16s. 4d. ſterling, how many pezzos of *ghorn*, exchange at 46⅝d. per pezzo?

(57.) *London* is indebted to *Leghorn* 7439 pezzos, or ſtres, 9 ſolidi 3 denari; what ſterling money ſtands as equivalent in the *London* merchant's books, the exchange ing at 48⅛d. per piaſtre?

(58.) A bill of 574l. 15s. is remitted to *Florence*, to be paid in *piaſtres* of 6 lires each, exchange at 54d. per piaſtre; how many will be received?

Examples to Prop. 6.

(59.) *London* remits to *Ireland* 574l. 15s. ſterling, how much currency of *Ireland* muſt be received, exchange at 7l. 10s. per cent.?

As 100l. : 107l. 10s. :: 574l. 15s. : 617l. 17s. 1$\frac{1}{2}$d. anſwer.

(60.) The value of 694l. 18s. 6d. ſterling is required in *Iriſh* currency, exchange at £5$\frac{3}{4}$ per cent.

(61.) *London* receives a bill of exchange from *Carolina* for 917l. 18s. ſterling; for how much currency was London indebted, exchange at 76 per cent.?

Examples to Prop. 7.

(62.) *Dublin* draws upon *London* for 879l. 6s. 6$\frac{1}{4}$d. *Iriſh*, exchange at 11$\frac{5}{8}$ per cent. How much muſt *London* pay *Dublin* to diſcharge the bill?

As 111$\frac{5}{8}$l. : 100l. :: 879l. 6s. 6$\frac{1}{4}$d. : 787l. 15$\frac{5}{2679}$ſh.

(63.) What muſt be paid in *London* for a remittance of 6747l. 14s. Iriſh, exchange at 11$\frac{1}{2}$ per cent.?

(64.) *Jamaica* remits to *London* 475l. 14s. 10d. currency; what ſterling money muſt be received for it, exchange being at £135 currency for £100 ſterling?

CLASS II. *exerciſing the 6th and 7th Propoſitions.*

(65.) A merchant in *London* conſigns to his factor, in *Jamaica*, goods amounting to 734l. 14s. 9d. ſterling, which are ſold for 900l. currency; what ſterling ought the factor to remit, after deducting 5 per cent. for his commiſſion and charges; and whether does the merchant gain or loſe, and how much; the exchange being at 25 per cent.?

(66.) My factor at *Barbadoes* bought goods for me to the amount of 7150l. 14s. currency; what is the value in ſterling money, allowing the factor 2$\frac{1}{2}$ per cent. for commiſſion, the exchange being at 35 per cent.?

(67.) A merchant at *Boſton* ſtands indebted to his correſpondent in *London* 7549l. 18s. 4d. currency; what ſterling ſum ſtands as an equivalent in the *London* merchant's books, exchange at 57 per cent.?

(68.) Sold ſugars in *London* for my employer in *Jamaica* to the amount of 1757l. ſterling; what currency ought I to remit, after deducting $2\frac{1}{2}$ per cent. for commiſſion, the exchange between *London* and *Jamaica* being £157 currency for £100 ſterling?

Examples to Prop. 8.

(69.) *London* draws upon *Holland* for a ſum of money when the exchange is at 35s. 6d. Flemiſh per £ ſterling, and afterwards draws again when the exchange is at 34s. 6d. What does *London* loſe or gain *per cent.* by this negociation when compared with the former?

If 35s. 6d. : 100l. :: 34s. 6d. : 97l. 3s. $7\frac{67}{71}$d.

Then 100l.—97l. 3s. $7\frac{67}{71}$d.=2l. 16s. $4\frac{4}{71}$d. loſs per cent.

(70.) *London* draws upon *Amſterdam* for a ſum of money when the exchange is at 34s. 6d. Flemiſh per £ ſterling, and afterwards draws again when the exchange is at 35s. 6d. How much does *London* gain or loſe *per cent.* by this tranſaction when compared with the former?

If 34s. 6d. : 100l. :: 35s. 6d. : 102l. 17s. $11\frac{135}{207}$d.

Then 102l. 17s. $11\frac{135}{207}$d.—100.=2l. 17s. $11\frac{135}{207}$d. gain per cent.

(71.) If the par of exchange between *London* and *Amſterdam* be *$37\frac{1}{9}$s. Flemiſh per £ ſterling, what does *London* gain or loſe per cent. by drawing bills upon Holland at 33s. 4d. Flemiſh per £ ſterling?

(72.) Suppoſe *London* exchanges with *Holland* when the courſe of exchange is at 35s. 6d. per £ ſterling, what will be the gain or loſs *per cent.* to *London*, admitting the *par* of exchange to be 33s. 4d. per £ ſterling?

(73.) A bill of exchange was drawn upon *Amſterdam* when the courſe of exchange was 34s. 3d. Flemiſh per £ ſterling; and, ſome time after, another was drawn, when the courſe of exchange was 33s. 6d. Flemiſh per £ ſterling; what was gained or loſt *per cent.* by this negociation when compared with the former?

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* The table, given at page 387 of the Negociator's Magazine, is calculated upon this principle.

CLASS II.

(74.) The *par* of exchange between *London* and *Portugal* is 5s. $7\frac{1}{2}$d. ſterling per milre; what is gained or loſt *per cent.* in London, when the courſe of exchange is 5s. $2\frac{1}{2}$d. per milre?

(75.) Suppoſe *London* exchanges with *Portugal* for the milre at 5s. 6d. ſterling; and afterwards at 5s. $1\frac{1}{2}$d.— What is gained or loſt per *cent.* by the latter negociation when compared with the former?

(76.) Suppoſe the courſe of exchange between *London* and *Madrid* be $41\frac{7}{8}$d. ſterling per piaſtre, at which time a bill of exchange is drawn by *London*; what would have been the gain or loſs per *cent.* to *London*, had the bill been drawn when the exchange was at $53\frac{1}{2}$d. ſterling per piaſtre, by comparing the latter negociation with the former?

ARBITRATION OF EXCHANGE.

Definition. By Arbitration, or the compariſon of exchange, is to be underſtood a method of remitting to, or drawing upon, foreign places in ſuch a manner as ſhall be moſt advantageous to the merchant.

I. *Simple Arbitration.*

Definition. When the exchanges among three places *only* are concerned, it is called *ſimple* Arbitration, and the *arbitrated* price is ſuch a rate of exchange between two of the places as ſhall be in proportion with the rates aſſigned between each of them and a third.

Note. All queſtions in ſimple arbitration may be ſolved with a little conſideration by one or more ſtatings in the direct or inverſe rule of three. —If a gain or loſs per cent. is mentioned, after you have found the proportional gain or loſs by the rule of three, the gain or loſs per cent. by a variation of exchange, may be found by propoſition 8, preceding, if it has no regard to time. But, if time, commiſſion, brokerage, &c. are conſidered, the ſeveral allowances to be made for theſe purpoſes muſt be calculated by the rules of intereſt, commiſſion, brokerage, &c. previous to the operation for the gain or loſs per cent.

II. *Compound Arbitration.*

Definition. *Compound Arbitration* has reſpect to the exchanges of four or more countries or cities, and its utility conſiſts in diſcovering the beſt and moſt advantageous method of negociating exchanges with different places.

Propoſition. To determine whether a direct or circular exchange will be preferable, having the courſe of exchange between ſeveral places given.

Rule. Diſtinguiſh the ſeveral courſes of exchange into *antecedents* and *conſequents:* place the *antecedents* in one column, and the *conſequents* in another, to the right hand of the *antecedents*, in ſuch a manner that the firſt *conſequent* may be of the ſame name and denomination as the ſecond *antecedent*, and the ſecond *conſequent* as the third *antecedent*, &c. through the whole. Then multiply all the *antecedents* together for a diviſor, and all the *conſequents* together for a dividend; the quotient produced from this diviſor and dividend will be the value of the ſum required. Then calculate the value of the ſum by the direct exchange, or by any other circular exchange; and by comparing theſe values together, may be ſeen which will be the moſt advantageous.

Note. By this rule the weights, meaſures, &c. of different countries may be compared. If an allowance for commiſſion, &c is to be made from place to place, the moſt certain method will be to find the value of the *ſum*, at each place, by the rule of three, and deduct the commiſſion therefrom as you proceed.

Examples in Simple Arbitration.

(77.) When *London* exchanges with *Paris* at 31½d. ſterling per ecu, or crown, and with *Amſterdam* at 34s. 8d. Flemiſh per £ ſterling; what ought the courſe of exchange to be between *Paris* and *Amſterdam*, that a merchant in *London* may remit a ſum of money to *Amſterdam* by way of *Paris*, inſtead of remitting immediately from *London* thither, without loſs?

If 240d. ſterling : 34s. 8d. Flemiſh :: 31½d. ſterling : 54⅗d. Flemiſh.

Therefore the courſe of exchange between Paris and Amſterdam ought to be at 54⅗d. Flemiſh per ecu.

(78.) If *Amſterdam* exchanges with *London* at 33s. 7d. per £ ſterling, and with *Liſbon* at 51½d. Flemiſh for the cruſade of 20 vintins, or 4 teſtoons, how ought the exchange to go between *London* and *Liſbon?*

(79.) A merchant of *Amſterdam* orders his factor at *London* to remit to his correſpondent at *Paris* at 53d. ſterling per crown; and to draw upon *Rotterdam* for the value at 32s. Flemiſh per £ ſterling; but, when the order came to hand, the exchange was on *Paris* at 54d. per *crown.* At what rate of exchange ought the factor to draw upon Rotterdam to execute his orders without loſs to his employer?

(80.) A factor in *London* is ordered to remit to *Venice* at 50d. per ducat, and to draw for the value upon *Madrid* at 42d. per dollar; but, on receipt of the order, bills upon *Venice* were at 53½d. At what rate muſt he draw upon *Spain* to compenſate this loſs?

CLASS II.

(81.) A merchant at *London* is deſirous of transferring a ſum of money to *Amſterdam* in the moſt advantageous manner, either directly to *Amſterdam,* or through *Paris,* at a time when the courſe of exchange between *London* and *Amſterdam* is 34s. 5d. per £ ſterling, and between *London* and *Paris* 31¼d. ſterling per *crown;*—by advice, he finds the courſe of exchange between *Paris* and *Amſterdam* to be 52d. Flemiſh per crown, upon which he remits directly to *Amſterdam,* and draws for the value upon *Paris.* What does he gain per cent. by theſe means; and what would he have loſt per cent. had he remitted the money to *Amſterdam* by way of *Paris,* and then drawn upon *Amſterdam* for the value, ſuppoſing he had received no advice of the courſe of exchange between *Paris* and *Amſterdam?*

(82.) A Spaniſh merchant ordered his factor in *London* to remit the value of 900 ducats to *Venice,* at 50¼d. per ducat, and to draw upon him at *Madrid* for the value at 41d. per piaſtre. When the order arrived, the exchange at *Venice* was at 51d. per ducat, and at *Spain* at 42¼d. per piaſtre; whether did the merchant gain or loſe by this negociation?

(83.) A mer-

(83.) A merchant in *London* remitted to *Amſterdam* 500l. ſterling, at the rate of 18d. ſterling per guilder; his correſpondent at *Amſterdam* was to remit the ſame, by order, to *Bourdeaux*, at 3 guilders per crown, rebating $\frac{1}{3}$ per cent. for his commiſſion: but, when he received this order, the exchange between *Amſterdam* and *Bourdeaux* was at $3\frac{1}{5}$ guilders per crown. The merchant at *London*, not apprized of this, drew upon *Bourdeaux* at 55d. ſterling per crown; whether did he gain or loſe, and how much per cent.?

(84.) A merchant at *Amſterdam* was indebted to another at *Paris* a bill of 3000 florins current, agio 4 per cent. and exchange at $90\frac{1}{2}$d. per ecu of 60 *ſous Tournois*; but, when this bill became negociable, the exchange was down at $89\frac{1}{2}$d. per *crown*, and the agio advanced to 5 per cent. Did the *Paris* merchant gain or loſe by this turn of affairs?

Examples in Compound Arbitration.

(85.) Sold goods to a houſe in *Amſterdam* to the amount of £824 Flemiſh, which my correſpondent adviſes me he will remit; but, as the exchange on *Amſterdam* was ſo low as 34s. 4d. per £ ſterling, I have deſired him to remit it to *France*, at 48d. Flemiſh per crown; thence he orders it to be remitted to *Vienna*, at 100 crowns for 60 ducats; thence to *Hamburgh*, at 100d. Flemiſh per ducat; thence to *Liſbon*, at 50d. Flemiſh per cruſade of 400 rez; and laſtly, from *Liſbon* to *England* at 5s. 8d. ſterling per milre. Whether ſhall I gain or loſe by the circular exchange?

By the circular exchange.

Antecedents.		Conſequents.
48d. Flemiſh	=	1 crown.
100d. crowns	=	60 ducats.
1 ducat	=	100d. Flemiſh,
50d. Flemiſh	=	400 rez, or 1 cruſade.
1 milre, or 1000 rez,	=	68d. ſterling.
How many pounds ſterling	=	824l. Flemiſh.

$$\frac{1\times\cancel{60}\times\cancel{100}\times\overset{2}{\cancel{400}}\times 68\times\overset{103}{\cancel{824}}}{\underset{8}{\cancel{48}}\times\cancel{100}\times 1\times\cancel{50}\times\underset{5}{\cancel{1000}}}=\frac{103\times 68\times 2}{5\times 5}=560l.\ 6s.\ 4\tfrac{4}{5}d.$$

By the direct exchange.

As 34s. 4d. Flemiſh : £1 ſterling :: £824 Flem. : £480 ſterling. Then 560l. 6s. $4\frac{4}{5}$d. − £480 = 80l. 6s. $4\frac{4}{5}$d. advantage by the circular exchange.

(86.) A banker

(86.) A banker in *Paris* remits to his factor at *Amſterdam* 7547 crowns 15 ſols Tournois; firſt to *London* at 30d. per crown; thence to *Rome*, at 65d. per ſtampt crown; thence to *Venice*, at 100 ſtampt crowns for 142 ducats bank; thence to *Leghorn*, at 105 ducats bank for 100 piaſtres; and from *Leghorn* to *Amſterdam*, at 87d. Flemiſh per piaſtre. How many guilders bank will be received at *Amſterdam*, and what will the banker gain, ſuppoſing the direct exchange between *Paris* and *Amſterdam* to be 51d. Flemiſh per ecu?

(87.) A merchant in *London* is deſirous to remit £759 ſterling to *Genoa*. He can remit by way of *Paris*, at 56d. per ecu; thence to *Venice*, at 100 crowns for 60 ducats bank; thence to *Rome*, at 140 ducats bank for 100 ſtampt crowns; and from *Rome* to *Genoa*, at 115 ſtampt crowns for 125 pezzos.—He can likewiſe remit by way of *Amſterdam*, at 33s. Flemiſh per £ ſterling; thence to *Frankfort*, at 2 rix-dollars for 16s. Flemiſh; thence to *Venice*, at 12 ducats for 11 rix-dollars; thence to *Rome*, &c. as above. How many pezzos by each of theſe methods will the merchant have for his money, and which method will be the more advantageous?

CLASS II.

(88.) A merchant in *London* has credit at *Leghorn* for 7547 piaſtres, whence he receives advice that a remittance can be made at 52d. per piaſtre. The merchant upon this orders them to be remitted to *Venice*, at 95 piaſtres for 100 ducats bank; thence to *Cadiz*, at 321 maravedis per ducat; thence to *Liſbon*, at 631 rez per piaſtre; thence to *Amſterdam* at 50d. Flemiſh per cruſade; thence to *Paris*, at 56d. Flemiſh per ecu; and, laſtly, from *Paris* to *London*, at $31\frac{1}{2}$d. per crown. What ought to be the arbitrated price between *London* and *Leghorn*; whether will the merchant gain or loſe, and how much per cent, by the circular exchange?

(89.) I have ordered my factor at *Amſterdam* to remit 1757l. 15s. Flemiſh (the exchange between *London* and *Amſterdam* being 34s. 7d. Flemiſh per £ ſterling) to *France*, at 54d. Flemiſh per ecu; thence to *Venice*, at 100 crowns for 56 ducats bank; thence to *Hamburgh*, at 100d. Flemiſh per ducat; thence to *Portugal*, at 45d. Flemiſh per cruſade;

and thence to *London*, at 63d. per milre. How much ſterling money ought I to receive, allowing my factor $\frac{1}{3}$ per cent. for commiſſion at each place; and whether will be the more advantageous,—the circular or the direct exchange?

(90.) If 100 ℔ weight of *England* make 88 ℔ at *Rouen*, 78 ℔ at *Rouen* 94 ℔ at *Lyons*, 69 ℔ at *Lyons* 53 ℔ at *Geneva*, 72 ℔ at *Geneva* 100 ℔ at *Marſeilles*, 121 ℔ at *Marſeilles* 100 ℔ at *Hamburgh*, 103 ℔ at *Hamburgh* 101 ℔ at *Paris*,—What is the difference between the weight of a pound at *London* and *Paris*?

INVOLUTION.

Definition 1. *When any given number* is multiplied by itſelf, and that product by the ſame number, and ſo on to any aſſigned number of products, the proceſs is called *Involution*, or the involving a number to any aſſigned power.

2. The given number is called the root, or firſt power; the firſt power multiplied by itſelf gives the ſecond power, or ſquare; the ſecond power multiplied by the firſt, gives the third power, or cube; the third power multiplied by the firſt, gives the fourth power, or biquadrate, &c.

3. *The number denoting the power* is called the *index*, or *exponent*, of that power. Thus, if a number is to be involved to the fourth power, then 4 is the index of the power.

4. *Powers are generally denoted* by writing the exponent over the firſt. Thus the ſquare of 205 is written $\overline{205}\,^{2}$, the cube $\overline{205}\,^{3}$; alſo the fourth power of $705 \times 9{\cdot}15$ may be expreſſed thus, $\overline{705 \times 9{\cdot}15}\,^{4}$, &c.

Note 1. A *general* rule for the practice of Involution is evidently contained in the 2d *definition*. A fraction may be involved to any power by a continual multiplication of its terms in a ſimilar manner.

Propoſition. To find the power of any number, above the cube, without finding all the intermediate powers.

Rule. Find, by the ſecond *definition*, two or more ſuch powers of the given number as that the ſum of their indices may make the index of the power required. Th multiply theſe powers continually together, and the l product will be the power required.

2. If any number end with 5 or 6, all the powers of that number w end with 5 or 6.

3. The *ſum* of any two numbers, differing by an unit, is equal to th *difference* of the *ſquares* of thoſe numbers.

4. The *ſum* of any two numbers, multiplied by their *difference*, is equ to the *difference* of the *ſquares* of the ſame numbers.

5. The *ſum*, or *difference*, of any two numbers will meaſure the *ſum*, *difference*, of the cubes of the ſame numbers, *viz.* the ſum will meaſu the ſum, and the difference the difference.

6. The product of two ſquare numbers is a ſquare number, and of tw cubes a cube-number, and ſo on for higher powers; likewiſe every pow of a ſquare number is a ſquare number, and every power of a cube-numb a cube-number, &c.

Examples.

(1.) Involve 1·05 to the 9th power.

1·05×1·05×1·05×1·05×1·05×1·05×1·05×1·05×1·05= 1·551328215978515625, the power required.

Or thus, by the rule, page 175.

1 + 1 + 1 = 3 index of the power.
1·05×1·05×1·05 = 1·157625 power.
3 + 3 = 6 index of the power.
1·157625×1·157625 = 1·340095640625 power.
6 + 3 = 9 index of the power
1·340095640625×1·157625 = 1·551328215978515625 power

(2.) Square 1754.
(3.) Square 549.
(4.) Cube 3·1416.
(5.) Cube ·7854.
(6.) Involve 57·5 to the 4th power.
(7.) Involve 1·732 to the 5th power.
(8.) What is the 9th power of 735?
(9.) Involve 365 to the 6th power?

EVOLUTION.

Definition 1. *The method of finding* the firſt power, or root, by having the *ſecond*, third, &c. power given, is called *Evolution*, or the extraction of roots, and is exactly the e reverſe of Involution. Though, in Involution, there no number whereof we cannot find the exact power, yet, Evolution, there are many numbers of which we cannot d the preciſe root.

2. *The roots which are perfectly accurate* are called *rational ots*, and thoſe roots, which are continually approximating earer to the truth, yet never arrive at it, are called *ſurd-oots.*

3. *Roots are ſometimes denoted* by writing the character √ efore the power, with the index of the root in it; or by utting the index of the root above the power in the form f a fraction. Thus the ſquare-root of 21 may be ex- reſſed by √ 21, or $\overline{21}|^{\frac{1}{2}}$; and the cube-root of 24+7 y $\sqrt[3]{24+7}$, or $\overline{24+7}|^{\frac{1}{3}}$ &c.

Note. There is no ſuch thing, according to our preſent notation of num- ers, as the exact ſquare-root of 2, 3, 5, 6, 7, 8, 10, &c. nor the exact ube-root of 2, 3, 4, 5, 6, 7, 9, &c. Hence, if the root of any number not compoſed of ſome of the natural ſeries, 1, 2, 3, 4, 5, 6, 7, &c. d *infinitum*, it is a *ſurd.*

SQUARE ROOT.

Propoſition 1. *To extract the Square-root of any whole number, or a pure or mixed decimal.*

Rule 1. Put a point over the unit's place, and alſo upon every ſecond figure, from the right hand to the left, in whole numbers, and from the left hand to the right in decimals.

2. Find a ſquare number equal to, or the next leſs than, the left-hand period, and put the root thereof in the quotient; ſubtract this ſquare from the left-hand period, and to the remainder bring down the next period for a dividend.

3. Double the quotient for a diviſor, then conſider what figure muſt be annexed to the right hand thereof, ſo that, if the reſult be multiplied by *that* figure, the product may be equal to, or the neareſt leſs number than, the dividend, and it will be the ſecond figure in the root. Then bring down the next period, double the figures in the quotient for a diviſor, and proceed in all reſpects as above till you have finiſhed the operation.

For the proof. Square the root found, and to that p duct add the remainder, if any; and that ſum will be t ſame as the number given to be extracted.

Squares	1	.	4	.	9	.	16	.	25	.	36	.	49	.	64	.
Roots	1	.	2	.	3	.	4	.	5	.	6	.	7	.	8	.

Hence we may obſerve, that, if any number end with 2, 3, 7, o the ſquare-root of that number can never be exactly found.

Prop. 2. To extract the Square-root of a vulgar fracti

Rule. Multiply the numerator by the denominator, a extract the ſquare-root of the product. The numerator o the given fraction, written above this root, or the deno minator written below it, will expreſs the root of an fraction when reduced to its loweſt terms.

If the product of the numerator by the denominato does not extract even, annex ciphers to the right-ha thereof and continue the root as far as is neceſſary, which divide by the denominator of the fraction to obtain its tru root.

Thus $\sqrt{\frac{N}{D}} = \frac{\sqrt{N}}{\sqrt{D}} = \frac{N}{\sqrt{N \times D}} = \frac{\sqrt{N \times D}}{D}$, whether $\frac{N}{D}$ repreſen a proper or an improper fraction. The product of the numerator by the denominator will always extract even, when the fraction is not a ſurd. Vide note 6, page 176.

Examples to Propoſition 1.

(1.) Extract the ſquare root of 1340095640625.

```
 . . . . . . .
1340095640625 ( 1157625, the root.
1
---
21)34
   21
-------
225)1300
    1125
--------
2307)17595
     16 49
----------
23146)144664
      138876
------------
231522)578806
       46 044
-------------
2315245 11476225
        11476225
```

(2.) Extract the ſquare root of 5678·243.

```
  . .  . .
  5678·2430 &c. ( 75·35 root.
  49
 ----
145)778
    ----
1503)5324
     ----
15065)81530
      -----
       6205 rem.
```

Note. After you have found the root to five or ſix figures, two or three more may be found by plain, or contracted, diviſion.

(3.) What is the ſquare root of 393129?

(4.) Extract the ſquare root of 3272869681.

(5.) Extract the ſquare root of 15241578750190521.

(6.) Required the ſquare root of 57132.

(7.) What is the ſquare root of 75·347?

(8.) Required the ſquare root of 1788·57′.

(9.) What is the ſquare root of ·4325?

(10.) Required the ſquare root of 5·3′.

Examples to Prop. 2.

(11.) What is the ſquare root of $\frac{2025}{2116}$?

2025×2116=4284900, the ſquare root of which is 2070; then $\frac{2025}{2070}=\frac{45}{46}$, or $\frac{2070}{2116}=\frac{45}{46}$, the root required.

(12.) Extract the ſquare root of $\frac{7}{9}$.

7×9=63, the ſquare root of which is 7·9372539332; this root, divided by 9, the denominator, gives ·8819171036, &c. for the ſquare root of $\frac{7}{9}$.

(13.) What is the ſquare root of $\frac{448}{567}$?

(14.) Required the ſquare root of $\frac{276}{739}$.

(15.) Required the ſquare root of $\frac{45}{94}$.

(16.) What is the ſquare root of $15\frac{5}{8}$?

(17.) Required the ſquare root of $29\frac{4}{25}$?

CLASS II.

(18.) A gentleman hired a number of labourers, at a ſhilling per day each, to dig a fiſh-pond. When they had finiſhed their work, their wages amounted together to 120l. 1s. What was the wages of one man, each man worked as many days as there were men in company?

(19.) A num-

(19.) A number of men, drinking porter in Lond ſpent, at a reckoning, half a crown and a farthing; wh they came to pay the landlord, they found that each m had as many farthings to pay as there were men in com pany. Pray how many men were there, and what qua tity of porter did each drink, the price of a pint of port being 1¾d. and the price of half a pint 1d.?

Note. The following examples depend upon the 47th propoſition of firſt book of *Euclid*, where it is demonſtrated that *the ſum of the ſquare two right lines, drawn perpendicular to each other, are perfectly equal to ſquare of a right line joining their extremities.*

(20.) The wall of a town, which is ſurrounded by moat 24 feet wide, is 18 feet high; what length muſt ladder be made to reach from the outer edge of the mo to the top of the wall?

(21.) A ladder, 50 feet long, will reach to a windo 30 feet from the ground on one ſide of the ſtreet; and without moving the foot, will reach a window 40 feet hig on the other ſide. The breadth of the ſtreet is required.

(22.) There are three towers, A, B, and C, ſtanding i a direct line, the heights whereof are 64, 90·249, and 5 feet reſpectively. The diſtance between the top of th tower A and that of B is 97 feet; and the diſtance betwe the bottom of the tower B and that of C is 76 feet. B theſe data it is required to find the diſtances the tops an bottoms of the towers are from each other?

(23.) A gentleman has a garden in the form of an equi lateral triangle, the ſides whereof are each 50 feet: at eac corner of the garden ſtands a tower;—the height of A i 30 feet, that of B 34 feet, and that of C 28 feet. At what diſtance from the bottom of each of theſe towers mu a ladder be placed that it may juſt reach the top of each tower, and what will be the length of the ladder, the ground of the garden being horizontal?

CUBE ROOT.

Prop. 1. *To extract the Cube root of any whole number, o a pure or mixed decimal.*

Rule 1. Put a point over the unit's place, and alſo upo every third figure from the right-hand to the left, in who

numbers

umbers, and from the left-hand to the right in decimals.

2. Find the neareſt leſs cube to the left-hand period, and ſubtract it therefrom: put the root in the quotient, and bring down the figures in the next period for a *reſolvend.*

3. Find a diviſor by multiplying the ſquare of the quotient by 300; ſeek how often it is contained in the *reſolvend,* nd put the anſwer in the quotient.

4. Cube the laſt figure in the quotient, multiply all the igures in the quotient by 30 except the laſt, and that product by the ſquare of the laſt; laſtly, multiply the diviſor by the laſt figure, and the ſum of theſe products will give the *ſubtrahend,* which take from the *reſolvend*; to the remainder bring down the next period, and repeat the work.

For the proof. Cube the root found, and to the product add the remainder, if any, and that ſum will be the ſame as the number given to be extracted.

Cubes	1	8	27	64	125	216	343	512	729.
Roots	1	2	3	4	5	6	7	8	9.

Note. The above rule is the ſame in principle as thoſe given by *Hill, Dilworth, Walkinghame, Vyſe,* and ſeveral other authors; but, when the number to be extracted is large, or has not a *rational root,* and is required to be extracted to ſeveral figures, the operation by this rule is very tedious. The following rule, which I have deduced from the ſame principles as the rule given by Mr. *Ward,* at page 131, 8th edit. of his Math. Guide, will be found preferable in thoſe caſes.

Rule II. 1. Point off the given number as before; ſeek the neareſt leſs cube to the firſt ſignificant left-hand period, and ſubtract it therefrom; put the root in the quotient, and to the remainder bring down *two* periods for a dividend, which divide by 300 times the root: reject the remainder, if any, and put a point over the unit's and hundredth's place of the quotient.

2. Let the root, found above, be a diviſor to *this quotient,* and to the right-hand thereof place ſuch a number that, if the diviſor *thus increaſed* be multiplied thereby, the firſt figure in the product may fall under the left hand point of the *quotient,* and this number will be the ſecond figure in the root, which add to the laſt diviſor for a new one; bring down the two next figures, and divide as before. Then will the root contain 3 figures.

3. *For a ſecond operation.* Make the given number, to be extracted, conſiſt of as many periods as you intend to have figures in the root, (not exceeding 9;) cube the root already found, and ſubtract it therefrom, taking care to put the unit's place exactly under the point belonging to the third ſignificant period; divide the remainder by 3 times the root, with as many ciphers annexed to it as there are ſignificant periods in the given number, abating 3.—Point every ſecond figure in the quotient, beginning at the unit's place, till you have made as many points as there are ciphers in the diviſor. Then let the root be a diviſor to *this quotient*, &c. as above.—No regard need be had to decimals till the operation is finiſhed.

Or, Let N repreſent the number to be extracted, r the neareſt leſs root to the firſt period, with as many ciphers to the right hand thereof as there are periods of whole numbers, abating one; or as many ciphers to the left hand as there are periods of ciphers to the left hand of the ſignificant figure, if a pure decimal.

Then will $\frac{1}{2}r+\sqrt{\frac{N-r^3}{3r}+\frac{1}{4}r^2}$ be the root as above, deduced from problem 91, page 266, of Mr. *Emerſon*'s Algebra. The operation here is to be repeated after you have found three figures; and, if the cube of theſe three figures exceed the number to be extracted, as will ſometimes happen, the right hand figure muſt be diminiſhed by an unit, and then recubed. The firſt operation, in either of the rules, will give 3 figures, the ſecond 9, and the third 27, &c. always triple.

Prop. 2. *To extract the cube root of a vulgar fraction.*

Rule. Reduce the fraction to its loweſt terms, then extract the cube root of the numerator for a new numerator, and the cube root of the denominator for a new denominator; but, if the terms will not extract even, multiply the numerator by the ſquare of the denominator, and the cube root of the product, divided by the denominator, will give the root required.—Or reduce the fraction to a decimal, and then extract the root.

Here $\sqrt[3]{\frac{N}{D}}=\frac{\sqrt[3]{N}}{\sqrt[3]{D}}=\frac{N}{\sqrt[3]{N\times D^2}}=\frac{\sqrt[3]{N\times D^2}}{D}$ as in the ſquare root.

Examples

Examples to Prop. 1, *Rule* 1.

(1.) Extract the cube root of 48627·125.

48627·125(36·5 root.
3 cubed = 27

3 ſquared × 300 = 2700)21627 reſolvend.

6 cubed = 216
3 × 30 × 6 ſquared = 3240
Diviſor 2700 × laſt figure 6 = 16200

19656 ſubtrahend.

36 ſquared × 300 = 388800)1971125 reſolvend.

5 cubed = 125
36 × 30 × 5 ſquared = 27000
Diviſor 388800 × laſt figure 5 = 1944000

1971125 ſubtrahend.

(2.) Required the cube root of 122615327232.
(3.) Required the cube root of 41421736.
(4.) Extract the cube root of 705·919947284.
(5.) Required the cube root of 17·54.
(6.) What is the cube root of 254358061056000?
(7.) The cube root of ·57345 is required.
(8.) Extract the cube root of 75·3857.
(9.) What is the cube root of ·7854?
(10.) Required the cube root of 517·375475.
(11.) Extract the cube root of 20874107909304.
(12.) Extract the cube root of 1551328·215978515625.

Examples to Rule 2.

(13.) Extract the cube root of 98003·449 to 6 places of decimals.

the root.
98003·449 &c. (461·049037, &c.
4 × 4 × 4 = 64

300 × 4 = 1200) 34003449 dividend.

46) 28336 *quotient.*

```
                     . .
            46 ) 28336 quotient.
             6   276
           ----  -----
           521 )  736
                  521
                 -----
                  215
                 .   .   .   .   .   .   .   .   .
                 98003·449000000000000000000
    461×461×461=97972181
                -------------
461×3000000=1383000000)31 268 000 000 000 000 000 000 dividend
                                      . . . . . .
              46104   ) 22 608 821 402 747 quotient.
                  4     184416
              -----   --------
             461089  ) 4167221
                  9    4149801
            --------  ---------
           46109803 )  174204027
                  3    138329409
           --------   -----------
          461098067 )  3587461847
                       3227686469
                       ----------
                       359775378 &c.
                       ----------
```

(14.) What is the cube root of 7154·10916753?

(15.) Extract the cube root of 8302348000000· to four places of decimals.

(16.) Extract the cube root of 2 to eleven places of decimals.

(17.) Extract the cube root of ·0001357 to ten places of decimals.

(18.) Extract the cube root of 13·6′ to nine places of decimals.

(19.) Extract the cube root of 92398647506217 to four places of decimals.

Examples to Prop. 2.

(20.) Extract the cube root of $\frac{243}{576}$.

First $\frac{243}{576}=\frac{27}{64}$; the cube-root of 27 is 3, and that of 64 is 4 therefore the cube root of $\frac{27}{64}$ is $\frac{3}{4}$.

(21.) Extract the cube root of $\frac{2}{3}$.

2×3 ſquared=18, the cube root of which (according to the preceding mples) is 2·620741, &c. This, divided by 3, gives ·87358, &c. the cube root of $\frac{2}{3}$; or $\frac{2}{3}$=·66666, &c. the cube root of which is 87358, &c. as before.

(22.) What is the cube root of $\frac{175}{476}$?

(23.) Required the cube root of $\frac{1912}{3078}$.

(24.) What is the cube root of $\frac{81}{375}$.

(25.) Required the cube root of $\frac{5}{9}$?

(26.) What is the cube root of $\frac{23}{24}$?

CLASS II.

The following queſtions depend upon the 12th and 18th propoſitions of the 12th book of *Euclid*, and the 33d propoſition of the 11th book; where it is demonſtrated, *that all ſolid bodies are in proportion to each other as the cubes of their ſimilar ſides, diameters, lines,* &c.

(27.) If the diameter of a globe be 1 inch, its ſolidity will be ·5236 inch; what will be the ſolidity of a globe of 15 inches diameter?

(28.) The ſolid content of a block of marble is 31185 inches; what will be the ſide of a cubical piece of equal ſolidity?

(29.) A maltſter agreed with a carpenter to make him a cubical bin, to hold 60 quarters of barley; what will be the internal length of one of its ſides, 2150·42 cubic inches being a *Wincheſter* buſhel?

(30.) If a ſtone, 20 inches long, 15 inches broad, and 8 inches thick, weighs 217lb. what will be the length, breadth, and thickneſs, of a ſimilar ſtone that weighs 9000lb.?

(31.) Admit the length of a ſhip's keel to be 125 feet, the breadth of the mid-ſhip beam 25 feet, and the depth of the hold 15 feet; required the dimenſions of two other ſhips, of a ſimilar conſtruction, the one to carry 3 times, the other $\frac{1}{2}$, the burthen of that given above?

(32.) A ſugar-loaf, in the form of a cone, the perpendicular height whereof is 20 inches, is to be divided into 3 equal parts; what will be the perpendicular height of each part?

TO EXTRACT ANY ROOT OF A POWER.

Rule. Point the root into periods as the queſtion requires. Find the neareſt root to the firſt period, and ſubtract its power therefrom; to the remainder bring down the firſt figure in the next period for a *dividend.* Involve the root to the next lower power, and multiply it by the index of the given power for a *diviſor*, the quotient is the next figure in the root. Then involve the whole root as before, and ſubtract. Repeat the operation till all the figures are brought down.

1. For a fraction $\sqrt[n]{\frac{N}{D}} = \frac{\sqrt[n]{N}}{\sqrt[n]{D}} = \frac{N}{\sqrt[n]{N\times D^{n-1}}} = \frac{\sqrt[n]{N\times D^{n-1}}}{D}$ univerſally, where N = the numerator, D = the denominator, and n the index of the root.

2. When the index of the power to be extracted is a compoſite number, the work may be performed more conciſely than by this general rule. Indeed, rules of this kind will never be made uſe of, except by thoſe who have not acquired ſuch a knowledge of the mathematics as will enable them to make uſe of better methods. Thus, the ſquare root of the ſquare root = the biquadrate, or fourth, root, for $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$. The cube root of the ſquare root, or the ſquare root of the cube root, = the ſixth root, for $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$. The ſquare root of the fourth root = the eighth root, for $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$, or $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$. The cube root of the cube root = the ninth root, for $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$, &c.

Examples.

(33.) Extract the 5th root of 30768282110671 5625.

$\dot{3}07\dot{6}8282\dot{1}1067\dot{1}5625$ (3145 root.

$\overline{3}\rceil^{5}=3\times3\times3\times3\times3=\underline{243}$ ſubrtahend.

$\overline{3}\rceil^{4}\times5=405$) $\underline{646}$ firſt dividend.

$\overline{31}\rceil^{5}=\underline{28629151}$ ſubtrahend.

$\overline{31}\rceil^{4}\times5=4617605$) $\underline{21391311}$ ſecond dividend.

$\overline{314}\rceil^{5}=\underline{3052447761824}$ ſubtrahend.

$\overline{314}\rceil^{4}\times5=48605856080$) $\underline{243804492431}$ third dividend.

$\overline{3145}\rceil^{5}=\underline{30768282110671 5625}$ ſubtrahend.

(34.) Extract the ſquare root of 2.

(35.) Required the cube, or third, root of 5.

(36.) What is the 4th root of 1728?

(37.) Required the 5th root of 57·54.

(38.) Required the 6th root of 3·1416?

(39.) Required the 7th root of 547·5.

(40.) What is the 8th root of 547·5?

(41.) Required the 9th root of 1·551328215978515625.

DUODECIMALS.

Definition. *Duodecimals* are ſo called becauſe every ſuperior place is 12 times its next inferior in that ſcale of notation. This way of conceiving an unit to be divided is chiefly in uſe among *artificers*, who generally take the linear dimenſions of their work in *feet*, *inches*, and *parts*.

Note, 12 Inches′ = 1 Foot, | 12 Thirds‴ = 1 Second″,
12 Seconds″ = 1 Inch, | 12 Fourths iv = 1 Third‴, &c.

Different works are computed by different meaſures, viz. glaſing, &c. by the foot; painting, plaſtering, paving, &c. by the yard; flooring, roofing, tiling, &c. by the ſquare of 100 feet; bricklayer's work, &c. by the rod of $16\frac{1}{2}$ feet, the ſquare of which is $272\frac{1}{4}$ feet.—I ſhall here obſerve, that bricklayers always value their work at the rate of $1\frac{1}{2}$ brick thick, therefore the content of the wall, &c. muſt be multiplied by the number of $\frac{1}{2}$ bricks it is in thickneſs, and then be divided by 3, before the value of the work is eſtimated. Several other obſervations, equally uſeful, might here be inſerted, but this part rather belongs to *menſuration* than *arithmetic*.—See my edition of *Hawney's Menſuration.*

A general rule for multiplying duodecimally, or ſquaring the dimenſions of artificers work.

Under the multiplicand write the correſponding denominations of the multiplier. Multiply each term in the multiplicand, beginning at the loweſt, by the feet in the multiplier; write each reſult under its reſpective term, obſerving to carry an unit for every 12, from each lower denomination to its next ſuperior. In the ſame manner multiply all the multiplicand by the inches in the multiplier, and write the reſult of each term one place removed to the right-hand of thoſe in the multiplicand. Work in a ſimilar manner with the ſeconds in the multiplier, ſetting the reſult of each term removed two places to the right-hand of thoſe in the multiplicand. Proceed in like man with the reſt of the denominations, and their ſum will gi the anſwer required.

Note. This may be performed by the rule of practice; thus, after have multiplied by the feet, take aliquot parts of the multiplicand, wi the inches, &c. Or the inches, &c. may be reduced to the fraction of foot by Prop. 10. Vulgar Fractions, and then multiplied together. O turn the inches, &c. into the decimal of a foot, by Prop. 2, Rule 2, i Reduction of Decimals, and then multiply them together by ſome of th rules in Multiplication of Decimals. By reducing the inches, &c. int decimals of a ſuperior name, it will often happen that theſe decimals wi be infinite; and hence the ſcholar may have a good opportunity of ex mining the truth and certainty of the rules I have laid down for managi recurring, or infinite, decimals; for, though the multiplier and multi plicand may be infinite in a decimal ſcale, yet they will be finite in a fra tional, or duodecimal, one.

Examples.

(1.) Multiply 4ft. 6in. 5 parts by 9ft. 4in. 7 parts.

	Ft	In.	Pts			
	4	6	5			
	9	4	7			
	40	9	9			prod. by 9 feet.
	1	6	1	8		ditto by 4 in.
		2	7	8	11	ditto by 7 pts.
Prod.	42	6	6′	4″	11‴	

By Practice.

		Ft	In.	Pts		
4 in.	$\frac{1}{3}$	4	6	5		
				9		
		40	9	9		
6′	$\frac{1}{8}$	1	6	1	8″	
1′	$\frac{1}{6}$		2	3	2	6‴
				4	6	5
		42	6	6	4	11

Note. The ſame anſwer may be *exactly* found either by fractions or decimals.

(2.) Mult. 7ft. 5in. by 4ft. 7in.
(3.) Mult. 9ft. 6in. by 8ft. 7in.
(4.) Mult. 3ft. 11in. by 9ft. 10in.
(5.) Mult. 25ft. 6in. by 34ft. 9in.
(6.) Mult. 15ft. 7in. by 5ft. 11in.
(7.) Mult. 297ft. 9in. by 7ft. 10in.
(8.) Mult. 77ft. 3in. 6pts by 54ft. 4in. 7pts.
(9.) Mult. 15ft. 3in. 6pts. 5″ by itſelf.
(10.) Multiply 10ft. 4in. 5pts. by 7ft. 8in. 9pts.
(11.) Multiply 25ft. 11in. 6pts. 8″ 7‴ by itſelf.

CLASS II.

(12.) If a window be 7ft. 3in. high, and 3ft. 5in. oad, how many ſquare feet of glaſing are contained erein?

(13.) There is a houſe with three tiers of windows, in a tier; the height of the firſt tier is 6ft. 11in. of the cond 5ft. 4in. and of the third 4ft. 3in. the breadth of ch window is 3ft. 6in. What will the glaſing come to 14½d. per foot?

(14.) What will the paving a court-yard come to at s. 4d. per yard, the length being 24ft. 5in. and breadth 2ft. 7in.?

(15.) What will be the expence of paving a rectan-ular court-yard, its length being 62ft. 7in. and breadth 4ft. 5in. and in which there is laid a foot-path the whole ength of it, and 5½ feet broad, with broad ſtones at 3s. er foot, the reſt being paved with pebbles at half a crown foot?

(16.) If the national debt be £239219796 7s. 9d. how ong a foot-path, of a yard wide, would this ſum pave f reduced to guineas?—a guinea being one inch in diameter?

(17.) What will be the expence of plaſtering a ceiling t 11½d. per yard, ſuppoſing the length 22ft. 7in. and breadth 13ft. 11in.?

(18.) A gentleman had a room painted at 8½d. per yard, the meaſure whereof is as follows: the height 11ft. 7in. the compaſs 74ft. 10in. the door 7ft. 6in. by 3ft. 9in. 5 window-ſhutters, each 6ft. 8in. by 3ft. 4in. the breaks in the windows 14in. deep and 8ft. high, the chimney 6ft. 9in. by 5ft. the ſhutters and doors being coloured on both ſides; what will the whole come to?

(19.) If a houſe meaſures 57ft. 7in. in length, and 31ft. 5in. in breadth, and if the roof be of a true pitch, what will it coſt roofing at half a guinea per ſquare?

(20.) How many ſquare rods are there in a wall 63½ feet long, 14ft. 11in. high, and 2½ bricks in thickneſs?

(21.) Admit the end-wall of a houſe to be 28ft. 10in. in breadth, and the height of the roof from the ground 55ft. 8in. the gable (or triangular part above the ſide walls)

walls) to riſe 42 courſes of bricks, reckoning 4 courſes a foot; and that 20 feet high be 2½ bricks thick, 20 fe more 2 bricks thick, and the remaining 15ft. 8in. 1½ bri thick. What will the work come to at 5l. 16s. per ro the gable being 1 brick in thickneſs?

AN USEFUL COLLECTION OF BILLS OF PARCELS, &c.

CLASS I.

Exercifing the Rules in COMPOUND MULTIPLICATION.

(1.) James Lamb, eſq.
Bought of John Simpſon, Jan. 1, 1798.

7½ lb. of green tea, at 10s. 4d. per lb. £
14¼ of fineſt bloom, at 14s. 8d. ——
10¼ of fine green, at 16s. 5d. ——
21 of hyſon at 10s. 10¾d. ——
19 of good hyſon, at 13s. 9¼d. ——
8¾ of bohea, at 6s. 9d. ——

£

(2.) Sir John Guchim,
Hull, 1798. To S. Jefferſon, Dr.

Jan. 11. For 37¾ yds of ſheeting, at 1s. 4½d. per yard —— £
Feb. 3. For 43⅛ yds of lace, at 4s. 0¾d. per yard —— ——
—— 16. For 75⅜ ells of Iriſh, at 2s. 3d. per ell
May 12. — 209 — of dowlas, at 9¼d. ——
—— 15. — 730 — of muſlin, at 7s. 0½d. ——

£

Received the contents,

S. Jefferſon.

(3.) Andrew Wines, eſq.
To W. Johnſon and Co. Dr.

ondon, 1798.
uly 11. 473⅛ gallons of Britiſh ſpirits, at 4s. 7½d.
per gallon ——— £
——— 308½ gallons of fine old rum, at 9s. 10d.
per gallon ——— ———
——— 610⅝ gallons of Holland's gin, at 5s. 2d.
per gallon ——— ———
Aug. 5. 207¼ gallons of rum, at 8s. 9¾d. per
gallon ——— ———
——— 119½ gallons of coniac brandy, at 10s. 0½d.
per gallon ——— ———
Sept. 22. 401⅛ gallons of Maidſtone gin, at 4s. 6d.
per gallon ——— ———

£

Received Dec. 24, 1798, the Contents, for ſelf and Co.
W. Johnſon.

(4.) George Veres, eſq.
Bought of Charles Weſt,

London, December 8, 1798.
A loin of lamb, weight 7¼lb. at 10¾d. per lb £
A fillet of veal, weight 16¾lb. at 6½d. ———
A buttock of beef, wt. 37½lb. at 4½d. ———
A pig, ——— weight 12¾lb. at 7¾d. ———
A leg of pork, weight 16¼lb. at 5½d. ———
A leg of mutton, weight 13¾lb. at 4¾d. ———

£

(5.) Hugh Abbot,
Bought of C. Hartley,
Lonaon, Aug. 19, 1798.
174$\frac{5}{8}$ ℔ of Quinquina, at 3l. 14s. 8d. per ℔ £
321$\frac{3}{5}$ ℔ of gum lac, at 5s. 9d. ——
607$\frac{1}{8}$ ℔ of rhubarb, at 12s. 4d. ——
720$\frac{1}{2}$ ℔ of maſtich, at 1s. 0$\frac{1}{4}$d. ——
509$\frac{1}{5}$ ℔ of ſaſſafras, at 6$\frac{1}{2}$d. ——

£

Received at the ſame time, the contents,
C. Hartley.

(6.) Miſs Evitt,
Bought of William Wilſon,
London, Sept. 22, 1798.
19$\frac{1}{4}$ yards of Flanders lace, at 9s. 8d. per yard £
27$\frac{5}{8}$ —— of Dreſden lace, at 15s. 5$\frac{1}{2}$d. ——
113$\frac{1}{2}$ —— of gauze, at 2s. 2$\frac{3}{4}$d. ——
215$\frac{1}{4}$ —— of muſlin, at 7s. 5$\frac{1}{4}$d. ——
25$\frac{5}{6}$ dozen of napkins, at 27s. 6d. per doz.
118 pair of kid gloves, at 1s. 8d. per pair.

£

(7.) Mr. Crowther,
Bought of Caroline Cockayne,
London, October 5, 1798.
114 yards of muſlin, at 8s. 4$\frac{1}{2}$d. per yard £
17$\frac{3}{5}$ —— of Holland, at 4s. 6d. ——
715$\frac{1}{2}$ —— of cambric, at 10s. 7d. ——
126$\frac{1}{4}$ ells of dowlas, at 1s. 2$\frac{1}{2}$d. per ell —
221$\frac{1}{4}$ —— of Iriſh, at 2s. 9$\frac{1}{2}$d. ——
419$\frac{3}{4}$ —— of chints, at 5s. 10d. ——

£

Received at the ſame time, the contents,
Caroline Cockayne.

(8.) Mrs. Mertown,

Bought of John Linſdall,

Wineſtead, July 5, 1798.

Tares, 104 buſhels, at 2s. 7d. per buſh. £

Peas, 127 buſh. 3 pecks, at 1s. 10$\frac{1}{2}$d. ——

Malt, 46$\frac{1}{4}$ quarters, at 1l. 14s. 4d. per qr.

Oats, 204$\frac{5}{8}$ —— at 18s. 6d. ——

Beans, 12$\frac{7}{8}$ —— at 1l. 17s. 3d. ——

£

(9.) Mr. Ochterlony,

To Hudſon and Co. Dr.

London, 1798.

Sept. 19. 170 pieces of Norwich crapes, at 2l. 7s. 8d. per piece — £

—— 204 pieces of Kendal cottons, at 1l. 16s. 3d. per piece ——

Oct. 5. 175$\frac{1}{3}$ yards of Lancaſhire ſheeting, at 1s. 2$\frac{3}{4}$d. per yard —

—— 698$\frac{2}{5}$ yards of Mancheſter velvet, at 7s. 8d. per yard — —

Nov. 7. 537$\frac{1}{8}$ ells of Yorkſhire drab, at 3s. 4d. per ell — —

—— 1001 ells of ditto foreſter, at 6s. 8d. per ell —— ——

£

(10.) Sir George Lovell,

Bought of Simpſon and Co.

London, Dec. 5, 1798.

2 pieces of fuſtian, each 27 yds, at 1s. 4d. pr yd £

5 —— of Iriſh, each 25 yds, at 2s. 4d. ——

7 —— of check, each 31 yds, at 10d. ——

8 —— of dowlas, each 29 yds, at 7$\frac{3}{4}$d. ——

12 —— of plaid, each 37 yds, at 1s. 10d. ——

11 —— of dimity, each 18 yds, at 2s. 1$\frac{1}{2}$d. ——

£

CLASS II.

Exerciſing the RULE OF THREE, *or* PRACTICE.

(11.) Mr. Meaſurewell,

Bought of Thomas O'Keefe,

Dublin, May 5, 1798, *Six parcels of muſlin, viz.*

	E.Flem.	qr.	n.		
No. 1.	24	2	1,	at 6s. 9d. per yard	£
2.	27	1	3,	at 7s. 2d.	———
3.	21	0	2,	at 8s. 4d.	———
4.	34	1	1,	at 5s. 3d.	———
5.	19	2	2,	at 7s. 9d.	———
6.	27	1	3,	at 7s. 2½d.	———

£

(12.) Mr. Cudworth,

Bought of Samuel Soar,

London, July 4, 1798.

Cwt.	qr.	lb.		
14	1	19	of tobacco, at 4l. 17s. 2d. per cwt.	£
17	2	17	of ſnuff, at 5l. 19s. 4d.	———
18	3	16	of tobac. in leaf, at 3l. 10s. 8d.	—
9	0	15	of ſugar, at 2l. 12s. 6d.	———
	3	10	of ſoap, at 2l. 17s. 4d.	———
49	1	9	of molaſſes, at 1l. 16s. 4d.	———

£

(13.) Anthony How, eſq.

Dr.

To 4 puncheons of Jamaica rum, each 48 gall.
at 12s. 9d. per gallon ——— £
To 7 pipes of mountain, at 6s. 5d. per gall.
To 5 hhds of malaga, at 8s. 6d. ———

£

Cr.

By our bill on Geo. Giles, efq. exchange at 4s. 2d½. per crown for 450 crowns £

By ditto on Monfieur Arbre, exchange at 4s. 5d. per crown, for 840 crowns — —

By ditto on Mr. Cordigon, exchange at 4s. 9d. per crown, for $1090\frac{4}{5}\frac{5}{7}$ crowns ——

Errors excepted, £

London, Geo. Keith and Co.

Sept. 22, 1798.

(14.) George Germaine, efq.

To Fairbank, Elmer, and Co. Dr.

London, 1798.

May 15. 4½ pieces of muflin, each 37¼ yds, at 10s. 7½d. per ell Englifh £

—— 7¾ pieces of chints, each 47½ yds, at 4s. 6¼d. per ell Eng. ——

June 11. 4⅛ pieces of Holland, each 25½ ells Fl. at 3s. 10d. per yard — —

—— 10⅜ pieces of ferge de Nifme, each 19⅓ ells Fr. at 2s. 9¼d. per yard

July 1. 1749⅛ yds of Kendal cottons, at 9⅛d. per ell Flemifh —— —

—— 947¼ yds of Manchefter ftuff, at 10¾d. per ell Flemifh — ——

£

(15.) William Weft, efq.

Bought of Daniel Jones,

London, May 19, 1798,	oz.	dwt.	gr.		
A punch bowl, weight	24	10	4	at 6s. 5d. per oz.	£
A tankard, ——	15	7	6	at 5s. 10d.	——
A tea-pot and lamp, –	35	8	9	at 5s. 7½d.	——
12 plates, ——	126	2	7	at 5s. 8¾d.	——
18 fpoons, ——	41	0	16	at 6s. 1d.	——
A waiter, ——	16	8	0	at 6s. 2½d.	——

£

(16.) Theodore

(16.) Theodore King, eſq.
Bought of James Bird,
Hull, June 15, 1798.
Shalloon, 174 ells Eng. at 14$\frac{1}{4}$d. per yard £
Muſlin, 24$\frac{3}{4}$ ells Flem. at 4s. 9d. per yard
Ruſſia tick, 17 pieces, each 71$\frac{1}{2}$ ells French, at
1s. 1$\frac{1}{8}$d. per yard —— ——
Callico, 20 pieces, each 34$\frac{1}{4}$ yards, at 4s. 9$\frac{3}{4}$d.
per ell Engliſh —— ——
Camblet, 174 yards, at 1s. 0$\frac{1}{2}$d. per ell Eng.
Yorkſhire drab, 1000 yards, at 3$\frac{1}{6}$s. ——

£

Received at the ſame time, the contents,
James Bird.

(17.) Mr. Torin,
To Norris and Co. Dr.
Beverley, 1798.
Feb. 5. Palm-ſack, 12$\frac{1}{2}$ doz. at 2l. 7s. 8d. per doz. £
May 11. Port, red, 1$\frac{3}{4}$ hhd, at 6s. 9d. per gall.
—— Claret $\frac{1}{2}$ — at 10s. 11d. ——
June 5. Liſbon, white, 31$\frac{1}{4}$ gal. at 1s. 10d. per qt
—— Rheniſh 17$\frac{3}{4}$ — at 2s. 7$\frac{1}{2}$d. ——
July 4. Sherry 25$\frac{1}{2}$ — at 6s. 4d. per gal.

£

Received, Sept. 22, 1798, the contents, in full,
J. Norris.

(18.) Valentine Fawkes, eſq.
Bought of William Vickerman,
Hull, Auguſt 5, 1798.
194 yds 1qr. 2n. of muſlin, at 5s. 9d. per yard £
1761 1 3 of linen, at 2s. 3d. ——
47 ells Eng. 1qr. 1n. of velvet, at 10s. 7$\frac{1}{2}$d. ——
10 pieces of chints, ea. 27$\frac{1}{4}$ yds, at 3s. 8d. per ell E.
7 pieces of cotton, ea. 31$\frac{1}{2}$ yds, at 1s. 10d. ——
9 pieces of ditto, each 34 ells E. at 1s. 9$\frac{1}{2}$d. per yd

£

(19.) Mr. Carpenter,

To George Minot, Dr.

ondon, 1798.

an. 1ſt. For 17cwt. 1qr. 15lb. of Virginia tobacco, at 10l. 10s. per cwt. £

7. For 14cwt. 2qr. 4lb. of Jamaica ſugar, at 4l. 11s. 2d. per cwt. ——

arch 9. For 15cwt. 11lb. of Barbadoes ditto, at 4l. 4s. per cwt. ——

—— For $179\frac{3}{4}$lb. of Jamaica pepper, at $7\frac{1}{4}$d. per lb. —— ——

ay 15. For 4 puncheons of rum, each 84 gall. at 5s. 10d. per gallon —

£

eceived, Aug. 5, 1798, the contents,

George Minot.

(20.) Mr. Willet,

Bought of William Winch,

London, Nov. 4, 1798.

47 pieces of ſhalloon, ea. $47\frac{1}{8}$yds, at 1s. $9\frac{1}{2}$d. pr ell £
16 — of camblet, ea. $39\frac{1}{4}$ — at 1s. $6\frac{3}{4}$d. ——
$27\frac{1}{2}$ — of drugget, ea. $27\frac{1}{4}$ — at 2s $2\frac{1}{4}$d. pr E.Fr.
49 — of calimanco, ea. $34\frac{1}{4}$ — at 1s. $8\frac{1}{2}$d. ——
$19\frac{1}{8}$ — of callico, ea. 42 ells E. at 2s. $11\frac{1}{2}$d. pr yd
17 — of chints, ea. $41\frac{1}{2}$ —— at 4s. $10\frac{1}{4}$d. ——

£

CLASS III.

Exercising the RULE of THREE, *or* PRACTICE; *and* TARE and TRET.

(21.) Mr. Cole,

Bought of George Mitchell,

London, May 1, 1798.

cwt.	qr.	℔.		
16	1	19	groſs of ſugar, tare 124℔. at 3l. 10s. per cwt. neat —	£
21	2	17	—— of ditto, tare 137℔. at 4l. 4s. per cwt. neat	——
19	1	21	—— of raiſins, tare 96℔. at 2l. 7s. per cwt. neat	——
11	3	14	—— of currants, tare 85℔. at 2l. 10s. 4d. per cwt. neat —	
5	1	17	—— of Pimento, tare 47℔. at 5l. 5s. per cwt. neat	——
7	2	19	—— of ginger, tare 74℔. at 5l. 6s. 6d.	
				£

Received at the ſame time, the contents,

George Mitchell.

(22.) Mr. George Lane,

Bought of James Khuff, 5 bags of cotton, *viz.*

London, June 5, 1798.

	Cwt.	qr.	℔.		qr.	℔.	
No. 1.	5	1	4	groſs, tare	1	4	l. s. d.
2.	7	2	11	—— —	2	5½	at 4 18 11
3.	4	3	9	—— —		21¼	per cwt.
4.	5	0	14	—— —	1	19¾	neat £
5.	6	2	17	—— —	2	14½	
							£

(23.) Meſſrs. Langton and Co.

To Stephen Memprize, Drs.

ull, 1798.

pril 8. To 17cwt. 2qr. 24℔ groſs of lump ſugar, tare 14℔ per cwt. at 4l. 17s. 6d. per cwt. neat —— £

—— To 27cwt. 1qr. 19℔ groſs of double refined ſugar, tare 16℔ per cwt. at 5l. 5s. per cwt. neat —— —

ay 10. To 19cwt. 3qr. 16℔ groſs of rice, tare 8℔ per cwt. at 1l. 10s. 4d. per cwt. neat

— 17. To 10cwt. 8℔ groſs of Malaga raiſins, tare 14℔ per cwt. at 3l. 1s. 5d. per cwt. neat —— ——

une 6. To 8cwt. 3qr. 7℔ groſs of currants, tare 7℔ per cwt. at 2l. 17s. 8d. per cwt. neat —— ——

—— To 1cwt. 1qr. 21℔ of pepper, tare 12℔ per cwt. at 6l. 8s. 2d. per cwt. neat —— ——

£

Received, July 17, 1798, 50l. 10s. 6d. in part of this ill,

Stephen Memprize.

(24.) Mr. Henry Chapman,

Bought of George Evitt, 5 barrels of indigo,

ondon, May 1, 1798.

	Cwt.	qr.	℔					
No. 1. qt	10	2	14	groſs, tare	7℔	per cwt.	at 2s.4⅛d. per lb. neat.	£
2. —	11	3	12	——	— 7	—		
3. —	12	1	17	——	— 8	—		
4. —	9	2	14	——	— 8	—		
5. —	10	1	14	——	— 7	—		

£

(25.) Mr. Amutie,

Bought of William Wilſon,

London, March 5, 1798.

7 hhds of ſugar, each 10cwt. 1qr. 12lb. groſs, tare 17lb. per hhd, at 2l. 8s. 10d. per cwt. neat £

3 hhds of pimento, each 4cwt. 7lb. groſs, tare 21lb. per hhd, at 5l. 1s. 6d. per cwt. neat ——

5 hhds of ginger, each 7cwt. 3qr. groſs, tare 13lb. per hhd, at 6l. 7s. 4d. per cwt. neat ——

6 hhds of pepper, each 3cwt. 2qr. 9lb. groſs, tare 19lb. per hhd, at 5l. 7s. 3d. per cwt. neat ——

8 hhds of tobacco, each 12cwt. 1qr. 24lb. groſs, tare 29lb. per hhd, at 6l. 6s. 8d. per cwt. neat

£

(26.) Francis Clarke, eſq.

Bought of John Jenkins,

London, April 9, 1798.

Five buts of currants, *viz.*

No. 1. 4cwt. 1qr. 12lb. groſs, tare 19lb. per cwt. tret 4lb. per 104lb.

2. 9cwt. 2qr. 17lb. groſs, tare 21lb. per cwt. tret 4lb. per 104 —

3. 8cwt. 3qr. groſs, tare 9lb. per cwt. tret 4lb. per 104 —

4. 7cwt. 11lb. groſs, tare 47lb. in the whole, tret 4lb. per 104 —

5. 9cwt. 1qr. 9lb. groſs, tare 7lb. per cwt. tret 4lb. per 104 —

at £2$\frac{1}{12}$ per cwt. neat

£

(27.) Granville King, eſq.

Bought of John Ruſſell,

ndon, May 10, 1798.

obacco in leaf, 19cwt. 1qr. 27lb. groſs, tare 149lb. at 5l. 0s. 4d. per cwt. neat — £
itto in rolls, 12cwt. 3qr. 19lb. groſs, tare $48\frac{1}{2}$lb. at 5l. 17s. 8d. per cwt. neat — —
mento, 4cwt. 2qr. 25lb. groſs, tare $17\frac{3}{4}$lb. at 7l. 13s. 5d. per cwt. neat — —
otton, 16cwt. 0qr. 17lb. groſs, tare 125lb. at 4l. 15s. 4d. per cwt. neat — —
gar, 21cwt. 1qr. 2lb. groſs, tare $158\frac{1}{4}$lb. at 2l. 1s. 7d. per cwt. neat — —
utmegs, 3cwt. 0qr. 6lb. groſs, tare $12\frac{3}{4}$lb. at 15l. 8s. 9d. per cwt. neat — —

£

Received at the ſame time, the contents,

John Ruſſell.

(28.) Mr. John Grant, Dr.

To J. H. Wicks, for 5 bags of Pimento,

ndon, Oct. 8, 1798. *Viz.*

		Cwt.	qr.	lb.		lb.	
o. 21.	Wt. groſs	0	3	19,	tare	$7\frac{1}{2}$	at 4l. 19s. 2d. per cwt. neat £
22.	——	1	0	4,	—	$9\frac{1}{4}$	
36.	——	0	3	2,	—	$8\frac{1}{4}$	
37.	——	1	1	5,	—	10	
41.	——	1	0	9,	—	$9\frac{1}{4}$	

£

(29.) Wilmer Willet, eſq.

Bought of Francis Duke, 6 buts of madd

London, Nov. 4, 1798.

No. 1. Wt. groſs 11cwt. 2qr. tare 14lb. per cwt. tret 4lb. per 104lb. and cloff 2lb. for every 3cwt. at 3l. 5s. per cwt. neat £

2. Wt. groſs 10cwt. 1qr. 14lb. tare 7lb. per cwt. tret 4lb. per 104lb. and cloff 2lb. for every 3cwt. at ditto — —

3. Wt. groſs 9cwt. 3qr. tare 16lb. tret 4lb. per 104lb. and cloff 2lb. for every 3 cwt. at ditto — —

4. Wt. groſs 12cwt. 16lb. tare 8lb. per cwt. tret 4lb. per 104lb. and cloff 2lb. for every 3cwt. at ditto — —

5. Wt. groſs 9cwt. 1qr. 14℔. tare 12lb. per cwt. tret 4lb. per 104lb. and cloff 2lb. for every 3cwt. at ditto — —

6. Wt. groſs 10cwt. tare 10lb. per cwt. tret 4℔ per 104℔. and cloff 2℔ for every 3cwt. at ditto — —

£

CLASS IV. *Exerciſing the Rules of* COMMISSION, DISCOUNT, &c.

(30.) Mr. Fenton,

London, 1798. To Timothy Broker, Dr.

Jan. 9. For transferring 1749l. 17s. 8d. bank-ſtock, commiſſion $2\frac{7}{8}$ per cent. £

Mar. 12. For transferring 2176l. 7s. 6d. 3 per cent. conſols. commiſſion $2\frac{1}{6}$s. per cent. —

—— 29. For transferring 954l. 18s. South ſea ſtock, commiſſion $6\frac{2}{3}$s. per cent. —

Apr. 4. For laying out 192l. 10s. 2d. in ſilks, commiſſion $2\frac{5}{8}$l. per cent. —

May 19. For laying out 551l. 18s. 9d. in cottons, commiſſion $2\frac{7}{8}$l. per cent. —

—— 21. For laying out 789l. 17s. in callicoes, commiſſion $3\frac{1}{8}$l. per cent. —

£

(31.) Sir James Lawſon,

To Richard Burton, Dr.

don, 1798.

b. 10. To purchaſing 8764l. 17s. 6d. India ſtock, commiſſion 2s. 6d. per cent. £

— To purchaſing 7694l. 11s. 4d. 3 per cents, commiſſion 2⅞s. per cent. —

— 19. To purchaſing 1875l. 16s. 9d. 4 per cents, commiſſion 2⅛s. per cent. —

r. 12. To diſcounting 147l. 19s. 11d. in bills payable at 3 months, at 5l. per cent.

— To diſcounting 910l. in bills payable at 15 days, at 4½l. per cent. —

ril 5. To diſcounting 200l. in notes payable at 10 days, at 5l. per cent. —

£

(32.) Mr. Adams,

Bought of William Birmingham,

ffield, Nov. 7, 1798.

groſs of buttons, at 18s. 6d. per groſs, to pay at 10 months — £

o dozen of knives, at 9s. per dozen, to pay ½ at 4 months, and the other half at 6 months —

dozen of metal buckles, at 1l. 12s. per dozen, and 10 dozen of plated ditto, at 1l. 14s. to pay at 20 days — —

£

If Mr. Birmingham allows Mr. Adams 5 per cent. diſ-
unt for ready money, what will be the value of his bill?

(33.) Mr. L. Thirlwall,

London, May 5, 1798. Bought of Richard Thwaite

457 ells Flem. of Holland, at 4s. 7½d. per yard, to pay ½ at 3 months, and the other ½ at 5 months £

43 pieces of Ruſſia ſheeting, each 50 yards, at 11¾d. per yard, to pay at 20 days ——

70 pieces of Iriſh, at 3l. 10s. per piece, to pay ¼ preſent, ½ the remainder at 5 months, and the other ½ at 8 months —— ——

Received at the ſame time, the contents, allowing 5 per cent. for ready-money. £

Rd. Thwaites.

(34.) Lumley Tutt, eſq.

To John Keith, Dr.

Burlington, March 5, 1798.

For diſcounting 71l. 10s. 4d. due at 4 months, at 4⅛ per cent. —— £

For diſcounting 4 bills, viz. one of 21l. 11s. 6d. one of 36l. 0s. 9¼d. one of 19l. and one of 40l. 17s. 11½d. due 7 months hence, at 5⅝l. per cent. per annum —— ——

For diſcounting a bond of 475l. one third being due at 10 days, one fourth at 2 months, and the reſt at 2 years, at 5 per cent. per annum ——

£

CLASS V. *Exerciſing* DUODECIMALS, &c.

(35.) Sir Leonard Hurt,

Sproatley, 1798. To John Simpſon, Dr.

To flooring three rooms, each 17ft. 9in. 4pts. by 14ft. 9in. the fire-place in each being 4ft. 4in. by 3ft. 7in. at 7l. 4s. 6d. per ſquare £

To wainſcoting 4 rooms, each 47ft. 9in. round, and 9ft. 7in. high, the 4 window-ſhutters each 6ft. by 3ft. 7in. and the doors 7ft. by 4ft. are reckoned work and half, at 6s 9½d. per ſq. yd.

To 74½ days work, at 2s. 9d. per day ——

Received, December 5, the contents,

John Simpſon. £

(36.) Mr. Greſham,

To William Milner, Dr.

Sproatley, Jan. 1, 1798.

To raiſing a brick-wall, 174ft. 6in. long, 6ft. 11in. high, and 3½ bricks thick, at 5l. 17s. per rod £

To the brick-work of a houſe, viz. 56ft. round, and 28ft. 7in. high, 15ft. 4in. of which is 4½ bricks thick, and 13ft. 3in. 3½ bricks thick, at 6l. 11s. per rod — ——

To paving a court-yard, 61ft. 6in. by 54ft. 9in. at 3½d. per foot —— ——

To tiling 15 ſquares 75ft. by 54ft. 9in. at 2l. 2s. per ſquare —— —— —

£

(37.) John Carttar, eſq.

Bought of Thomas Adamſon,

Hull, June 5, 1798.

15 planks of beech, each 10ft. 11in. long, and 15 inches broad, at 2¼d. per ſquare foot £

27 fir ditto, each 12ft. 8in. long, and 14in. broad, 1½d. per ſquare foot —— —

7½ loads of oaken timber, at 2l. 14s. per ton —

10½ ton of elm ditto, at 2l. 12s. per load —

1437 deal boards, at 5l. 8s. 4d. per hundred —

£

Received at the ſame time; the contents,

Thomas Adamſon.

(38.) Meſſrs. Mount and Son,

To Henry Edmons, Dr.

London, April 4, 1798.

To paving a court-yard, 50ft. 9in. by 40ft. 7in. 8pts. at 2s. 8½d. per yard —— £

To paving a ſtable with clinkers, 17ft. 10in. 5pts. by 10ft. 4in. 11pts. at 4s. 2¼d. per yard —

To a brick-wall 294ft. by 8ft. 10in. 2pts. and 2½ bricks thick, at £5 9s. per rod ——

To ceiling 5 rooms, each 12ft. 4in. 10pts. by 8ft. 0in. 8pts. at 8¾d. per yard ——

To ſlating a houſe, 29ft. by 18ft. 10in. the roof of a true pitch, and the eaves-boards projecting 15 inches, at 6l. 9s. per ſquare ——

£ ——

(39.) Mr. Conſtant,

To Benjamin Lancaſter, Dr.

York, May 5, 1798.

To flooring 3 rooms, each 17ft. 6in. by 13ft. 4in. at 6l. 17s. per ſquare —— ——

To wainſcoting ditto, being 61ft. 8in. round, and 10ft. high, (including the cornice and moulding) at 6s. 2d. per ſquare yard —— ——

To 174 oaken planks, at 7l. 12s. 6d. per hundred —— —— ——

To 215 deal planks, at 4l. 17s. 2d. per hundred —— —— ——

To flooring an out room, being 19ft. 7in. 4pts. by 11ft. 2in. at 4l. 12s. 1d. per ſquare ——

£ ——

(40.) Mr. Craven,

To Francis Oldfield, Dr.

Hemſley, June 4, 1798.

To ſlating a barn, length 27ft. breadth 15ft. the eaves-boards projecting 1ft. 4in. at 6l. 7s. 4d. per ſquare —— £

To plaſtering a room 19ft. 10in. by 9ft. 6in. at 1s. 3½d. per yard —— ——

To white-waſhing 3 rooms, each 9ft. high, 27ft. long, and 18ft. wide, the 3 doors each 6ft. 6in. by 3ft. 9in. and 9 windows each 6ft. by 4ft. 9in. at 2½d. per yard —— ——

£ ——

CLASS VI. INVOICES, &c.

Dublin, Aug. 7, 1798.

(41.) Invoice of 547 firkins of butter and 70 barrels of pork, laden by me, James Donegall, on board the Cork, Patrick Fitzgerald, maſter, for the proper account and riſk of Thomas Saunders, merchant in London, under the mark per margin, contents, coſts, and charges, viz.

547 firkins of butter, bought of James O'Brien, weight 56lb. each neat, at 1l. 4s. 7d. per cwt —— } £

70 barrels of pork, bought of Patrick O'Neille, at 19s. 4d. per barrel }

	Charges.	l.	s.	d.
	To cuſtom of the butter	2	11	0
V	Ditto of the pork ——	1	14	0
Λ	For 547 firkins ——	15	19	11
	Cooperage, hoops, heading, &c.	8	4	0
T. S.	For 70 barrels, cooperage, &c. of ditto ——	4	5	0
	Lighterage and Wharfage	0	17	9
	Cartage and portage ——	0	19	7

To my commiſſion at 2½ per cent.

——

Errors excepted, £

James Donegall. ——

Queſt. What ſterling money does this invoice amount to, exchange at 12 per cent.?

Kingſton, Jamaica, June 5, 1798.

(42.) Invoice of 14hhds of tobacco, laden on board the Speedwell, George Panton, maſter, conſigned to James Porter, merchant in London, for his proper account and riſk, marked as per margin; contents, coſts, and charges, viz.

To 14 hhds of the beſt tobacco, weight 84cwt. 1qr. 14lb. Tare 2qr. 4lb. per hhd, tret 4lb. per 104lb. at 9¼d. per ℔ } £

Charges.

		£	s	d
No. 8.	For 14 empty hhds	£1	8	0
to 22.	Cooperage, hooping, and heading, &c. ——	1	1	0
J. P.	Warehouſe-room ——	0	17	9
	Boatage and ſtowage —	0	14	5
	Charges at ſhipping ——	0	19	4

For my commiſſion at 3½ per cent.

£

Errors excepted,

Per George Minot.

Queſt. What ſterling money does the above invoice amount to, exchange at 25 per cent.?

Amſterdam, Jan. 11, 1798.

(43.) Invoice of 12 pieces of Holland, 11 pieces of cambric, and 10 pieces of Ghentiſh cloth, ſhipped by me, Abraham Van Schooten, on board the Nancy, Robert Cooke, maſter, for the proper account and riſk of John Harriſon, merchant at Hull, marked as per margin; contents, coſts, and charges, viz.

To

	G. S. P.
To 12 pieces of Holland, qt 479⅓ ells Fl. at 1 guild. 4½ ſtiv. per ell —	
To 11 pieces of cambric, qt. 347½ ells Flem. at 1 guild. 3 ſtiv. 12 p. per ell —	
To 10 pieces of Ghentiſh cloth, qt. 117½ ells Flemiſh, at 18½ ſtivers per ell —	

Charges.

		G.	S.
	To cuſtoms and brokerage of the Hollands, at 3 guild. per piece	36	0
No. 1	To charges in buying —	4	5
to 33.	To cuſtom of cambric and Ghentiſh		
J.H.	cloth —	19	12
	To canvas, folding, and tacking	4	11
	To warehouſe-room —	3	14
	To boatage aboard —	1	11

To my commiſſion at 2½ per cent.

£

Errors excepted,

Abraham Van Schooten.

Queſt. What ſterling money does the above invoice amount to, exchange at 34s. 6d. Flemiſh per £ ſterling?

Bourdeaux, Nov. 4, 1798.

(44.) Invoice of half a tun of wine and 25 puncheons of prunes, ſhipped on board the Friendſhip, John Sampſon, maſter, for the account and riſk of James Lloyd, merchant in London, marked as in the margin; contents, coſts, and charges, viz.

Liv. s. den.

To 2 hhds of claret, at 50 crowns per tun —— ——

To 25 puncheons of prunes, viz. —

	No.	℔	No.	℔
	1, wt	1000	14, wt	955
	2, —	1120	15, —	960
	3, —	750	16, —	1710
	4, —	594	17, —	1410
	5, —	740	18, —	940
J.LL.	6, —	1140	19, —	310
	7, —	943	20, —	412
	8, —	1110	21, —	1101
	9, —	541	22, —	941
	10, —	742	23, —	375
	11, —	494	24, —	948
	12, —	175	25, —	549
	13, —	419		

Tare 79½lb. per caſk.

at 2 liv. 17s. 7d. per quintal, neat.

Charges.

Liv. s. den.

To cuſtom and brokerage of the wine, at 20 liv. per tun ——

Ditto of prunes, 4 liv. 15s. per puncheon —— ——

To ſledage and boatage of the wine —— —— 0 15 0

To ditto for the prunes, at 9 ſols per puncheon ——

To the ſhip-broker for the prunes, 11 ſols per ton ——

To average poor's box, 27 ſols per ton —— ——

To my commiſſion at 2½ per cent.

Errors excepted,

Jean Jacques d'Anville.

Queſt. What ſterling money does the above invoice amount to, exchange at 54½d. per ecu?

CLASS VII.

BILLS of EXCHANGE, PROMISSORY NOTES, RECEIPTS, &c.

1. *Inland Bills of Exchange**.

(45.) *Hull*, June 5, 1798.

Sir,

Pay Mr. Thomas Strange, or bearer, one hundred and fifty pounds, and place it to my account.

To Mr. Saunders, C. Hartley.
merchant, *London.*

London, Feb. 10, 1798.

(46.) Meſſrs. Jones and Co.

Pay William Simpſon, or bearer, ninety pounds on account.

Joſeph James.

Briſtol, Feb. 11, 1798.

(47.) At ſight, pay Mr. John Ruſſel the ſum of fifty pounds, the value received of Mr. John Hill, and place it to account, as per advice from

To Mr. Stephen Munn, James Trueman.
grocer, *Strand, London.*

£300 *Newcaſtle*, April 5, 1798.

(48.) At fifteen days ſight, pay Mr. Richard Thorpe, or order, the ſum of three hundred pounds, for value received of Sir James Jukes, and place it to account, as per advice from

To Mr. John Harriſon, Rd. Hutton.
merchant at *Hull.*

---

* Every bill of exchange, draft, or order, payable otherwiſe than on demand, where the ſum ſhall amount to £2 and not exceed £30 muſt be on a *ſtamp of* 8 *pence*. Above £30 and not exceeding £50, *one ſhilling*. Above £50 and not exceeding £100, *one ſhilling and four-pence*. Above £100 and not exceeding £200, *two ſhillings*. Above £200, *two ſhillings and eight-pence*. The ſtamp duty to be paid by the perſon who makes or ſigns the bills, &c.

£500 *Glaſgow*, May 4, 179

(49.) Two months after date, pay to Sir Chriſtophe Sykes, or order, five hundred pounds, value received o the Right Hon. the Lady Dundas, and place it to accoun as per advice from

To Sir James Allpay, Collin M'Donald
merchant in *London*.

2. *Foreign Bills of Exchange**.

(50.) For £571 18s. ſterling, at 34s. 4d. Flem. pe
£ ſterling at uſance.

London, Sept. 22, 1798

At uſance, pay this my firſt bill of exchange to Jacob Vanderberghauſen, or order, five hundred ſeventy-one pounds eighteen ſhillings ſterling, at thirty-four ſhilling and four-pence Flemiſh per £ ſterling, value received o Samuel James, eſq. and place it to account, as per advic from

Your humble ſervant,
To Mr. Van Schwellyingberg, James Willis
merchant, *Amſterdam*.

Queſt. What is the value of this bill in Flemiſh money?

(51.) For 7494 guild. 14 ſtiv. at 35s. 4d. pe
£ ſterling, at uſance.

Amſterdam, June 2, 1798

At uſance, pay this my ſecond bill of exchange, my firſt not paid, to Charles Johnſon, or order, ſeven thouſand four hundred and ninety-four guilders fourteen ſtivers, at thirty-five ſhillings and four-pence Flemiſh per £ ſterling, value received of Herman Vanbeck, and place it to account, as per advice from

To James Hall, eſq. Your humble ſervant,
merchant in *London*. Simon Van Buſching

Queſt. What is the value of this bill in ſterling money?

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* Foreign bills of exchange drawn in ſets, according to the cuſtom of merchants, where the ſum ſhall not exceed £100, are charged with a *ſtamp duty of eight-pence*. Exceeding £100 and not £200, *one ſhilling*. And exceeding £200, *one ſhilling and four-pence*. Every ſingle bill of each ſet is chargeable with the duty.

(52.) For 5000 crowns, at 4s. 3d. double uſance.

Paris, Sept. 17, 1798.

At double uſance, pay this my firſt bill of exchange to ames Philips, or order, the ſum of five thouſand crowns, four ſhillings and three-pence each, value received, and lace it to account, as per advice of

To Mr. Wm. King, Your humble ſervant,
Merchant in *London*. Jean Du Faur.

Queſt. What is the value of this bill in ſterling money?

(53.) For 1576 pieces of eight of Mexico,
at 54½d. each, at three months.

Legborn, Feb. 14, 1798.

Three months after date, pay this my firſt bill of exhange to Mr. John La Motte, or order, one thouſand five ndred and ſeventy-ſix pieces of eight of Mexico, for the lue received of himſelf, at 54½d. ſterling per piece, and lace it to account, as per advice from

Your humble ſervant,
To Mr. Wm. Hintz, James Moreni.
merchant in *London*,

Queſt. What is the value of this bill in ſterling money?

(54.) For 1749l. 18s. ſterling, at 54¼d. per
ducat bank, at uſance.

London, Jan. 5, 1798.

At uſance, pay this my third bill of exchange, my firſt and ſecond not paid, to Mr. Joſhua Sommers, or order, one thouſand ſeven hundred and forty-nine pounds eighteen ſhillings ſterling, in ducats, at fifty-four pence farthing each, and place it to the account of

Your humble ſervant,
To Mr. Michael Taſſoni, James Lamb.
merchant at *Venice*.

Queſt. What is the value of this bill in ducats bank?

3. *Promiſſory Notes* *.

(55.) I promiſe to pay to Chriſtopher Humphries, order, the ſum of fifty pounds, on demand, for value ceived. Witneſs my hand this fifth day of July, 1798.

Francis Oldfiel

£ 50.

Bromley, June 4, 179

(56.) Three months after date, I promiſe to pay to M William Simpſon, the ſum of twelve pounds, for val received by me,

James Surepa

£ 12.

4. *Receipts* †.

(57.) Received, June 4. 1798, of Mr. Abraham Wi kinſon, the ſum of eighteen pounds, in full of all demand by me,

John Cl

£18.

(58.) Received, December 30, 1798, of Mr. Thom Keith, the ſum of fifty pounds eleven ſhillings and f pence, for half a year's rent, due at Michaelmas laſt, t the poor and truſtees of Emanuel hoſpital—for whoſe u it is received by me,

John Foake

£50 11 6

---

* Promiſſory, or other notes, payable otherwiſe than to the *bear* *demand*, muſt be on a ſtamp of the ſame value as an inland bill of e change, draft, or order. See the note, page 211.

† Every receipt, diſcharge, or acquittance for money, amounting to £ and under £20, muſt be on a *twopenny ſtamp*. £20 and under £50, *fou* *pence*. £50 and under £100, *ſix-pence*. £100 and under £500, *ſhilling*. £500 and upwards, *two ſhillings*. To be paid by the perſ who *requires* the receipt; ſalaries, penſions, debts, or other ſums payab from the crown excepted; in ſuch caſes the perſon *giving* the receipt mu find the ſtamp.

(59.) Receive

(59.) Received, July 11, 1798, of Mr. John Elmer,
e ſum of twenty pounds nine ſhillings, in part of pay-
nt of ſeventy pounds, due to me on bond from the ſaid
hn Elmer.—I ſay received by me,

£20 9 Robert Foſter.

(60.) Received, May 4, 1798, of Mr. George True-
n, a promiſſory-note for the ſum of twenty-five pounds,
yable to me, or order, two months after date; which
n, when paid, is in full of all demands.—I ſay received
me,

William Keith.

£25

END OF THE FIRST PART.

THE

THE
COMPLETE PRACTICAL
ARITHMETICIAN.

PART II.

ALLIGATION.

DEFINITION. When different ſorts of wine, co ſpices, metals, &c. or any number of ſimples, different qualities are required to be mixed together, th method of proportioning ſuch a mixture is called Alliga tion, from the quantities being generally linked, or joined together by lines.

Note 1. The firſt propoſition and rule is uſually called Alligatio medial; the ſecond Alligation alternate, and is the reverſe of Alligatio medial; the third Alligation partial; and the fourth Alligation total.

Propoſition 1. *Given the particular quantities mixed, an their reſpective rates, or prices, to find the mean rate, or pric of the compound.*

Rule. Multiply the quantities of the mixture by the reſpective rates, or prices, reduced to one denomination, and divide the ſum of the products by the ſum of the quantities, the quotient will be the mean rate, or price.

The method of proof. Find the whole value of the mixture at the mean price, and if it be the ſame with the total value of the ſeveral ingredients, at *their* reſpective prices, the work is right.

Prop. 2. *Given the rates, or prices, of ſeveral ingredients to find the quantities thereof, ſo that the mixture may be ſold at a given rate, or price.*

Rule 1. Reduce the particular rates to the ſame denomination as the mean rate; write them orderly under each other, beginning with the greateſt, and place the mean rate to the left-hand of them.—Then connect the ſimple rates together, ſo that each rate *leſs* than the mean may be coupled with *one* greater, or with *each* greater; and each rate *greater* than the mean with *one* leſs, or with *each* leſs.

2. Take the difference with each ſimple rate and the mean rate, and place it *alternately*; that is, againſt the rate with which it is linked.—Then, if only one difference ſtand againſt any rate, it will be the quantity belonging to that rate; but, if there be ſeveral, their ſum will be the quantity.

Queſtions under this and the following rules may be proved by the rule to the firſt propoſition.

Note. Queſtions that fall under this and the following propoſitions are called by *algebraiſts* indeterminate, or unlimited, problems, becauſe they will admit of an infinite number of different anſwers; for finding which, *algebra* furniſhes us with an univerſal rule. But the rule given above is limited, in its immediate effect, to the different anſwers obtained by the various methods of linking the ſimples; viz. juſt as many different anſwers may be obtained as there are different ways of linking together a greater and leſs rate than the mean.—Though the preceding rule is in ſome meaſure limited, yet an infinite number of anſwers may be deduced from it; for, after the rates are coupled, and their ſeveral differences taken, inſtead of any, or every, couple of ſuch differences, we may take any *equimultiples* thereof, and place them *alternately*; and theſe, or other quantities proportional to them, will be the quantities required.

Prop. 3. *Given the rates, or prices, of ſeveral ingredients, the quantity of one, and the mean rate, to find the ſeveral quantities of the reſt in proportion to that given.*

Rule. Take the difference between each rate and the mean, as before.—Then, as the difference ſtanding againſt the price of the given quantity is to *that* quantity, ſo are the ſeveral other differences to their reſpective quantities.

Prop. 4. *Given the rates, or prices, of ſeveral ingredients, the mean rate, and the whole quantity of the mixture, to find the particular quantities of each ſort.*

Rule. Take the difference between each rate and the mean, as before.—Then, as the ſum of theſe differences is to the whole quantity of the mixture, ſo is each particular difference to its reſpective quantity.

Examples to Propoſition 1.

(1.) A vintner would mix 2 gallons of wine, at 14s. per gallon, with 1 gallon at 12s. two gallons at 9s. and 4 gallons at 8s. per gallon. What will be the worth of a gallon of this mixture?

2 gallons multiplied by 14s. gives 28 product.
1 ——— by 12s. ——— 12
2 ——— by 9s. ——— 18
4 ——— by 8s. ——— 32
—— ——
9 Sum of the products 90 this, divided by 9, (the ſum of the quantities) gives 10s. the value, or *mean rate*, of a gallon, anſwer.

(2.) A grocer would mix 4 cwt. of ſugar at 2l. 18s. per cwt. 7cwt. 2qr. at 2l. 13s. per cwt. 5cwt. 1qr. at 1l. 19s. per cwt. and 3cwt. 3qr. at 1l. 14s. per cwt. together; what is the worth of a cwt. of this mixture?

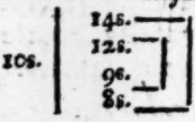
(3.) A tobacconiſt mixed 50lb. of tobacco, at 11½d. per lb. with 40lb. at 14d. per lb. 27lb. at 2s. 6d. per lb. and 87lb. at 3s. per lb. What was the worth of 1lb. of this mixture?

(4.) A farmer mixed 2qr. 4buſh. of corn, worth 2l. per quarter; 4qr. 4buſh. of an inferior kind, worth 1l. 4s. per quarter; and 5qr. of a third kind, worth only 16s. per quarter: required the value of a quarter of this mixture.

Examples to Prop. 2.

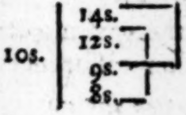
(5.) A vintner would mix four ſorts of wine, of different prices, together, viz. at 14s. 12s. 9s. and 8s. per gallon; what quantity of each ſort muſt he put into the compound, that he may be enabled to ſell it at 10s. per gallon?

Anſwer.



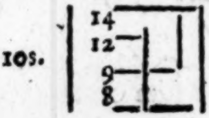
2 gal. at 14s.
1 — at 12
2 — at 9
4 — at 8

Or thus,



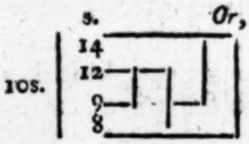
1 gal. at 14s.
2 — at 12
4 — at 9
2 — at 8

Otherwiſe,



2+1=3 gal. at 14s.
2 =2 — at 12
4 =4 — at 9
4+2=6 — at 8

Or,



2+1=3 gal. at 14s.
1+2=3 — at 12
2+4=6 — at 9
4+2=6 — at 8, &c. &c.

(6.) A grocer wiſhes to mix ſugar at 4d. 6d. and 10d. per lb. ſo that he may ſell the mixture at 8d. per lb. What quantity of each may he take?

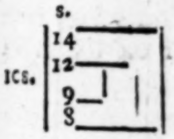
(7.) A goldſmith would mix gold of 23 carats fine with gold of 20 carats, ſome of 18, ſome of 17, and ſome of 14 carats fine; how much of each ſort muſt he melt together to form a compoſition of 19 carats fine?

(8.) A provider for the army, deſirous of mixing wheat, at 4s. per buſhel, with rye at 3s. per buſhel, barley at 2s. per buſhel, peaſe at 1s. 4d. per buſhel, and oats at 1s. per buſhel, wiſhes to be informed how to proportion the mixture that it may be worth 1s. 8d. per buſhel?

Note. By reducing the ſeveral rates into pence, 24 anſwers, in whole numbers, may be obtained to this queſtion by the different methods of linking the ſimples *only*.

Examples to Prop. 3.

(9.) A merchant propoſes to mix four ſorts of wine together, viz. 2 gallons of one ſort, at 14s. per gallon, with others at 12s. 9s. and 8s. per gallon; how many gallons of each ſort muſt he take to make a compoſition worth 10s. per gallon?



diff.
2 =2.
1+2=3. 2 diff. : 2 gal. :: 3 diff. : 3 gal.
2 =2. 2 diff. : 2 gal. :: 2 diff. : 2 gal. } anſw.
4+2=6. 2 diff. : 2 gal. :: 6 diff. : 6 gal.

Note. Different anſwers may be obtained by linking the quantities differently.

(10.) A diſtiller would mix 80 gallons of brandy, at 12s per gallon, with another ſort at 7s. and a third at 4s. per gallon; what quantity of each ſort muſt he take to make a compoſition worth 8s. per gallon?

(11.) A grocer would mix teas at 12s. 8s. and 6s. per lb. with 28lb. at 4s. 6d. per lb. What quantity of each muſt he take to make a compoſition worth 7s. per lb.

(12.) A perſon is deſirous of mixing corn, at 4, 3, and 2 ſhillings per buſhel, with 24 buſhels of an inferior kind worth 1s. 6d. per buſhel; how many buſhels of each muſt he take that he may afford to ſell the mixture at 3s. 4d. per buſhel?

Examples to Prop. 4.

(13.) A merchant propoſes to mix four ſorts of wine; the beſt at 14s. per gallon, the ſecond at 12s. the third at 9s. and the fourth at 8s. per gallon. How many of each will make a mixture of 12 gallons worth 10s. per gallon?

	s.	diff.		
10s.	14	1 =1	As 12d. : 12 gal. :: 1d. : 1 gal.	Anſwer.
	12	1+2=3	12d. : 12 — :: 3d. : 3 —	
	9	4+2=6	12d. : 12 — :: 6d : 6 —	
	8	2 =2	12d. : 12 — :: 2d. : 2 —	

Sum of the differences 12

Note. Other anſwers may be obtained by linking the ſimples differently. —In order to give the ſcholar a clearer idea of this ſubject, I have given the *ſame* example to each of the propoſitions.

(14.) A grocer would mix four ſorts of ſugar, viz. at 2s. 1s. 8d. 1s. and 8d. per lb. What quantity of each muſt he take to make a compoſition of 72lb. at 1s. 4d. per lb.?

(15.) It is required to mix brandy at 8s. 7s. and 1s. per gallon, with water at 0s. per gallon, ſo that a compoſition of 16 gallons thereof may be worth 5s. per gallon?

(16.) How much gold, of 8, 9, and 24, carats fine, muſt be mixed together to make a compoſition of 64 oz. of 14 carats fine?

POSITION.

Definition. *The Rule of Poſition*, or trial and error, is ſo called becauſe we ſuppoſe ſome uncertain number, or numbers; and, by reaſoning from them according to the nature of the queſtion, and paying proper attention to the error, or errors, obtain a true anſwer.

SINGLE

SINGLE POSITION.

Definition. *By ſingle Poſition*, or a ſingle ſuppoſition, are olved thoſe queſtions wherein the reſults are proportional o their ſuppoſitions.

Rule.

Suppoſe ſome convenient number, and proceed with it ccording to the nature of the queſtion; then, if the eſult be either too much or too little, ſay, as the falſe umber reſulting is to the true number given, ſo is the hole, or any part, of the ſuppoſed number to the whole, r correſponding part, of the required number.

Examples.

(1.) A drover being aſked how many ſheep he had got, replied, If, ſir, you add $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$, of the number together, the ſum will be 18. How many had he?

Suppoſe he had 12.

Then $\frac{1}{3}$ of 12 = 4
$\frac{1}{4}$ of ditto = 3
$\frac{1}{6}$ of ditto = 2

The ſum is 9, but ſhould be 12.
Hence, as 9 : 18 :: 12 : 24 ſheep, anſwer.

(2.) Three perſons are to pay a reckoning of 20s. A is to pay $\frac{1}{2}$, B $\frac{1}{3}$, and C $\frac{1}{4}$; how much muſt each perſon pay of the reckoning?

(3.) A can do a piece of work in 7 days, B can do the ſame in 5, and C in 6. Set them all at work together, in what time will they finiſh it?

(4.) One-fifth part of an army were killed in battle, $\frac{1}{8}$ part were taken priſoners, and $\frac{1}{10}$ part died by ſickneſs; if 4000 men were left, how many men did the army at firſt conſiſt of?

(5.) I have a ciſtern which has three cocks, D, E, and F. Now, if D be opened by itſelf, when the ciſtern is full, it will empty it in 9 hours; if E be opened by itſelf, it will empty the ciſtern in 11 hours; and, if F be opened by itſelf, it will empty the ciſtern in 13 hours. In what time will they empty the ciſtern if I ſet them all open together?

(6.) A perſon delivered to another a ſum of money, to receive intereſt for the ſame at 4 per cent. per annum, (ſimple intereſt). At the end of 3 years he received for principal and intereſt 176l. 8s. What was the ſum lent?

DOUBLE POSITION.

Definition. ***By Double Poſition***, or two ſuppoſitions, are ſolved thoſe queſtions wherein the errors are in proportion, to the difference between the true number, and each ſuppoſition.

Rule.

Suppoſe any two convenient numbers, and proceed with them according to the nature of the queſtion, marking the errors (with + or —) according as they exceed or fall ſhort of the truth.

Then,

Multiply the firſt ſuppoſition by the ſecond error, and the ſecond ſuppoſition by the firſt error, and divide the ſum of the products by the ſum of the errors, if they are differently marked; or the difference of the products by the difference of the errors if they are marked alike, and the quotient will be the number ſought.

Or, II.

Multiply the difference between the two ſuppoſed numbers by the leſs error, and divide the product by the ſum of the errors, if they are differently marked; or by the difference if they are marked alike; and the quotient will be a correction of the number belonging to the *leſs* error, and muſt be added to it, if *that* error be leſs than the truth, or ſubtracted, if it be greater.

Note 1. Mr. *Ward*, Mr. *Malcolm*, and ſeveral other writers of note, have omitted the rule of poſition, becauſe all queſtions that can be ſolved by it are more readily ſolved by a ſimple equation in *algebra*. Though this obſervation be true, yet poſition has its uſe; for, it may frequently be applied to the ſolution of adfected and exponential equations in *algebra*, better than any other method, (particularly the ſecond rule) for, by repeating the proceſs, the anſwer will continually approximate to the true number

within

ithin any aſſigned degree of exactneſs. For this reaſon it is of eſſential rvice in the more abſtruſe parts of the mathematics; for, in many diffi- lt problems, there is hardly any other way to obtain a ſolution.

2. *In any enquiry where it is poſſible to prove the anſwer when diſcovered.* ake two ſuppoſitions as near the truth as you are able to gueſs, and from hem deduce an anſwer as directed in the *ſecond* rule: then take the nearer f the two ſuppoſitions and the reſult above gained, as two other ſuppo- ſitions; and, in like manner, deduce another anſwer: proceed thus till ou have obtained an anſwer ſufficiently exact.

Examples.

(1.) What number is that, which, being multiplied by 3, the product increaſed by 4, and that ſum divided by 8, the quotient may be 32?

Suppoſe 12
×3
36
+4
8)40
Quotient 5
ſhould be 32
Error — 27

Again, ſuppoſe 108
×3
324
+4
8)328
Quotient 41
ſhould be 32
Error + 9

By Rule 1.

its error
1ſt ſuppoſition 12 ⨯ —27
2d ſuppoſition 108 ⨯ + 9
27 its error 12
2916 108
108
27+9=36)3024(84 anſwer.
144

By Rule 2.

$\frac{\overline{108-12\times 9}}{27+9}=24$, correction of the number (108) belonging to the leſs error. Hence 108—24=84, as before.

(2.) A man has 2 excellent horſes;—and a ſingle-horſ chaiſe and furniture, worth 150l. Now, if the firſt horſ be put in the chaiſe, his value, with the furniture, &c. wil be three times that of the ſecond horſe without it; but, i the ſecond horſe be put in the chaiſe, his value will b double that of the firſt. What are the horſes worth?

(3.) A perſon being aſked the time of the day, replied, the day is now 16 hours long, and the ſun riſes at 4 o'clock. Now, if you add $\frac{1}{2}$ of the hours that have paſſed ſince the ſun roſe to $\frac{1}{4}$ of thoſe which muſt elapſe before the ſun ſets, you will have the *exact* time of the day.

(4.) A perſon received 11 crowns and 7 dollars for a debt of 4l. 10s. 10d. and at another time received 4 crowns and 3 dollars for a debt of 1l. 15s. What was the value of a crown and a dollar in Engliſh money?

(5.) A perſon diſtributed in charity 2d. a piece among ſeveral poor children, and had 4d. left. He would have given them 3d. a piece, but wanted 10d. to be able to do it. What was the number of children?

Examples exerciſing Note 2.

(6.) If $g\times\overline{4372-g}\big|^{6}=4\times\overline{4372-4}\big|^{6}$ what is g? *Anſwer* 2916.

(7.) If $\frac{4372\times r}{4}-r^{7}=\frac{4372-4}{4}$, what is r? *Anſw.* 3.

(8.) If $\frac{882\cdot1248\,r}{80}-r^{9}=\frac{802\cdot1248}{80}$, what is r? *Anſ.* 1·05 nearly.

(9.) If $\frac{80+568\cdot6257\,r^{9}}{568\cdot6257}-r^{10}=\frac{80}{568\cdot6257}$, what is r? *Anſw.* 1·05 nearly.

(10.) If $r^{3}+12r=300$, what is r? *Anſw.* 6·098, &c.

Note. By the 2d note, and a table of logarithms, the moſt intricate adfected equations, in algebra, may be ſolved.—Should any perſon be at the trouble to prove the queſtions in geometrical progreſſion, and in annuities in arrears and preſent worth of annuities, at compound intereſt, by all the different theorems, he will meet with examples ſimilar to the 6th, 7th, 8th, and 9th preceding; for which reaſon they are inſerted here.

ARITH-

ARITHMETICAL PROGRESSION.

Definition. When a ſeries of numbers increaſes, or de-reaſes, by an equal exceſs, or difference, thoſe numbers re ſaid to be in *arithmetical progreſſion*; ſuch as

2, 4, 6, 8, 10, &c. or 15, 14, 13, 12, 11, &c.

nd the numbers which form ſuch ſeries are called the terms f the progreſſion. The firſt and laſt terms are uſually lled the extremes.

Note 1. If *three* numbers are in arithmetical progreſſion, the ſum of e extremes will be equal to double the mean; and the product of the tremes, increaſed by the ſquare of the common difference, will be equal the ſquare of the mean. Thus, if 5, 7, 9, are in arithmetical pro-reſſion, then will $5+9=7\times2$, and $9\times5+2\times2=7\times7$.

2. If *four* numbers are in arithmetical progreſſion, the ſum of the two tremes will be equal to the ſum of the means.

Thus, if 2, 5, 8, 11, are in arithmetical progreſſion,
Then will $2+11=5+8$.

3. If a ſeries of numbers, (conſiſting of any number of terms) are in ithmetical progreſſion, the ſum of the extremes will always be equal to e ſum of any two means equidiſtant from the extremes; or to double e mean if the terms be odd.

Thus, if 3, 5, 7, 9, 11, 13, &c. are in arithmetical progreſſion,
Then will $3+13=5+11=7+9$.
Or, if 1, 4, 7, 10, 13, &c. are in arithmetical progreſſion, then will $1+13=4+10=7\times2$.

4. If out of any ſeries of numbers in arithmetical progreſſion there be taken any ſeries of *equidiſtant* terms, *that* ſeries will alſo be in arithmetical rogreſſion.

Thus, if 2, 4, 6, 8, 10, 12, 14, &c. are in arithmetical progreſſion,
Then will 4, 8, 12, &c. be in arith. prog.

5. In any ſeries of numbers in arithmetical progreſſion the common exceſs, or difference, is as often repeated as there are terms in the progreſ-ſion, wanting one; viz. every term, except the firſt, is continually in-creaſed, or diminiſhed, by the common exceſs, or difference.

6. The rules for an aſcending or deſcending ſeries are the ſame; for, a deſcending ſeries becomes an aſcending one by beginning at the leaſt term.

Propoſition 1. *Given the leaſt term, the greateſt term, and the number of terms, of an arithmetical progreſſion, to find the ſum of the terms.*

Rule. To the leaſt term add the greateſt, multiply the ſum by half the number of terms, and the product will be the ſum of the terms.

Prop. 2. *Given the leaſt term, the greateſt term, and number of terms, to find the common exceſs, or difference.*

Rule. Divide the difference between the greateſt a the leaſt term by the number of terms leſs unity, and t quotient will be the common exceſs, or difference.

Prop. 3. *Given the leaſt term, the greateſt term, and t common exceſs, or difference, to find the number of terms.*

Rule. Divide the difference between the *greateſt* an the *leaſt* term, by the common exceſs, or difference, th quotient, increaſed by an unit, will give the number o terms.

Prop. 4. *Given the greateſt term, the number of terms, an the common exceſs, or difference, to find the leaſt term.*

Rule. Multiply the common exceſs, or difference, by the number of terms leſs 1: ſubtract the product from the greateſt term, and the remainder will be the leaſt term.

Prop. 5. *Given the number of terms, the common exceſs, or difference, and the ſum of the terms, to find the leaſt term.*

Rule. Divide the ſum of the terms by the number of terms; and, from the quotient, ſubtract half the product of the common exceſs, or difference, by the number of terms leſs 1, the remainder will be the leaſt term.

Prop. 6. *Given the leaſt term, the number of terms, and the common exceſs, or difference, to find the greateſt term.*

Rule. Multiply the number of terms by the common exceſs, or difference, and to that product add the leaſt term; from this ſum ſubtract the common exceſs, or difference, and the remainder will be the greateſt term.

Note. The following propoſitions and theorems contain the whole practice of arithmetical progreſſion, (including the propoſitions and rules already given.)

Where
l = the leaſt term.
g = the greateſt term.
n = the number of terms.
s = the ſum of the terms.
d = the common exceſs, or difference;

poſition 1. *Given l, g, and n, to find s and d.*

heorem I. $\overline{l+g}\times\frac{n}{2}=s.$ *Theo.* II. $\frac{g-l}{n-1}=d.$

op. 2. *Given l, g, and s, to find n and d.*

eo. III. $\frac{2s}{g+l}=n.$ *Theo.* IV. $\frac{\overline{g+l}\times\overline{g-l}}{2s-\overline{g+l}}=d.$

op. 3. *Given l, g, and d, to find n and s.*

eo. V. $\frac{g-l}{d}+1=n.$ *Theo.* VI. $\overline{\frac{g-l}{d}+1}\times\frac{g+l}{2}=s.$

rop. 4. *Given l, n, and s, to find g and d.*

eo. VII. $\frac{2s}{n}-l=g.$ *Theo.* VIII. $\frac{\overline{s-ln}\times 2}{\overline{n-1}\times n}=d,$

Prop. 5. *Given l, n, and d, to find g and s.*

Theo. IX. $l+\overline{\overline{n-1}\times d}=g.$ *Theo.* X. $\overline{\overline{nd-d}+2l}\times\frac{n}{2}=s.$

Prop. 6. *Given l, s, and d, to find g and n.*

Theo. XI. $\frac{1}{2}\sqrt{8ds+\overline{2l-d}|^{2}}-\frac{1}{2}d=g.$

Theo. XII. $\frac{\sqrt{8ds+\overline{2l-d}|^{2}}-\overline{2l-d}.}{2d}=n.$

Prop. 7. *Given g, n, and s, to find l and d.*

Theo. XIII. $\frac{2s}{n}-g=l.$ *Theo.* XIV. $\frac{\overline{ng-s}\times 2}{\overline{n-1}\times n}=d.$

Prop. 8. *Given g, n, and d, to find l and s.*

Theo. XV. $g-\overline{\overline{n-1}\times d}=l.$ *Theo.* XVI. $\overline{\overline{2g+d}-nd}\times\frac{n}{2}=s.$

Prop. 9. *Given g, s, and d, to find l and n.*

Theo. XVII. $\frac{1}{2}\sqrt{\overline{2g+d}|^{2}-8ds}\pm\frac{1}{2}d=l.$

Theo. XVIII. $\frac{\overline{2g+d}\mp\sqrt{\overline{2g+d}|^{2}-8ds}}{2d}=n.$

Prop. 10. *Given n, s, and d, to find l and g.*

Theo. XIX. $\frac{s}{n}-\overline{n-1}\times\frac{1}{2}d=l.$ *Theo.* XX. $\frac{s}{n}+\overline{n-1}\times\frac{1}{2}d=$

The application of the preceding theorems is very evident and eaſy: rules might here be inſerted, were they of any uſe, for finding the ſum *polygonal* and *figurate* numbers, conſtituting part of the ancient *Pythag rean* ſpeculations about numbers, &c. Should any perſon wiſh to becom acquainted with ſuch numbers, he may conſult Mr. *Malcolm*'s Arithmeti from page 396 to 441.

Examples to Propoſition 1.

(1.) If the leaſt term of a ſeries of numbers in arith metical progreſſion be 4, the greateſt 100, and the numbe of terms 17, what is the ſum of the terms?

4+100=104, and 104×$\frac{17}{2}$=884, anſwer.

(2.) If the leaſt term be 3, the greateſt 108, and th number of terms 14, what is the ſum of the terms?

(3.) How many ſtrokes does the hammer of a cloc ſtrike in 12 hours?

(4.) If 100 ſtones be laid in a right line, and exactly the ſpace of a yard be left between one ſtone and another, how far muſt a perſon travel who gathers up theſe ſtones ſingly, returning with every one to a baſket a yard diſtant from the firſt?

Examples to Prop. 2.

(5.) If the leaſt term of a ſeries of numbers in arith-metical progreſſion be 4, the greateſt 100, and the number of terms 17, what is the common difference between each term?

100—4=96 diviſor, and 17—1=16 dividend; hence 96 divided by 16 gives 6, the common difference.

(6.) If the leaſt term be 3, the greateſt 108, and the umber of terms 14, what is the common difference?

(7.) A perſon travelled from London to a certain place n 8 days; he travelled 2 leagues the firſt day, and every ay he travelled farther than he did the preceding by an qual number of leagues; the laſt day he travelled 23 eagues: how far did he travel every day?

Examples to Prop 3.

(8.) The leaſt term of a ſeries of numbers in arithmeical progreſſion is 4, the greateſt 100, and the common ifference between each term is 6; what is the number of erms?

100—4=96 dividend, which, divided by 6, gives 16 for the quotient; hich, increaſed by an unit, gives 17 for the number of terms.

(9.) If the leaſt term be 3, the greateſt 108, and the common difference 5, what is the number of terms?

(10.) A man, going a journey, travelled the firſt day 2 leagues, and the laſt day 23; he increaſed his journey every day 3 leagues; how many days did he travel?

Examples to Prop. 4.

(11.) The greateſt term of a ſeries of numbers in arithmetical progreſſion is 100, the number of terms 17, and the common difference between each term 6; what is the leaſt term?

$\overline{17-1}\times 6=96$; then $100-96=4$, anſwer.

(12.) If the greateſt term be 108, the number of terms 22, and the common difference 5, what is the leaſt term?

(13.) A man in 6 days went from London to a certain place; every day's journey was greater than the preceding one by 4 miles; his laſt day's journey was 40 miles: what was his firſt?

Examples to Prop. 5.

(14.) The number of terms is 17, the common difference 6, and the ſum of the terms, of a ſeries of numbers in arithmetical progreſſion, is 884; what is the leaſt term?

$884 \div 17 = 52$, and $\overline{17-1} \times 6 = 96$; then $52 - \frac{96}{2} = 4$, the leaſt ter

(15.) If the number of terms be 22, the common di ference 5, and the ſum of the terms 1221, what is th leaſt term?

(16.) A man is to receive 300l. at 12 payments, eac ſucceeding payment to exceed the former by 4l. Wh will his firſt payment be?

Examples to Prop. 6.

(17.) If the leaſt term of a ſeries of numbers in arith metical progreſſion be 4, the number of terms 17, and th common difference 6, what is the greateſt term?

$17 \times 6 = 102$, and $102 + 4 = 106$. then $106 - 6 = 100$, the greate term.

(18.) If the leaſt term be 3, the number of terms 22 and the common difference 5, what is the greateſt term?

(19.) A man bought 100 yards of cloth; the firſt yar coſt him 2s. and each ſucceeding yard 1s. more to the laſt what did the laſt yard ſtand him in?

GEOMETRICAL PROGRESSION.

Definition. When a ſeries of numbers increaſes by a common multiplier, or decreaſes by a common diviſor, thoſe numbers are ſaid to be in *geometrical progreſſion*; ſuch as 2, 4, 8, 16, &c. or 27, 9, 3, 1, &c. The firſt and laſt terms are uſually called the extremes, and the common multiplier or diviſor the *ratio*.

Note 1. If *three* numbers are in geometrical progreſſion, the product of the two extremes will be equal to the ſquare of the mean.

Thus, if 3, 9, 27, are in geometrical progreſſion,
Then will $3 \times 27 = 9 \times 9$.

2. If *four* numbers are in geometrical progreſſion, the product of the two extremes will be equal to the product of the means.

Thus, if 2, 4, 8, 16, are in geometrical progreſſion,
Then will $2 \times 16 = 4 \times 8$.

3. If a ſeries of numbers (conſiſting of any number of terms) is in geometrical progreſſion, the product of the two extremes will be equal to the product of any two means equidiſtant from the extremes; or to the ſquare of the mean, if the terms be odd.

Thus, if 1, 2, 4, 8, 16, 32, &c. are in geometrical progreſſion,
Then will $1 \times 32 = 2 \times 16 = 4 \times 8$.

Or if 1, 2, 4, 8, 16, &c. are in geometrical progreſſion,
Then will $1\times16=2\times8=4\times4$.

4. If, out of any ſeries of numbers in geometrical progreſſion, there be ken any ſeries of *equidiſtant* terms, *that* ſeries will likewiſe be in geo- trical progreſſion.

Thus, if	2,	4,	8,	16,	32,	64,	&c. are in geometrical progreſſion,
Then will		4,	·	16,		64,	&c. be in geometrical progreſſion.

Propoſition 1. *In any ſeries of numbers in geometrical progreſſion, where the firſt, or leaſt, term is equal to the ratio:—Given the firſt, or leaſt, term, the number of terms, and the ratio, to find the greateſt, or any remote, term, without finding all the intermediate ones.*

Rule. Write down a few of the leading terms in the geometrical ſeries, over which place the arithmetical ſeries, 1, 2, 3, 4, 5, &c. as *indices:* find what figures of theſe indices, added together, will give the index of the term wanted in the geometrical ſeries; then multiply the numbers, ſtanding under ſuch indices, into each other, and their product will be the term ſought.

Thus,	1,	2,	3,	4,	5,	6.	&c. indices.
	3,	9,	27,	81,	243,	729,	&c. geometrical ſeries.

Prop. 2. *In any ſeries of numbers in geometrical progreſſion, where the firſt, or leaſt, term is different from the ratio:—Given the firſt, or leaſt, term, the number of terms, and the ratio, to find the greateſt, or any remote, term, without finding all the intermediate ones.*

Rule. Write down a few of the leading terms in the geometrical ſeries, over which place the arithmetical ſeries, 0, 1, 2, 3, 4, 5, 6, &c. as *indices:* add together the moſt convenient indices to make an index, an unit leſs than the index of the term required; then multiply the numbers ſtanding under ſuch indices into each other, taking care to divide the product of every two by the firſt term in the geometrical ſeries, and the laſt quotient will be the term required.

Thus,	0,	1,	2,	3,	4,	5,	&c. indices.
	5,	10,	20,	40,	80,	160,	&c. geometrical ſeries.

Prop. 3. *Given the firſt, or leaſt, term, the ratio, and the number of terms, to find the ſum of the terms.*

Rule. Find the laſt, or greateſt, term, by one of the preceding rules, from which ſubtract the firſt, or leaſt, term, and divide the remainder by the ratio, leſs 1; th quotient, increaſed by the laſt, or greateſt, term, will gi the ſum of the ſeries.

Prop. 4. *In any ſeries of numbers in geometrical progreſſio decreaſing, ad infinitum,—given the firſt term and the ratio find the ſum of the ſeries.*

Rule. Subtract the ſecond term from the firſt; the ſquar of the firſt term, divided by this difference, will give th ſum of the ſeries.

See the 7th note, in circulating decimals, Part I. *page* 92.

Note. If l = the leaſt term,
g = the greateſt,
n = the number of terms,
s = the ſum of the terms,
r = the ratio,
log. = logarithm of any letter.

Then will the following theorems exhibit all the poſſible caſes of geome trical progreſſion, including thoſe already given.

Propoſition 1. *Given l, g, and n, to find s and r.*

Theo. I. $$g+\frac{g-l}{\overline{\frac{g}{l}}^{\frac{1}{n-1}}-1}=s.$$

Theo. II. $$\overline{\frac{g}{l}}^{\frac{1}{n-1}}=r.$$

Prop. 2. *Given l, g, and s, to find n and r.*

Theo. III. $r^{n-1}=\frac{g}{l}$, *or*, $\frac{log.\ g-log.\ l}{log.\ \overline{s+l}-log.\ \overline{s-g}}+1=n.$

Theo. IV. $\frac{s-l}{s-g}=r$, *or*, $log.\ \overline{s-l}-log.\ \overline{s-g}=log.\ r.$

Prop. 3. *Given l, g, and r, to find n and s.*

Theo. V. $r^{n-1}=\frac{g}{l}$ *or*, $\frac{log.\ g-log.\ l}{log.\ r}+1=n.$

Theo. VI. $\frac{\overline{r\times g}-l}{1}=s$, *or*, $g+\frac{g-l}{r-1}=s.$

Prop. 4. *Given l, n, and s, to find g and r.*

Theo. VII. $$g\times\overline{s-g}^{\,n-1}=l\times\overline{s-l}^{\,n-1}.$$

Theo. VIII. $\frac{sr}{l}-r^{n}=\frac{s-l}{l}.$ The value of g in the firſt equation, and the value of r in the ſecond, muſt be found as directed in the 2d note, in *Double Poſition*.

Prop. 5. *Given l, n, and r, to find g and s.*

Theo. IX. $lr^{n-1}=g$. Theo. X. $\dfrac{r^{n}-1}{r-1}\times l=s$.

Prop. 6. *Given l, s, and r, to find g and n.*

Theo. XI. $\dfrac{\overline{s\times\overline{r-1}+l}}{r}=g$.

Theo. XII. $r^{n}=\dfrac{s\times\overline{r-1}}{l}+1$; or, log. $\dfrac{s\times\overline{r-1}}{l}+1\div\log.\ r=n$.

Prop. 7. *Given g, n, and s, to find l and r.*

Theo. XIII. $l\times\overline{s-l}^{\,n-1}=g\times\overline{s-g}^{\,n-1}$

Theo. XIV. $\dfrac{sr^{n-1}}{s-g}-r^{n}=\dfrac{g}{s-g}$.

Where the value of *l* in the firſt equation, and the value of *r* in the ſecond, muſt be found as directed in the 2d note, in *Double Poſition*, being the moſt difficult propoſition in geometrical progreſſion.

Prop. 8. *Given g, n, and r, to find l and s.*

Theo. XV. $\dfrac{g}{r^{n-1}}=l$. Theo. XVI. $\dfrac{r^{n}-1}{r^{n}-r^{n-1}}\times g=s$.

Prop. 9. *Given g, s, and r, to find l and n.*

Theo. XVII. $\overline{rg+s}-rs=l$.

Theo. XVIII. $r^{n-1}=\dfrac{g}{rg+s-rs}$, or, $\dfrac{\log.\ g-\log.\ rg+s-rs}{\log.\ r}+1=n$.

Prop. 10. *Given n, s, and r, to find l and g.*

Theo. XIX. $\dfrac{sr-s}{r^{n}-1}=l$. Theo. XX. $\dfrac{r^{n}-r^{n-1}}{r^{n}-1}\times s=g$.

The above theorems will anſwer for any *finite* ſeries of numbers, either increaſing or decreaſing; (ſee note 6th, Arithmetical Progreſſion.) But, if the ſeries decreaſe, *ad infinitum*, then *n* will be infinite, or greater than any aſſignable number, and $l=0$. Hence the three following theorems anſwer all the poſſible caſes of an infinite decreaſing geometrical progreſſion.

Prop. 11. *Given g and r, to find s.*

Theo. XXI. $\frac{rg}{r-1}=s$. Or proceed by Prop. 4th in the text.

Prop. 12. *Given r and s, to find g.*

Theo. XXII. $\frac{s\times\overline{r-1}}{r}=g$.

Prop. 13. *Given g and s, to find r.*

Theo. XXIII. $\frac{s}{s-g}=r$. In the three preceding theorems, if the ratio be a fraction, then r muſt repreſent the reciprocal of that fraction. Thus if the ratio be $\frac{7}{9}$, then $r=\frac{9}{7}$, &c.

Examples to Propoſition 1.

(1.) The firſt, or leaſt, term of a ſeries of numbers in geometrical progreſſion is 3, the ratio 3, and the number of terms 14, what is the greateſt, or laſt term?

1 . 2 . 3 . 4 . 5, &c. indices.
3 . 9 . 27 . 81 . 243, &c. leading terms.
5 + 5 + 4 = 14, index to the 14th term.
243 × 243 × 81 = 4782969, laſt, or 14th, term.

(2.) If the firſt, or leaſt, term be 2, the ratio 2, and the number of terms 19, what is the laſt, or greateſt, term?

(3.) A draper ſold 20 yards of cloth; the firſt yard for 3d. the ſecond for 9d. the third for 27d. &c. in triple proportion geometrical; what did he ſell the laſt yard for?

Examples to Prop. 2.

(4.) The firſt, or leaſt, term of a geometrical ſeries is 5, the ratio 3, and the number of terms 12; what is the laſt, or greateſt, term?

0 . 1 . 2 . 3 . 4, &c. indices.
5 . 15 . 45 . 135 . 405, &c. leading terms.
4 + 4 = 8 index to the 9th term.

$$\frac{405\times405}{5}=32805 \text{ the 9th term.}$$

8 + 3 = 11 index to the 12th term.

$$\frac{32805\times135}{5}=885735 \text{ the 12th term.}$$

(5.) If the firſt, or leaſt, term be 7, the ratio 2, and the mber of terms 19, what is the laſt, or greateſt, term?

(6.) A thraſher worked 20 days for a farmer, and re- eived (by agreement) for the firſt day's work 4 barley- orns, for the ſecond 12, for the third 36, &c. in triple roportion geometrical; what did he receive for his laſt ay's work, admitting 7680 barley-corns to fill a pint mea- ure, and the barley to be worth 2s. 6d. per buſhel?

Examples to Prop. 3.

(7.) If the firſt term of a ſeries of numbers in geome- rical progreſſion be 5, the ratio 3, and number of terms 2, what is the ſum of the terms?

The laſt, or greateſt, term (by example 4,) is 885735.
Then 885735 — 5 = 885730 dividend;
And 3 — 1 = 2 diviſor. Hence 885730 ÷ 2 = 442865;
nd 442865 + 885735 = 1328600, ſum of the terms.

(8.) If the firſt term be four, the ratio 3, and the num- ber of terms 7, what is the ſum of the terms?

CLASS II. *Exerciſing all the preceding propoſitions.*

(9.) What would a horſe be ſold for that has 4 ſhoes on, with 8 nails in each ſhoe, at 1 farthing for the firſt nail, 2 for the ſecond, 4 for the third, &c. And, ſuppoſing another horſe to be ſold with only two ſhoes on, on the ſame conditions, what would be the difference in their prices?

(10.) If a ſervant ſhould agree with his maſter to ſerve him 11 years, without any other reward than the produce of a wheat-corn for the firſt year; and, for the ſecond year, ground ſufficient to ſow his firſt year's produce on, &c., from year to year, till the end of the time: what would his wages amount to, admitting each wheat-corn to yield ten by ſowing, 7680 wheat-corns to fill a pint-meaſure, and that he could ſell his wheat at 4s. per buſhel?

(11.) A nobleman dying, left 10 ſons, to whom and to his executor he bequeathed his eſtate as follows; to his executor he gave 1024l. the youngeſt ſon was to have as much and half as much, and every ſon to ſucceed the next younger in the ſame ratio of 1½; what was the eldeſt ſon's fortune, and what did the nobleman die worth? *Anſwer, the eldeſt ſon's fortune was £59049, and the nobleman died worth £175099.*

Examples to Prop. 4.

(12.) Required the ſum of $\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+\frac{3}{1000}$ &c. *ad infinitum.*

$\frac{3}{10}-\frac{3}{100}=\frac{27}{100}$, difference between the firſt and ſecond terms. $\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$, ſquare of the firſt term. Then $\frac{9}{100}\div\frac{27}{100}=\frac{1}{3}$ anſwer.—Hence we may infer, that, if a ball were put in motion by force which moved it $\frac{3}{10}$ of a league, or 1584 yards, the firſt minu or any equal ſpace of time) $\frac{3}{100}$ of a league, or $158\frac{2}{5}$ yards, the ſecond &c. for *ever*, it would go no farther than 1 mile! For, it is evident tha $\frac{3}{10}+\frac{3}{100}$, &c. *ad infinitum*, $=\cdot3333$, &c. *ad infinitum*; and this is equa to $\frac{1}{3}$ preciſely, by the nature of vulgar fractions and infinite decimals.

(13.) Required the ſum of $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$, &c. *ad infinitum.*

(14.) Required the ſum of $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}$, &c. *ad infinitum.*

(15.) If a body be put in motion by a force which moves 10 miles the firſt equal ſpace of time, 9 miles the ſecond, and ſo on (in the ratio of $\frac{9}{10}$) for *ever*, how many miles will it paſs over?

VARIATIONS.

Definition. By Variations are meant the different ways any number of things may be altered, or changed with reſpect to their places. Theſe are ſometimes called *Changes, Permutation, Alternation*, &c.

Propoſition 1. *To find the number of changes that can be made of any given number of things, all different from each other.*

Rule. Multiply continually together the numbers 1, 2, 3, 4, 5, &c. to the number of terms; and the laſt product will be the anſwer.

Prop. 2. *Given any number of different things, to find how many changes can be made of them, by taking any given number of them at a time.*

Rule. Multiply the number of things by itſelf leſs 1, and that product by the ſame number leſs 2, &c. diminiſhing each ſucceeding multiplier, by an unit, till you have

ave made as many products (abating one) as there are hings taken at a time; the laſt product will be the an-ſwer.

Examples to Propoſition 1.

(1.) How many changes may be rung by 8 bells?

$$1\times2\times3\times4\times5\times6\times7\times8=40320, \text{ anſwer.}$$

(2.) How many changes may be rung on 9 bells?

(3.) An arithmetician aſked a farmer with whom he odged, what he ſhould give him per annum for *board* and odging; the farmer aſked him 25l. The arithmetician aid that was ſomewhat dear; however, he would give im that ſum if he would find him with board and lodging o long as he could place himſelf and the honeſt farmer's amily (conſiſting of 6 perſons) in a different poſition at dinner. How long might he ſtay for 25l.?

(4.) How many changes may be rung on 12 bells, and how long would they take in ringing once over, ſuppoſing 10 changes to be rung in a minute, and the year to conſiſt of 365 days 6 hours?

Examples to Prop. 2.

(5.) How many changes may be rung with 4 bells out of 8?

$$8\times\overline{8-1}\times\overline{8-2}\times\overline{8-3}=8\times7\times6\times5=1680, \text{ anſwer.}$$

(6.) How many changes may be rung with 7 bells out of 12?

(7.) Required the number of words that can be made with 5 letters of the 26 in the alphabet, allowing any 5 letters to make a word?

COMBINATIONS.

Definition. *By Combinations* muſt be underſtood a method of taking a leſs number of quantities out of a greater, as often as poſſible, without reſpect to their places, and combining them together.

Propoſition. To find the combinations of a leſs number things out of a greater, all different.

Rule. Take the ſeries 1, 2, 3, 4, 5, &c. up to the leſ number of things, and multiply them continually together then take a ſeries of as many terms, decreaſing by an unit from the greater number of things, and multiply *them* continually together.—Divide the latter product by the former and the quotient will be the anſwer.

Examples.

(1.) How many combinations can be made with 5 letter out of the 26 of the alphabet?

$$1\times2\times3\times4\times5=120 \text{ diviſor.}$$

$26\times\overline{26-1}\times\overline{26-2}\times\overline{26-3}\times\overline{26-4}=7893600$ dividend; an $7893600\div120=65780$, anſwer.

Or,

$$\frac{\overset{13}{\cancel{26}}\times\overset{5}{\cancel{25}}\times\overset{\overset{2}{\cancel{8}}}{\cancel{24}}\times23\times22}{1\times\cancel{2}\times\cancel{3}\times\cancel{4}\times\cancel{5}}=13\times5\times2\times23\times22=65780.$$

(2.) A ſucceſsful general was aſked by his ſovereign what reward he ſhould confer upon him for his ſervices; the general modeſtly aſked only a farthing for every file of 10 men in a file which he could make with a body of 100 men; what ſterling money will this amount to?

Thoſe, who wiſh for farther information in the *doctrine of combinations, permutations*, &c. may conſult Mr. *Emerſon*'s Treatiſe on the ſubject.

SIMPLE INTEREST by DECIMALS.

Put $p=$ the principal, or ſum put to intereſt.
$r=$ the ratio, being the rate per cent. divided by 100.
$t=$ the time, or years, the money is at intereſt.
$i=$ the intereſt for the time t.
$a=$ the amount.

Then, at 2½ per cent. $r=\cdot025$	At 4 per cent. $r=\cdot04$
— 3 — $r=\cdot03$	— 4½ — $r=\cdot045$
— 3½ — $r=\cdot035$	— 5 — $r=\cdot05$, &c.

And

		And			Decimals.
1 day	=	$\frac{1}{365}$	of a year	=	·002739726, &c.
1 week	=	$\frac{7}{365}$	of a year	=	·019178082, &c.
1 month	=	$\frac{1}{12}$	of a year	=	·083
1 quarter	=	$\frac{1}{4}$	of a year	=	·25
1 half	=	$\frac{1}{2}$	of a year	=	·5

Hence the decimal parts of a year, for any number of days, weeks, months, &c. may be readily found.

Propoſition 1. *Given the principal, time, and rate per cent. to find the intereſt or the amount.*

Rule. Multiply the principal, time, and ratio, together, the laſt product will be the *intereſt*; to which add the principal to find the amount.

Theorem, $ptr = i$, and $ptr + p = a$. When p, t, and r, are given.

Prop. 2. *Given the amount, (or the intereſt) time, and rate, to find the principal.*

Rule. Multiply the time by the ratio, and add an unit to the product; by this ſum divide the *amount*, and the quotient will be the principal.—*Or*, divide the *intereſt* by the product of the time and ratio, and the quotient will be the principal.

Theo. $\frac{a}{tr+1} = \frac{i}{tr} = p$. When a, (or i,) t, and r, are given.

Prop. 3. *Given the principal, time, and amount (or the intereſt) to find the rate per cent.*

Rule. Divide the difference between the amount and the principal (*viz.* the intereſt) by the product of the principal and time, and the quotient will be the ratio, which multiply by 100 to obtain the rate per cent.

Theo. $\frac{a-p}{pt} = \frac{i}{pt} = r$, when p, t, and a, (or i,) are given.

Prop. 4. *Given the principal, rate, and amount, (or intereſt) to find the time.*

Rule. Divide the difference between the amount and the principal (*viz.* the intereſt) by the product of the principal and ratio, and the quotient will be the time.

Theo. $\frac{a-p}{pr} = \frac{i}{pr} = t$. When p, r, and a, (or i,) are given.

Examples to Propoſition 1.

(1.) What will 567l. 10s. amount to in 9 years, at 4 p cent. per annum?

$$\begin{array}{rl}
£567{\cdot}5 & = p \\
9 & = t \\
\hline
5107{\cdot}5 & = pt \\
{\cdot}04 & = r \\
\hline
204{\cdot}300 & = ptr = i \\
567{\cdot}5 & = p \\
\hline
£771{\cdot}800 & = ptr + p = a. \\
20 & \\
\hline
\text{s. } 16{\cdot}000 &
\end{array}$$

Anſwer 771l. 16s.

(2.) What is the amount of 235l. at ſimple intereſt, fo 3¾ years, at 5 per cent. per annum?

(3.) What is the intereſt of 550l. for 5 years, at 3½ pe cent. per annum?

(4.) What will 700l. 10s. amount to in 5¼ years, at per cent. per annum?

(5.) What will 715l. 15s. amount to in 7½ years, at 4 per cent, per annum?

(6.) What is the intereſt of 715l. 15s. for 240 days, a 5 per cent. per annum?

(7.) What is the intereſt of 357l. 10s. for 65 days, a 5 per cent. per annum?

(8.) What will 510l. amount to in 5 years 120 days, a 5 per cent. per annum?

Examples to Prop. 2.

(9.) What principal, in 9 years, will amount to 771l. 16s at 4 per cent. per annum?

$$\begin{array}{rl}
9 & = t \\
{\cdot}04 & = r \\
\hline
{\cdot}36 & = tr \\
1{\cdot} & \\
\hline
1{\cdot}36 & = tr + 1 \text{ diviſor.}
\end{array}$$

Now $a = 771$l. 16. $= £771{\cdot}8$, dividend.

Hence $771{\cdot}8 \div 1{\cdot}36 = £567{\cdot}5 = 567$l. 10s. anſwe

(10.) What principal, put to intereſt for 9 years, will ain 204l. 6s. intereſt, at 4 per cent. per annum?

9 = *t* Now *i* = £204 6s. = £204·3, dividend.
·04 = *r* Hence 204·3 ÷ ·36 = £567·5 = 567l. 10s. anſwer.

·36 = *tr* diviſor.

(11.) What principal, in $3\frac{3}{4}$ years, will amount to 279l. 1s. 3d. at 5 per cent. per annum?

(12.) What principal, put to intereſt for 5 years, at $3\frac{1}{2}$ per cent. per annum, will gain 96l. 5s. intereſt?

(13.) What principal, put to intereſt for $5\frac{1}{4}$ years, will amount to 810l. 16s. $6\frac{3}{4}$d.$\frac{3}{5}$, at 3 per cent. per annum?

(14.) What principal will amout to 943l. 17s. $10\frac{3}{4}$d.$\frac{1}{2}$ in $7\frac{1}{2}$ years, at $4\frac{1}{4}$ per cent. per annum?

(15.) What principal, put to intereſt for 240 days, at 5 per cent. per annum, will gain 23l. 10s. $7\frac{1}{2}$d. $\frac{8}{73}$?

(16.) What principal, put to intereſt for 65 days, at 5 per cent. per annum, will gain 3l. 3s. $7\frac{3}{4}$d.$\frac{65}{73}$ intereſt?

(17.) What principal, put to intereſt for 5 years and 120 days, at 5 per cent. per annum, will amount to 679l. 8s. $4\frac{1}{4}$d.$\frac{7}{73}$?

Examples to Prop. 3.

(18.) At what rate per cent. will 567l. 10s. amount to 771l. 16s. in 9 years time?

£771 16 = £771·8 = *a*
567 10 = 567·5 = *p*

204·3 = *a* — *p* = *i*, dividend.
567·5 × 9 = 5107·5 = *pt*, diviſor.
Then 204·3 ÷ 5107·5 = ·04. Hence the rate is 4 per cent.

(19.) At what rate per cent. will 235l. amount to 279l. 1s. 3d. in $3\frac{3}{4}$ years?

(20.) At what rate per cent. will 550l. gain 96l. 5s. intereſt in 5 years?

(21.) At what rate per cent. will 700l. 10s. amount to 810l. 16s. $6\frac{3}{4}$d.$\frac{3}{5}$ in $5\frac{1}{4}$ years?

(22.) At what rate per cent. will 715l. 15s. amount to 943l. 17s. $10\frac{3}{4}$d. $\frac{1}{2}$ in $7\frac{1}{2}$ years?

(23.) At what rate per cent. will 715l. 15s. gain 23l. 10s. $7\frac{1}{2}$d. in 240 days?

(24.) At what rate per cent. will 357l. 10s. gai 3l. 3s. $7\frac{3}{4}$d $\frac{65}{73}$ in 65 days?

(25.) At what rate per cent. per annum will £51 amount to 679l. 8s. $4\frac{1}{4}$d.$\frac{7}{73}$ in 5 years and 120 days?

Examples to Prop. 4.

(26.) In what time will 567l. 10s. amount to 771l 16 at 4 per cent. per annum?

£771 16 = £771·8 = a
567 10 = 567·5 = p

204·3 = $a-p=i$, dividend.
567·5 × ·04 = 22·700 = pr, diviſor.
Then 204·3 ÷ 22·7 = 9 years, the time required.

(27.) In what time will 235l. amount to 279l. 1s. 3d at $3\frac{1}{2}$ per cent. per annum?

(28.) In what time will 550l. gain 96l. 5s. at $3\frac{1}{2}$ pe cent. per annum?

(29.) In what time will 700l. 10s. amount to 810l. 16s. $6\frac{3}{4}$d.$\frac{3}{5}$ at 3 per cent. per annum?

(30.) In what time will 715l. 15s. amount to 943l. 17s. $10\frac{3}{4}$d.$\frac{1}{2}$ at $4\frac{1}{4}$ per cent. per annum?

(31.) In what time will 715l. 15s. gain 23l. 10s. 7d$\frac{1}{2}$.$\frac{15}{73}$ at 5 per cent. per annum?

(32.) In what time will 357l. 10s. gain 3l. 3s. $7\frac{3}{4}$d.$\frac{65}{73}$ at 5 per cent. per annum?

(33.) In what time will 510l. amount to 679l. 8s. $4\frac{1}{4}$d.$\frac{7}{73}$ at 5 per cent. per annum?

DISCOUNT at SIMPLE INTEREST, by DECIMALS.

Propoſition. Any ſum of money, due ſome time hence, being given to find its preſent value to the creditor, diſcounting at any rate per cent.

Rule. As the amount of £1, for the given time, is to £1, ſo is the intereſt of the debt for the ſaid time to the diſcount required.—Subtract the diſcount from the debt, and the remainder will be the preſent value.

Theo. $\frac{Dtr}{tr+1}$ = the diſcount; and, $\frac{D}{tr+1}$ = the preſent worth.
Where D = the debt, and t and r as before.

Examples.

(1.) What is the diſcount and preſent worth of 795l. 11s. 2d. for 11 months, at 6 per cent. per annum?

$$\pounds 795\ 11\ 2 = \pounds 795{\cdot}5583' = D.$$

$$11 \text{ months} \times {\cdot}083' = {\cdot}916' \text{ year} = t.$$

$795{\cdot}5583' \times {\cdot}916 \times {\cdot}06 = 43{\cdot}7557083' = Dtr$, intereſt of the debt for 11 months.

$\overline{{\cdot}916' \times {\cdot}06} + 1 = 1{\cdot}055 = tr + 1$, amount of 1l. for 11 months.

As $1{\cdot}055$: 1l. :: $43{\cdot}7557083'$: £41·4746 = 41l. 9s. 5¼d. $= \frac{Dtr}{tr+1}$, the diſcount of the debt. Then 795l. 11s. 2d. — 41l. 9s. 5¼d. = 754l. 1s. 8d. preſent worth.

Or,

$795{\cdot}5583' \div 1{\cdot}055 = 754{\cdot}083728 =$ 754l. 1s. 8d. the preſent worth.

(2.) What is the diſcount of 495l. 18s. for 5 months, at 3½ per cent. per annum?

(3.) What is the preſent worth of 1507l. 14s. 9d. due 7 months hence, at 5 per cent.?

(4.) What is the diſcount of 7147l. 14s. due 175 days hence, at 3¾ per cent.?

(5.) What ready money will diſcharge a debt of 1789l. 13s. 4d. due 3 years 3 quarters and 29 days hence, diſcounting at 4⅓l. per cent. per annum?

EQUATION OF PAYMENTS AT SIMPLE INTEREST,

BY DECIMALS,

ON MALCOLM's PRINCIPLES.

Propoſition. Having two *debts, due at different times, to find the equated time for paying the whole at once, without loſs either to the debtor or creditor.*

Rule 1. Divide the ſum of the debts by twice the firſt payment, multiplied by the ratio; to the quotient add half the time between the two payments, and call the ſum the firſt number found.

2. Multiply the ſecond payment by the time between the two payments, and divide the product by the firſt payment multiplied by the ratio; call the quotient the ſecond number found.

3. From the *ſquare* of the firſt-found number ſubtract the ſecond, and extract the ſquare-root of the differenc —The firſt-found number, diminiſhed by this root, wil give the equated time, reckoning from the time the fir payment is due.

Note. The above rule is the ſame as Mr. *Malcolm*'s, though expreſſe in a different, and, it is apprehended, more intelligible manner.—Thi rule is built upon a ſuppoſition 'That the ſum of the *intereſts* of the debt due *before* the equated time, from the time they become due to *that* time ought to be equal to the ſum of the *diſcounts* of the debts due *after* th equated time from *that* time to the time they become due.' According t this ſuppoſition, the rule given above is univerſally true for two payments. But, when three, or more, payments are to be equated for, Mr *Malcolm*'s directions for finding an equated time for the two that are fir payable, then their ſum, and a third, &c. is not ſtrictly true, accordin to the ſuppoſition on which his rule is founded; nor would it be an eaſ matter to give general rules, or theorems, for all the poſſible caſes, o account of the variation of the debts, and the difficulty of finding betwee which of the payments the equated time would fall. Beſides, in long an tedious operations, miſtakes are frequently made; and the anſwer, whe obtained, *admitting it to be true*, differs a mere trifle from the anſwe found by the old rule: hence, a rule, founded upon *ſimple intereſt*, an Mr. *Malcolm*'s principles, may, I think, with propriety be conſidered a an uſeleſs curioſity.

Examples.

(1.) A perſon has now due to him 320l. and at the end of 5 years, 96l. more will be due from the ſame debtor. Now both parties have agreed that the whole ſhall be paid at once, viz. at that time when the intereſt of the 320l. ſhall be equal to the diſcount of the 96l. both being calculated at 5l. per cent. per annum. The time of payment is required?

1ſt. $320+96=416$l. ſum of the debts.

$320\times2\times{\cdot}05=32$, the product of twice the firſt payment by the ratio.

$416\div32=13$, quotient. Then $13+\frac{5}{2}=15{\cdot}5$, *the firſt number found.*

2dly. $\overline{96\times5}\div\overline{320\times{\cdot}05}=30$, *the ſecond number found.*

3dly. $\sqrt{\overline{15{\cdot}5}|^{2}-30}=\sqrt{210{\cdot}25}=14{\cdot}5$, and $15{\cdot}5-14{\cdot}5=1$ year, the time which muſt elapſe (after the firſt payment is due) before the whole ought to be paid together, according to the conditions of the queſtion.

(2.) There is 100l. payable 1 year hence, and 105l. payable 3 years hence; what is the equated time, allowing ſimple intereſt, at 5 per cent. per annum?

(3.) At Michaelmas, 1788, I lent 320l. and at Michaelmas, 1793, 202l. will be due to me from the ſame perſon. Now, on what day, and in what year, may I receive both the debts together, viz. 522l. reckoning intereſt at 5 per cent. per annum?

COMPOUND INTEREST BY DECIMALS.

Put p = the principal, or money lent.
r = the ratio, or amount of £1 for a year, ½ year, ¼ year, &c. according as the payments are made yearly, ½ yearly, quarterly, &c.
t = the time, *or number of payments.*
a = the amount.

Rates per Cent.	The amounts of £1, or values of r, for		
	Yearly payments.	½-yearly payments.	Quarterly payments.
3	1·03	1·015	1·0075
3½	1·035	1·0175	1·00875
4	1·04	1·02	1·01
4½	1·045	1·0225	1·01125
5	1·05	1·025	1·0125
5½	1·055	1·0275	1·01375
6	1·06	1·03	1·015

The amounts, or values of r, in the preceding table, are calculated thus:

$100 : 100+3 :: 1 : 1{\cdot}03 = r$ for yearly payments.
$100 : 100+1\frac{1}{2} :: 1 : 1{\cdot}015 = r$ for ½-yearly payments.
$100 : 100+\frac{3}{4} :: 1 : 1{\cdot}0075 = r$ for quarterly payments.

This method is moſt commonly uſed.—Some writers find the value of r thus: let m = the amount of £1 for half a year, at 3 per cent. Then 1·03 is undoubtedly the true amount for a year; hence, according to the principles on which the rules of compound intereſt are found,

$1 : m :: m : 1{\cdot}03 \ \therefore\ m = \sqrt{1{\cdot}03} = 1{\cdot}014889$, &c. $= r$, for ½-yearly payments.

$1 : m^2 :: m^2 : 1{\cdot}03 \ \therefore\ m = \sqrt[4]{1{\cdot}03} = 1{\cdot}007417$, &c. $= r$, for quarterly payments.

Or, if m = the amount of £1 for $\frac{1}{n}$ of a year, at R per cent. then $r = \overline{\frac{R}{100}+1}\,^{\frac{1}{n}}$ *univerſally.*

And theſe values of r ſeem to me, to be more correct than thoſe given above; eſpecially in the calculation of annuities: for this reaſon I have inſerted the following table, that the reader may uſe which he pleaſes. Mr. *Ward*, in his *Clavis Uſuræ*, publiſhed in 1710, makes uſe of this method.

Rates per cent.	The amounts of £1, or value of r, for Yearly payments.	½-yearly payments.	Quarterly payments.
3	1·03	1·014889	1·007417
3½	1·035	1·017349	1·008637
4	1·04	1·019803	1·009853
4½	1·045	1·022252	1·011065
5	1·05	1·024695	1·012272
5½	1·055	1·027132	1·013475
6	1·06	1·029563	1·014673

Propoſition 1. *Given the principal, rate, and time, to find the amount or intereſt.*

Rule. Find the amount of £1 for the firſt payment, by ſimple intereſt, which involve to ſuch a power as is denoted by the number of payments.—This power, multiplied by the principal, will give the amount; from which deduct the principal, and the remainder will be the intereſt.

Or, *Theo.* I. $p \times r^{t} = a$, *when* p, r, *and* t, *are given.*

Logarithmically. log. p + log. $r \times t$ = log. a.

Prop. 2. *Given the amount, rate, and time, to find the principal.*

Rule. As the amount of £1, at the rate and for the time given, is to £1, ſo is the amount given to the principal required.

Or, Theo. II. $\frac{a}{r^{t}} = p$, *when* a, r, *and* t, *are given.*

Logarithmically. log. a — log. $r \times t$ = log. p.

Prop. 3. *Given p, a, and t, to find r.*

Theo. III. $\left.\overline{\frac{a}{p}}\right|^{\frac{1}{t}} = r.$

Logarithmically. $\frac{log.\ a - log.\ p.}{t} = log.\ r.$

Prop. 4. *Given p, a, and r, to find t.*

Theo. IV. $\frac{a}{p} = r^{t}.$ If *t* is not a whole number, it cannot be found without logarithms.

Logarithmically. $\frac{log.\ a - log.\ p}{log.\ r} = t.$

Examples to Propoſition 1.

(1.) What will 200l. amount to in 6 years, at 5 per cent. per annum, compound intereſt, and what intereſt will it gain?

Here the amount of 1l. for the firſt payment is £1·05, and 1·05× 1·05×1·05×1·05×1·05×1·05=1·340095640625 (=r^{t}) This, multiplied by 200, gives 268·019128125=£268 0 4½·363 (=$p \times r^{t}$) the amount; from which deduct 200l. the principal, and the remainder, 68l. 0s. 4½d.363, will be the intereſt.

(2.) What will 275l. amount to in 3 years, at 5 per cent. per annum, compound intereſt?

(3.) What is the compound intereſt of 700l. 15s. for 7 years, at 4 per cent. per annum?

(4.) What is the compound intereſt of 800l. for 9 years, at 5 per cent. per annum?

Examples to Prop. 2.

(5.) What principal, put to intereſt for 6 years, will amount to 268l. 0s. 4½d·363 at 5 per cent. per annum?

Firſt, £268 0 4½·363 = 268·019128125 (=a,) and 1·05 × 1·05 ×1·05×1·05×1·05×1·05 = 1·340095640625 (=r^{t}) amount of 1l. for 6 years. Hence

As 1·340095640625 : 1l. :: 268·019128125 : 200l. the principal required (=$\frac{a}{r^{t}}$)

(6.) What principal, put to intereſt for 3 years, wil amount to 318l. 6s. 11¼d. at 5 per cent. per annum?

(7.) What principal, put to intereſt for 4 years, at 4 per cent. per annum, will amount to £819 15s. 6d¾. ·2504832?

(8.) What principal, put to intereſt for 9 years, a 5 per cent. per annum, will amount to £1241 1s. 3·017467875d.?

Examples to Prop. 3. *Theorem* III.

(9.) At what rate per cent. will 200l. amount to £268 0 4½·363 in 6 years time?

Firſt, £268 0 4½·363 = £268·019128125 (= a,) and 268·019128125 ÷ 200 = 1·340095640625 (= $\frac{a}{p}$;) the 6th root of which, (by the rule page 186,) is 1·05 (= $\overline{\frac{a}{p}}\Big|^{\frac{1}{t}}$.)

Or, the ſquare root of 1·340095640625 *is* 1·157625, *and the cube of* 1·157625 *is* 1·05 *as above. Hence the rate is* 5 *per cent.*

(10.) At what rate per cent. will 275l. amount to 318l. 6s. 11¼d. in 3 years time?

(11.) At what rate per cent. will 700l. 15s. amount to £819 15 6¾d·2504832 in 4 years?

(12.) At what rate per cent. will 800l. amount to £1241 1s. 3·017467875d. in 9 years?

Examples to Prop. 4. *Theo.* IV.

(13.) In what time will £200 amount to 268l. 0s. 4½d·363, at 5 per cent. per annum?

£268 0 4½·363 = £268·019128125 (= a.)

Then 268·019128125 ÷ 200 = 1·340095640625 (= $\frac{a}{p} = r^t$) which, being divided by 1·05, (= r,) and the *quotient* by 1·05, &c. till nothing remains, the number of diviſions will ſhew the time, 6 years.

Note, This method of finding the time, by repeated diviſions, is made uſe of by Mr. Ward, *(ſee ex. 3rd, page* 255, *8th edit. of his Math. Guide,) and ſeveral writers have followed his example; but it is far from being an eligible, or correct method. It may ſerve to prove a queſtion when the time happens to be whole years. The beſt method of ſolving the queſtions in this and the preceding propoſition is by logarithms.*

(14.) In what time will 275l. amount to 318l. 6s. 11¾d. at 5 per cent. per annum?

(15.) In what time will 700l. 15s. amount to 819l. 15s. 6¾d·2504832, at 4 per cent. per annum?

(16.) In what time will 800l. amount to 1241l. 1s. 3·017467875d. at 5 per cent. per annum?

DISCOUNT AT COMPOUND INTEREST.

Propoſition. Any ſum of money, due ſome time hence, being given to find its preſent value to the creditor, diſcounting at any rate per cent. compound intereſt.

Theo. I. $\frac{D}{r^t}$ = the preſent worth; *Theo.* II. $\frac{r^t - 1}{r^t} \times D$ = the diſcount. Where D = the debt, and t and r as before.

Logarithmically. log. D—log. $r \times t$ = log. of the preſent worth, and log. $r^t - 1$ + log. D—log. $r \times t$ = log. of the diſcount.

Examples.

(1.) What is the preſent worth and diſcount of 243l. $2\frac{1}{40}$s. due 4 years hence, diſcounting at 5 per cent. per annum?

Firſt, £243 $2\frac{1}{40}$s. = 243·10125 (= D) the dividend.

1·05 × 1·05 × 1·05 × 1·05 = 1·21550625 (= r^t) the diviſor.

Hence 243·10125 ÷ 1·21550625 = £200, the preſent worth, and £243 $2\frac{1}{40}$s. — 200 = 43l. $2\frac{1}{40}$s. the diſcount.

Or, 1·21550625 — 1 = ·21550625 (= $r^t - 1$;) this, multiplied by 243·10125 (= D) gives 52·389838757812 5 (= $\overline{r^t - 1} \times D$) for a dividend, which divide by 1·21550625 (= r^t) and the quotient will be 43·10125 = 43l. $2\frac{1}{40}$s. the diſcount. Hence the preſent worth is 200l. as before.

(2.) What is the preſent worth and diſcount of a debt of 400l. due 4 years hence, at 5 per cent. per annum?

(3.) If 643l. 4s. 11d. be payable in 6 years, what is the preſent worth, diſcounting at 5 per cent. per annum?

(4.) A perſon has £500 due at 5 different times, vi 100l. at 1 year's end, 100l. more at the end of 2 years 100l. more at the end of 3 years, 100l. more at the end o 4 years, and 100l. more at the end of 5 years. What i the preſent worth of the whole, diſcounting at 6 per cent per annum?

EQUATION OF PAYMENTS AT COMPOUND INTEREST.

Propoſition. Having ſeveral debts, due at different times from one perſon, to find the TRUE *equated time for paying th whole at once, without loſs either to the debtor or creditor, allowing compound intereſt.*

Rule 1. Find the amount of each debt from the time i becomes due to the time of the laſt payment, [by Prop. 1, Compound Intereſt] add theſe amounts, together with the laſt payment, into one ſum.

2. Find in what *time* [by Prop. 4, Compound Intereſt] the ſum of the debts will amount to the ſum of the amounts found above:—this *time,* ſubtracted from the time the laſt payment becomes due, will give the *true* equated time.

Note. This rule, which is *Sir Samuel Moreland*'s, is founded on the ſame manner of reaſoning as the common rule, Part I. and will bring out the *ſame* anſwer, allowing ſimple intereſt inſtead of compound.—Were this a place for algebraical demonſtrations, it might eaſily be ſhewn that the above rule is univerſally true, allowing *compound* intereſt, whether we argue from *Burrow*'s, *Kerſey*'s, or *Malcolm*'s, principles, it being deducible from each.

Examples.

(1.) A owes to B 1000l. 200l. of which will be due 1 year hence, 200l. two years hence, 150l. three years hence, 300l. four years hence, and 150l. five years hence; ſhould theſe perſons agree to have the whole diſcharged at once, what will be the *true* equated time, reckoning intereſt at 5 per cent. per annum?

1·05

$\overline{1\cdot05}|^{4}\times200=243\cdot10125$ amount of the firſt payment.
$\overline{1\cdot05}|^{3}\times200=231\cdot525$ ditto of the ſecond.
$\overline{1\cdot05}|^{2}\times150=165\cdot375$ ditto of the third.
$1\cdot05\times300=315\cdot$ ditto of the fourth.
$150\cdot$ laſt payment.

£1105·00125 ſum of the amounts.

$200+200+150+300+150=$ £1000, ſum of the debts: Now we have to find in what time 1000l. will amount to £1105·00125, at 5 per cent. compound intereſt; *(and here we muſt be under the neceſſity of making uſe of logarithms, ſince the method made uſe of in Ex.* 13, *page* 248, *will by no means do.*

Firſt, $1105\cdot00125=\frac{884001}{800}$, then *log.* $\frac{884001}{800}$ — *log.* 1000 = 0·0433628 (= *log. a* — *log. p*;) this dividend by *log.* 1·05 (= *log. r*) = 0·0211893 gives 2·0464, &c. for the quotient, (= *t.*) Hence 5 years — 2·0464, &c. years = 2·9535, &c. the *true* equated time.

(2) A perſon has 320l. due to him; and, at the end of 5 years, 96l. more will be due from the ſame debtor: now both parties have agreed for the whole to be diſcharged at once. The *true* equated time is required, reckoning intereſt at 5 per cent. per annum?

(3.) There is 100l. payable one year hence; and 105l. payable three years hence; what is the *true* equated time, allowing compound intereſt at 5 per cent. per annum?

(4.) There is 100l. payable one year hence, 200l. two years hence, 300l. three years hence, and 500l. five years hence; required the *true* equated time for paying the whole at once, reckoning compound intereſt at 5 per cent. per annum?

ANNUITIES CERTAIN.

Definition 1. *Annuities certain* ſignify any intereſt of money, rents, or penſions, payable yearly, or from time to time, to *ſome* certain period, or for *ever*. They are divided into two parts, viz. annuities in *poſſeſſion*, or ſuch as are either entered upon, or are to be entered upon immediately; and annuities in *reverſion*, or ſuch as are not to be entered upon till ſome particular future event has happened, or till ſome given period of time has elapſed; an the time the purchaſer holds the annuity, after he *has* entered upon it, is called the *reverſion.*

2. *An annuity is ſaid to be in arrears* when the debto keeps it in his hands for any certain time after the term o payment; and the ſum of all the ſingle payments, togethe with the intereſt due upon each payment from the time o its becoming due to the time the whole is paid off, is calle the amount of ſuch annuity.

3. *When an annuity,* to be entered on immediately, o ſome time hence, is ſold for ready money, the price whic ought to be paid for it is called the preſent worth.

ANNUITIES AT SIMPLE INTEREST.

Let x = the annuity, penſion, rent, or *payment,* whether yearly, $\frac{1}{2}$ yearly, or quarterly, &c.

t = the time, *or number of payments.*

r = the intereſt of £1 for 1 year, $\frac{1}{2}$ year, or $\frac{1}{4}$ year, &c. according as the payments are made yearly, $\frac{1}{2}$-yearly, or quarterly.

a = the amount.

For	The intereſts of £1, or values of r, at				
	3 per cent.	$3\frac{1}{2}$ per cent.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent
1 year	·03	·035	·04	·045	·05
$\frac{1}{2}$	·015	·0175	·02	0225	·025
$\frac{1}{4}$	·0075	·00875	·01	·01125	·0125
1 m. $=\frac{1}{12}$	·0025	002916′	·003′	·00375	·00416′
1 w. $=\frac{7}{365}$	·0005754	·0006713	·0007672	·0008631	·000959
1 d. $=\frac{1}{365}$	0000822	·0000959	·0001096	·0001233	·000137

ANNUITIES IN ARREARS, AT SIMPLE INTEREST

Propoſition 1. *To find the amount of an annuity, at ſimpl intereſt, in arrears, or unpaid, for any number of payments.*

Rule. Take the ſeries of numbers, 1, 2, 3, 4, 5, &c to the number of years leſs 1, and add them together Multiply this ſum by one year's intereſt of the annuity and the product will be the whole *intereſt* due thereon; t which add the product of the annuity and time, and th ſum will be the amount.

Or, *Theo.* I. $\frac{\overline{tt-t}\times rn}{2}+tn=a.$ *When n, t, and r, are given.*

Prop. 2. *Given a, t, and r, to find n.*

Theo. II. $\frac{2a}{\overline{ttr+2t}-tr}=n.$

Prop. 3. *Given r, a, and n, to find t.*

Theo. III. $\sqrt{\frac{2a}{nr}+\left.\frac{\overline{2-r}}{2r}\right|^{2}}-\frac{\overline{2-r}}{2r}=t.$

Prop. 4. *Given a, t, and n, to find r.*

Theo. IV. $\frac{\overline{a-nt}\times 2}{\overline{t-1}\times tn}=r.$

Examples to Propoſition 1.

(1.) If an annuity, rent, or penſion, of 100l. be unpaid for 5 years, what will it amount to at 6 per cent. per annum?

$1+2+3+4=10$
One year's intereſt of 100l. $=6$
60 whole intereſt due.
$100\times 5=500$
£560 the amount required.

Or thus by Theorem I.

$5=t$	$100=n$	$100=n$
5	$\cdot06=r$	$5=t$
$25=tt$	$6\cdot00=rn$	$500=tn$
5		
$20=\overline{tt-t}$		

Then $\frac{20\times 6\cdot00}{2}=60$, and $60+500=560$l. ($=a$) the amount as before.

(2.) If an annuity of 80l. be unpaid for 9 years, what will it amount to at 5 per cent. per annum?

(3.) If the payment of a penſion of 560l. per annum be omitted for 7 years, what will it amount to, allowing ſimple intereſt at 4 per cent. per annum?

(4.) If an annuity of 350l. payable half-yearly, were unpaid for 4 years, what would it amount to in that time, reckoning intereſt at $4\frac{1}{2}$ per cent.?

(5.) If 70l. annuity, payable every quarter of a year, were unpaid for 5 years, what would it amount to in that time at 5 per cent.?

Examples to Prop. 2. *Theo.* II.

(6.) What annuity, unpaid for 5 years, will amount to 560l. allowing 6 per cent. ſimple intereſt?

$$\begin{array}{rrl} & 5 & = t \\ & 5 & \\ \cline{2-2} & 25 & = tt \\ & \cdot 06 & = r \\ \cline{2-2} & 1\cdot 50 & = ttr \\ 5\times 2 = & 10\cdot & = 2t \\ \cline{2-3} & 11.50 & = ttr + 2t \\ 5\times\cdot 06 = & \cdot 30 & = tr \\ \cline{2-3} & 11\cdot 20 & = ttr + 2t - tr,\ \text{diviſor.} \end{array} \qquad \begin{array}{rl} 560 & = a \\ 2 & \\ \hline 1120 & = 2a\ \text{dividend.} \end{array}$$

Hence $1120 \div 11\cdot 2 = 100$l. ($= n$) the annuity.

(7.) What annuity, unpaid for 9 years, will amount to 864l. allowing 5 per cent. ſimple intereſt?

(8.) What annuity, unpaid for 7 years, will amount to 4390l. 8s. at 4 per cent. per annum?

(9.) What annuity, payable half-yearly, will amount to 1510l. 5s. if unpaid for 4 years, reckoning intereſt at $4\frac{1}{2}$ per cent.?

(10.) What annuity, payable quarterly, will amount to 391l. 11s. 3d. if unpaid for 5 years, reckoning intereſt at 5 per cent.?

Examples to Prop. 3. *Theo.* III.

(11.) In what time will an annuity of 100l. amount to 560l. if unpaid, reckoning intereſt at 6 per cent.?

Here $a = 560$, $n = 100$, and $r = \cdot 06$.

$$\frac{560\times 2}{100\times\cdot 06} = \frac{1220}{6} = 186\cdot 6' = \frac{2a}{nr}.$$

$$\frac{2-\cdot 06}{2\times\cdot 06} = \frac{1\cdot 94}{\cdot 12} = 16\cdot 16' = \frac{2-r}{2r}.$$

16·16′

$$16{\cdot}16' \times 16{\cdot}16' = 261{\cdot}361' = \overline{\frac{2-r}{2r}}\Big|^{2}$$

Laſtly, $186{\cdot}6' + 261{\cdot}361' = 448{\cdot}027'$, the ſquare root of which is $21{\cdot}16'$; hence $21{\cdot}16' - 16{\cdot}16' = 5$ years, the time required.

(12.) In what time will an annuity of £80l. amount to 864l. (unpaid for that time) at 5 per cent. per annum?

(13.) In what time will an annuity of £560 amount to 4390l. 8s. at 4 per cent. per annum?

(14.) In what time will an annuity of 350l. payable half-yearly, amount to 1510l. 5s. (if unpaid for that time) reckoning intereſt at 4½ per cent.?

(15.) If an annuity of 70l. payable quarterly, amounts to 391l. 11s. 3d. unpaid for a certain time, reckoning intereſt at 5 per cent. the time is required.

Examples to Prop. 4. *Theo.* IV.

(16.) An annuity of 100l. unpaid for 5 years, amounted to 560l.; what rate per cent. was allowed?

$$\begin{array}{rl} 560 &= a \\ 100\times5=500 &= nt \\ \hline 60 &= \overline{a-nt} \\ 2 & \\ \hline 120 &= \overline{a-nt}\times 2\text{, dividend.} \end{array} \qquad \begin{array}{rl} 5 &= t \\ 1 & \\ \hline 4 &= \overline{t-1} \\ 100\times5=500 &= nt \\ \hline 2000 &= \overline{t-n}\times nt\text{ diviſor.} \end{array}$$

Then $120 \div 2000 = {\cdot}06$. Hence the rate is 6 per cent.

(17.) At what rate per cent. will an annuity of 80l. unpaid for 9 years, amount to 864l.?

(18.) At what rate per cent. will 560l. amount to 4390l. 8s. if unpaid for 7 years?

(19.) At what rate per cent. will an annuity of 350l. payable half-yearly, amount to 1510l. 5s. if unpaid for 4 years?

(20.) At what rate per cent. will an annuity of 70l. payable quarterly, amount to 391l. 11s. 3d. if unpaid for 5 years?

PRESENT WORTH *of Annuities at Simple Intereſt.*

Propoſition 1. *To find the preſent worth of an annuity at Simple Intereſt, the time of its continuance, and the rate per cent. being given.*

Rule. Find the amount of the annuity [by Prop. 1, page 252.] ſuppoſing it in arrears till the laſt payment is due. Then find the preſent worth of that amount, [by the rule in Diſcount, page 242.] and it will be the anſwer.

Or, *Theo.* I. $\frac{\overline{ttr+2t}-tr}{2tr+2}\times n=p$, the preſent worth, when n, r, and t, are given.

Note. The above rule is the ſame as thoſe given by *Sir Samuel Moreland*, *Ward*, &c. as a correction of Mr. *Kerſey*'s, which they conceived to be falſe. Mr. *Malcolm*, at page 600 of his Arithmetic, has endeavoured to ſhew the fallacy of *Moreland*'s arguments, and the truth of *Kerſey*'s rule.—As ſome *late* writers have copied Mr. *Malcolm*'s example, and part of his argument, I cannot diſmiſs this ſubject without offering ſome reaſons why I prefer *Moreland*'s rule, nor can I, without being guilty of a palpable defect, omit giving Mr. *Kerſey*'s rule likewiſe, that the reader may judge for himſelf, and make uſe of which he pleaſes.

Kerſey's rule. Find the preſent worth of each payment by itſelf, diſcounting from the time it falls due, (by the rule in Diſcount, page 242.) the ſum of all theſe is the preſent worth of the whole.

Example. What is the preſent worth of an annuity of £100, to continue 5 years, diſcounting at 6 per cent. *Anſwer*, £430 15s. $4\frac{8}{13}$d. by *Ward*'s, and £425 18s. $9\frac{1}{4}$d. by *Kerſey*'s rule.—Mr. *Malcolm* acknowledges the true preſent worth to be £430 15s. $4\frac{8}{13}$d. if the purchaſer receives nothing till the end of five years, (the debtor of the annuity paying him intereſt for his money at the rate of 6 per cent.) becauſe the annuity would, at the end of that time, amount to £560.

But, if the purchaſer receives the annuity as it becomes due, it is worth only £425 18s, $9\frac{1}{4}$d. I would aſk why the annuity is *not* worth as much to the purchaſer when he receives it annually, as when he receives nothing till the laſt payment is due? If the debtor of the annuity can make £560 of it by *keeping* it till the laſt payment, cannot the purchaſer make the ſame by *receiving* it as it becomes due, and putting it out to intereſt at the ſame rate per cent.?—The propriety of *Sir Samuel Moreland*'s correction, therefore, appears evident to me; thoſe, who wiſh for farther information on the ſubject, may conſult Mr. *Malcolm*'s Arithmetic, from page 599 to 604.—See alſo the *Scholium*, page 206, *Wood*'s Algebra.

Prop. 2. *Given p, r, and t, to find n.*

Theo. II. $\frac{2tr+2}{tr+2-r}\times\frac{p}{t}=n$.

Prop. 3. *Given p, r, and n, to find t.*

Theo. III. $\sqrt{\frac{2p}{rn}+\overline{\frac{n-rp}{rn}-\frac{1}{2}}^{2}}-\overline{\frac{n-rp}{rn}}-\frac{1}{2}=t.$

Prop. 4. *Given p, t, and n, to find r.*

Theo. IV. $\frac{nt-p}{2p+n-nt}\times\frac{2}{t}=r.$

Examples to Propoſition 1.

(1.) Required the preſent worth of an annuity of 100l. to be continued 5 years, allowing 6 per cent. ſimple intereſt?

$1+2+3+4=10$
one year's intereſt of 100l. $=6$

60 whole intereſt.
$100\times5=500$

560 amount of the annuity (*per Prop.* 1, *page* 252.) Then per the Theo. page 242,) $\overline{5\times\cdot06+1}=1\cdot3=tr+1$, and $560=D$; hence $560\div1\cdot3=430\cdot7'69230'=$ 430l. 15s. 4½d·4′61538′d. the preſent worth.

Or thus, per Theorem I.

$5=t$
5

$25=tt$
$\cdot06=r$

$1\cdot50=ttr$
$2\times5=10\cdot=2t$

$11\cdot50=\overline{ttr+2t}$
$5\times\cdot06=\cdot30=tr$

dividend $11\cdot20=\overline{ttr+2t-tr}$

$5=t$
$\cdot06=r$

$\cdot30=tr$
2

$\cdot60=2tr$
$2\cdot$

$2\cdot60=2tr+2$ diviſor.

Then $\frac{11\cdot2}{2\cdot6}\times100=430\cdot7'69230'=$ £430 15s. 4½d·4′61538′, the preſent worth as before.

(2.) Required the preſent worth of an annuity of 80l. to continue 9 years, at 5 per cent. per annum.

(3.) Required the preſent worth of an annuity of 560l. to continue 7 years, at 5 per cent. per annum.

(4.) What is the preſent worth of an annuity of 350l. payable half-yearly, to continue 4 years, at $4\frac{1}{2}$ per cent.?

(5.) What is the preſent worth of an annuity of 70l. payable quarterly, to continue 5 years, at 5 per cent. per annum?

Examples to Prop. 2. *Theo.* II.

(6.) What annuity, to continue 5 years, at 6 per cent. will be worth 430l. 15s. $4\frac{8}{13}$d. preſent money?

Firſt, 430l. 15s. $4\frac{8}{13}$d. $= 430{\cdot}7'69230' = p$, and $430{\cdot}7'69230' \div 5 = 86{\cdot}1'53846' = \frac{p}{t}$.

$$\begin{array}{rr}
5 = t & 5 = t \\
2 & {\cdot}06 = r \\
\hline
10 = 2t & {\cdot}30 = r \\
{\cdot}06 = r & 2{\cdot} \\
\hline
{\cdot}60 = 2tr & 2{\cdot}30 = tr + 2 \\
2{\cdot} & {\cdot}06 = r \\
\hline
2{\cdot}60 = 2tr + 2, \text{ divdend} & 2{\cdot}24 = \overline{tr + 2} - r, \text{ diviſor.}
\end{array}$$

Then $\frac{2{\cdot}6}{2{\cdot}24} \times 86{\cdot}1'53846' = \frac{224}{2{\cdot}24} = 100\text{l.} = n$, the annuity required.

(7.) What annuity, to continue 9 years, at 5 per cent. per annum, is worth 595l. 17s. 2d.3·58621, ready money?

(8.) What annuity or penſion, to continue 7 years, will 3339l. 5s. $2\frac{2}{9}$d. ready money buy, at 5 per cent. per annum, ſimple intereſt allowed for diſcount?

(9.) What annuity, (payable half-yearly) to continue 4 years, will 1279l. 17s. 5·492d. purchaſe, allowing ſimple intereſt at $4\frac{1}{2}$ per cent.?

(10.) What annuity, (payable quarterly) to continue 5 years, will 313l. 5s. ready money purchaſe, allowing the purchaſer 5 per cent. for his money?

Examples to Prop. 3. *Theo.* III.

(11.) What time may a perſon hold a penſion of 100l. per annum, if he pays 430l. 15s. $4\frac{8}{13}$d. ready money, and is allowed 6 per cent. for his money?

Firſt, 430l. 15s. $4\frac{8}{13}$d. $= 430\cdot7'69230' = p$

$$\frac{430\cdot7'69230' \times 2}{\cdot06 \times 100} = 143\cdot5'89743' = \frac{2p}{rn}$$

$$\begin{array}{rlcrl} 430\cdot7'69230' & = p & \qquad & \cdot06 & = r \\ \cdot06 & = r & & 100 & = n \\ \hline 25\cdot846'15384' & = rp & & 6\cdot00 & = rn \end{array}$$

$$\frac{100 - 25\cdot846'15384'}{6} - \frac{1}{2} = 11\cdot858'97435' = \overline{\frac{n-rp}{rn} - \frac{1}{2}};$$ and

$$11\cdot858'97435' \times 11\cdot858'97435' = \overline{\frac{n-rp}{rn} - \frac{1}{2}}^{\,2};$$ then

$$\sqrt{143\cdot5'89743' + 11\cdot858'97435' \times 11\cdot858'97435'} - 11\cdot858'97435' = 16\cdot858'97435' - 11\cdot858'97435' = 5$$ years, *(exactly)* the time required.

(12.) What time muſt an annuity of 560l. continue, at 5 per cent. to be worth 3339l. 5s. $2\frac{2}{9}$d. ready money?

(13.) What time muſt an annuity of 80l. continue, at 5 per cent. to be worth 595l. 17s. 2d.3·58621 ready money?

(14.) How long muſt an annuity of 350l. payable half-yearly, continue, at $4\frac{1}{2}$ per cent. to be worth 1279l. 17s. 5·492d.?

(15.) What time muſt an annuity of 70l. payable quarterly, continue, at 5 per cent. per annum, to be worth 313l. 5s.?

Examples to Prop. 4. *Theo.* IV.

(16.) At what rate per cent. will an annuity of 100l. to continue 5 years, be worth 430l. 15s. $4\frac{8}{13}$d. ready money?

Firſt, 430l. 15s. $4\frac{5}{13}$d. $= 430\cdot7'69230' = p$.

$$\begin{array}{rl} 100 & = n \\ 5 & = t \\ \hline 500 & = nt \\ 430\cdot7'69230' & = p \\ \hline 69\cdot2'30769' & = nt - p. \end{array} \qquad \begin{array}{rl} 430\cdot7'69230' & = p \\ 2 & \\ \hline 861\cdot5'38461' & = 2p \\ 100\cdot & = n \\ \hline 961\cdot5'38461' & = \overline{2p+n} \\ 500\cdot & \\ \hline 461\cdot5'38461' & = \overline{2p+n} - nt \end{array}$$

dividend $69\cdot2'30769' = nt - p$.

diviſor $461\cdot5'38461' = \overline{2p+n} - nt$

Then $69\cdot2'30769' \div 461\cdot5'38461' = \cdot15$; which, multiplied by ·4, ($= \frac{2}{t}$) gives $,06 = r$. Hence then the rate is 6 per cent.

(17.) At what rate per cent. will an annuity of 80l. b worth 595l. 17s. 2d.3·58621, preſent money, if it continue in arrears for 9 years?

(18.) At what rate per cent. will an annuity of 56ol to continue 7 years, be worth 3339l. 5s. 2$\frac{2}{9}$d. ready money?

(19.) At what rate per cent. will an annuity of 35ol payable half-yearly, to continue 4 years, be worth 1279l 17s. 5·492d. ready money?

(20.) At what rate per cent. will an annuity of 7ol payable quarterly, to continue 5 years, be worth 313l. 5s ready money?

PRESENT WORTH *of Annuities, in Reverſion, at Simple Intereſt.*

Here t=the *reverſion*, or the time the purchaſer holds the annuity, T= the time before he enters upon it; r, n, &c. as before.

Propoſition 1. *To find the preſent worth of an annuity i reverſion at ſimple intereſt.*

Rule. Find the preſent worth of the annuity (for th time of its continuance) as though it were to be entere upon immediately, (by Prop. 1. page 256;) then find wha principal put to intereſt, at the ſame rate per cent. for th time between the purchaſe and the commencement of th annuity, will amount to *that* preſent worth, (by Prop. 2 page 239) and it will be the anſwer.

Or, *Theo.* I. $\frac{\overline{ttr+2t}-tr}{\overline{2tr+2}\times\overline{Tr+1}}\times n=p$, the preſent worth, T, t, r and n, are given.

Prop. 2. *Given T, t, r, and p, to find n.*

Theo. II. $\frac{\overline{2tr+2}\times\overline{Tr+1}}{\overline{ttr+2t}-tr}\times p=n$.

Note. Though I have given rules, as it is cuſtomary, for calculating annuities at *ſimple intereſt*, I conſider them merely as an uſeleſs curioſity or exerciſe for the ſcholar, where he may have an opportunity of ſeeing the abſurdity of purchaſing annuities at ſimple intereſt.—E. g. An annuity of £50, to continue only 40 years, at 5 per cent. (by Prop. 1. Theo. 1.) would coſt £1316 13s 4d. Now this ſum would yie £65 16s. 8d. annual intereſt for ever! And the price of the ſame annuity (viz. £50,) were it to continue for *ever* at ſimple intereſt, would be infinitely great, or greater than any aſſignable number; whereas, if compound intereſt is allowed, and it is certainly more equitable, the price of the annuity in the former caſe would be £857 19s. 1d. and in the latter ly £1000.

Examples to Propoſition 1.

(1.) The *reverſion* of a leaſe of 175l. per annum, to continue 11 years, is to be ſold; but the purchaſer is to receive no benefit from it till the expiration of 9 years; what is it worth, allowing the buyer 6 per cent. per annum for ready money?

Firſt, $1+2+3+4+5+6+7+8+9+10=55$.

Then, $\overline{175\times{\cdot}06\times55}+\overline{175\times11}=2502{\cdot}5$, amount of the *reverſion*, and $2502{\cdot}5\div\overline{\overline{11\times{\cdot}06}+1}=2502{\cdot}5\div1{\cdot}66=1507{\cdot}53$, the preſent worth of the leaſe, were it to commence immediately. Again, $1507{\cdot}53\div\overline{\overline{{\cdot}06\times9}+1}=1507{\cdot}53\div1{\cdot}54=978{\cdot}9156=$ 978l. 18s. 3¼d. the preſent worth of the reverſion.

Or thus, by Theorem I.

$$\begin{array}{rlcl}
 & 11=t & & 11=t \\
 & \underline{11} & & \underline{2} \\
 & & 9=T & \\
 & 121=tt & \underline{{\cdot}06} & 22=2t \\
 & \underline{{\cdot}06=r} & & \underline{{\cdot}06=r} \\
 & & {\cdot}54=Tr & \\
 & 7{\cdot}26=ttr & \underline{1{\cdot}} & 1{\cdot}32=2tr \\
11\times2= & \underline{22{\cdot}\ =2t} & & \underline{2{\cdot}} \\
 & & 1{\cdot}54=Tr+1 & \\
 & 29{\cdot}26=ttr+2t & & 3{\cdot}32=2tr+2 \\
{\cdot}06\times11 & \underline{{\cdot}66}=tr & & \underline{1{\cdot}54=Tr+1} \\
 & 28{\cdot}60=\overline{ttr+2t}-tr & & 5{\cdot}1128\ \text{product.}
\end{array}$$

Then $\frac{28{\cdot}6}{5{\cdot}1128}\times175=978{\cdot}91566265=978\text{l. }18\text{s. }3^{d}{\cdot}3{\cdot}03615$, the preſent worth as before.

(2.) There is a legacy of 20l. per annum, for 8 years, left to a perſon of 16 years of age; the time of payment is to commence when he arrives at 21 years of age; but he, wanting money, wiſhes to ſell it, allowing the purchaſer 4 per cent. for preſent payment: what is it worth?

(3.) What is the preſent worth of the reverſion of a leaſe of 50l. per annum, to continue for 5 years, but not to commence till the end of 3 years, allowing 5 per cen for preſent payment?

(4.) What ought a perſon to pay in ready money f the reverſion of 1000l. a year, to continue 20 years, o a leaſe which cannot commence till the expiration of years, allowing the purchaſer ſimple intereſt at 5 p cent.?

Examples to Prop. 2. Theo. II.

(5.) The leaſe of a houſe, taken for 11 years, but commencing not till the end of 9 years, is ſold for 978l. 18 3d.3·03615 preſent money; what ought the yearly rent t be, the purchaſer being allowed 6 per cent. for h money?

Firſt, 978l. 18s. 3d.3·03615 $= 978\cdot91566265625 = p$.

$$\begin{array}{lll}
11 = t & 9 = T & 11 = t \\
\underline{2} & \underline{\cdot06 = r} & \underline{11} \\
22 = 2t & \cdot54 = Tr & 121 = tt \\
\underline{\cdot06 = r} & \underline{1\cdot} & \underline{\cdot06 = r} \\
1\cdot32 = 2tr & 1\cdot54 = Tr + 1 & 7\cdot26 = ttr \\
\underline{2\cdot} & & 11 \times 2 = \underline{22\cdot} = 2t \\
3\cdot32 = 2tr + 2 & & 29\cdot26 = \overline{ttr + 2t} \\
\underline{1\cdot54 = Tr + 1} & & \cdot06 \times 11 \quad \underline{\cdot66 = tr} \\
5\cdot1128 \text{ product.} & & 28\cdot60 = \overline{ttr + 2t} - tr
\end{array}$$

Then $\frac{5\cdot1128}{28\cdot6} \times 978\cdot91566265625 = 175\text{l.} = n$, the annuity, o yearly rent.

(6.) There is a legacy of a certain ſum per annum fo 8 years, left to a perſon of 16 years of age; but the time of payment is not to commence till the legatee is 21 years of age; he, in the mean time, wanting money, ſold the legacy for 115l. 3s. ·0´3´. allowing the purchaſer 4 per cent. for his money; what ought the legacy to be worth per annum?

(7.) What annuity, or yearly rent, to be entered upon 3 years hence, and then to continue 5 years, may be bought for 191l. 6s. 1·0435d. ready money, allowing the purchaſer 5 per cent.?

(8.) The reverſion of a leaſe, to be entered on 5 years ence, and then to continue 20 years, was ſold for 1180l. llowing the purchaſer 5 per cent. What is the yearly nt?

ANNUITIES AT COMPOUND INTEREST.

Let n = the annuity, penſion, rent, or *payment*, whether yearly, ½-yearly, or quarterly.
t = the time, or *number of payments.*
r = the ratio, or amount of £1 for a year, ½ year, ¼ year, &c. according as the payments are made yearly, ½-yearly, quarterly, &c. by either of the methods or tables given in compound Intereſt.
a = the amount.

ANNUITIES *in Arrears, at Compound Intereſt.*

Propoſitian 1. *Given the annuity, payable in whole years, at an equal number of payments, the rate per cent. and time, find the amount.*

Rule. Make an unit the firſt term of a geometrical ſeries, e amount of £1 for 1 year, ½ year, ¼ year, &c. the *ratio*, ccording as the payments are made yearly, ½-yearly, arterly, &c.—Carry the ſeries to as many terms as there e payments, and find its ſum, (by Prop. 3, of Geome-ical Progreſſion,) which multiply by the annuity, and e product will be the amount.

Or, *Theo.* I. $\frac{r^t-1}{r-1} \times n = a$, when n, r, and t, are given.

Logarithmically. $log.\ \overline{r^t-1} + log.\ n - log.\ \overline{r-1} = log.\ a.$

Prop. 2. *Given* a, r, *and* t, *to find* n.

Theo. II. $\frac{r-1}{r^t-1} \times a = n.$

Logarithmically. $log.\ \overline{r-1} + log.\ a - log.\ \overline{r^t-1} = log.\ n.$

Prop. 3. *Given a, r, and n, to find t.*

Theo. III. $\frac{\overline{r-1}\times a}{n}+1=r^{t}$. If t is not a whole number, cannot be found without logarithms.

Logarithmically. $\frac{\log.\ \overline{\overline{ar-a}+n}-\log.\ n}{\log.\ r}=t$.

Prop. 4. *Given a, n, and t, to find r.*

Equation. $\frac{ar}{n}-r^{t}=\frac{a-n}{n}$. After this equation is reduced to number the value of r muſt be found as directed in the 2d note in Double Poſition. If t be a mixed fraction, the value of r cannot be found withou logarithms.

Examples to Propoſition 1.

(1.) What is the amount of an annuity of 100l. to continue 5 years, at 6 per cent. per annum, compound intereſt?

$$1+1\cdot06+\overline{1\cdot06}^{2}+\overline{1\cdot06}^{3}+\overline{1\cdot06}^{4}=\frac{\overline{1\cdot06}^{4}-1}{1\cdot06-1}+\overline{1\cdot06}^{4}=$$

5·63709296; this, multiplied by 100, the annuity, gives 563·709296 = 563l. 14s. 2·23104d. for the amount required.

Or, by Theorem I.

$$1\cdot06\times1\cdot06\times1\cdot06\times1\cdot06\times1\cdot06=1\cdot3382255776=r^{t} \text{ and}$$

$1\cdot3382255776-1=\cdot3382255776=r^{t}-1$; alſo $1\cdot06-1=\cdot06=r-1$.

Hence $\frac{\cdot3382255776}{\cdot06}\times100=563\cdot709296$, the amount as before.

(2.) What is the amount of an annuity of 80l. unpaid, or in arrears, for 9 years, at 5 per cent.?

(3.) Required the amount of an annuity of 560l. to continue 7 years, at 5 per cent. per annnm?

*The following examples are performed by Logarithms**.

(4.) If a penſion of 356l. per annum, payable half-yearly, be unpaid for 9 years, what will it amount to at 6 per cent.?

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* Thoſe who wiſh to be made thoroughly acquainted with performing multiplication, diviſion, involution, and evolution, by logarithms, may conſult a ſynopſis of logarithmical arithmetic, annexed to the KEY to *The Complete Practical Arithmetician.*

Here $n = 178$, $r = 1{\cdot}03$, (by table I. page 245), and $t = 18$.

$log.\ r = log.\ 1.03 = 0{\cdot}0128372$
18

$log.\ r^{t} = 0{\cdot}2310696$, the number anſwering to which is $1{\cdot}702433 = r^{t}$

$log.\ r^{t} - 1 = log.\ {\cdot}702433 = -1{\cdot}8466048$
$log.\ n = log.\ 178 = +2{\cdot}2504200$

$+2{\cdot}0970248$
$log.\ r - 1 = log.\ {\cdot}03 = -2{\cdot}4771213$

log. of the amount $= +3{\cdot}6199035$, the number anſwering to which is $4167{\cdot}768$ = 4167l. 15s. 4·32d. anſwer.

(5.) What is the amount of an annuity of 350l. payable half-yearly, unpaid for 4 years, at 4½ per cent.?

(6.) If an annuity of 70l. per annum, payable quarterly, be unpaid for 5 years, what will it amount to at 5 per cent.?

Examples to Prop. 2. Theo. II.

(7.) What annuity unpaid for 5 years, will amount to 563l. 14s. 2·23104d. at 6 per cent.?

£563 14 2·23104 $= 563{\cdot}709296 = a$.
$1{\cdot}06 - 1 = {\cdot}06 = r - 1$ dividend.

$1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06 = 1{\cdot}3382255776 = r^{t}$, and $rt - 1 = {\cdot}3382255776$, diviſor.

Then $\frac{{\cdot}06}{{\cdot}3382255776} \times 563{\cdot}709296 = £100 = n$, the annuity.

(8.) What annuity, unpaid for 9 years, will amount to 882l. 2s. 6·04d. at 5 per cent.?

(9.) What annuity, in arrears, for 7 years, will amount to 4559l. 10s. 5d.3·7444, at 5 per cent.?

The following examples are performed by Logarithms.

(10.) What annuity, payable half-yearly, will amount to 4167l. 15s. 4·32d. at 6 per cent. if unpaid for 9 years?

£4167 15 4·32 $= 4167{\cdot}768 = a$, by the 4th queſtion, $r^{t} - 1 = \overline{1{\cdot}03)}^{18} - 1 = {\cdot}702433$.

$log.\ r - 1 = log.\ {\cdot}03 = -2{\cdot}4771213$
$log.\ a = log.\ 4167{\cdot}768 = +3{\cdot}6199035$

$+2{\cdot}0970248$
$log.\ {\cdot}702433 = -1{\cdot}8466048$

log. of ½ the annuity $= +2{\cdot}2504200$, the number anſwering to which is 178. Hence the annuity is 356l.

(11.) What annuity, payable half-yearly, will amount to 1515l. 6s. 1·1d. if unpaid for 4 years, at $4\frac{1}{2}$ per cent.?

(12.) What annuity, payable quarterly, will amount to 394l. 16s. $10\frac{1}{4}$d. at 5 per cent. if unpaid for 5 years?

Examples to Prop. 3. *Theo.* III.

(13.) To find the time in which an annuity of 100l. will amount to 563l. 14s. 2·23104d. at 6 per cent. per annum?

$$\text{£}563\ 14\ 2\cdot23104 = 563\cdot709296 = a.$$

$$1\cdot06 - 1 = \cdot06 = r - 1$$

$$563\cdot709296 \times \cdot06 = 33\cdot82255776 = \overline{r-1} \times a.$$

Then $\frac{33\cdot82255776}{100} + 1 = 1\cdot3382255776 = \overline{1\cdot06)}^{t}$ *which number being continually divided by* $r = 1\cdot06$ *(according to Mr. Ward and others) till nothing remains, the number of diviſions will be* 5, *the years required.*

(14.) In what time will an annuity of 80l. unpaid, amount to 882l. 2s. 6·04d. at 5 per cent.?

(15.) What time muſt an annuity of 560l. continue in arrears, at 5 per cent. to raiſe a ſtock of 4559l. 10s. 5d. 3·7444?

The following examples are performed by Logarithms.

(16.) What time muſt an annuity of 356l. payable half-yearly, be continued, at 6 per cent. to raiſe a ſtock of 4167l. 15s. 4·32d.?

$$\begin{aligned} \text{£}4167\ 15\ 4\cdot32 = 4167\cdot768 &= a \\ 1\cdot03 &= r \\ 4292\cdot80104 &= ar \\ 4167\cdot768 &= a \\ 125\cdot03304 &= ar - a \\ 178\cdot &= n \\ 303\cdot03304 &= \overline{ar - a + n} \end{aligned}$$

$$\begin{aligned} \log.\ 303\cdot03304 &= 2\cdot4814899 \\ \log.\ 178 &= 2\cdot2504200 \end{aligned}$$

$\log.\ r = \log.\ 1\cdot03 = \cdot0128372\ )\ 0\cdot2310699\ (\ 18 = t$, the number of payments; hence the time is 9 years.

(17.) How long muſt an annuity of 350l. payable half-yearly, remain in arrears at $4\frac{1}{2}$ per cent. to raiſe a ſtock of 1515l. 6s. 1·1d.?

(18.) How long muſt an annuity of 70l. payable quarterly, remain in arrears, at 5 per cent. to raiſe a ſtock of 394l. 16s. $10\frac{1}{4}$d.?

Examples to Prop. 4.

(19.) Required the rate per cent. compound intereſt, for an annuity of 100l. continuing 5 years, to raiſe an amount of 563l. 14s. 2·23104d.?

£563 14 2·23104 $= 563{\cdot}709296 = a$.

Then $\frac{563{\cdot}709296\,r}{100} - r^5 = \frac{563{\cdot}709296 - 100}{100}$

Or, $5{\cdot}63709296\,r - r^5 = 5{\cdot}63709296 - 1$. Now, by (note 2, page 223) Double Poſition, I find $r = 1{\cdot}06$; hence the rate is 6 per cent.

(20.) An annuity of 80l. in arrears for 9 years, amounted to 882l. 2s. 6·04d. what was the rate per cent.?

PRESENT WORTH *of Annuities in Arrears, at Compound Intereſt.*

Propoſition 1. *To find the preſent worth of an annuity at compound intereſt, the time of its continuance, and the rate per cent. being given.*

Rule. Find the amount of the annuity, (by Prop. 1, page 263.) ſuppoſing it in arrears till the laſt payment is due. Then find the preſent worth of that amount, (by the rule for Diſcount, page 249, or by Prop. 2, Compound Intereſt,) and it will be the anſwer.

Or,

Find the preſent worth of each payment by itſelf, diſcounting from the time it falls due, (by the rule for Diſcount, page 249,) the ſum of theſe preſent worths will be the preſent worth of the whole.

Or, *Theo.* I. $\frac{1 - \frac{1}{r^t}}{r - 1} \times n = p$, the preſent worth, when n, r, and t, are given.

Logarithmically. $\log.\ \overline{1-\frac{1}{r^t}} + \log.\ n - \log.\ \overline{r-1} = \log.\ p$.

Prop. 2. *Given p, r, and t, to find n.*

Theo. II. $\frac{r-1}{1-\frac{1}{r^t}} \times p = n$.

Logarithmically. $\log.\ \overline{r-1} + \log.\ p - \log.\ \overline{1-\frac{1}{r^t}} = \log.\ n$.

Prop. 3. *Given p, r, and n, to find t.*

Theo. III. $\frac{n}{\overline{n+p-pr}} = r^t$. If t be not a whole number, it cannot be found without logarithms.

Logarithmically. $\frac{\log.\ n - \log\ \overline{n+p-pr}}{\log.\ r} = t$.

Prop. 4. *Given p, n, and t, to find r.*

Equation. $\overline{\frac{n+p}{p} \times r^t - r^{t+1}} = \frac{n}{p}$. After this equation is reduced to numbers, the value of r muſt be found (as directed in the ſecond note, page 223), by Double Poſition. If t be not a whole number, the value of r, cannot be found without logarithms.

Examples to Propoſition 1.

(1.) Required the preſent worth of an annuity of 100l. to be continued 5 years, allowing 6 per cent. compound intereſt?

The amount of 100l. for 5 years is 563·709296 (*See example* I. *page* 264), and 1·06×1·06×1·06×1·06×1·06=equal 1.3382255776, the amount of 1l. for 5 years.

As 1·3382255776 : 1l. :: 563·709296 : 421·236378=421l. 4s. 8·731d. the preſent worth.

Or thus,

Ratio $= 1{\cdot}06$)100·(94·339622, preſent worth of 100l. for 1 year.
$\overline{1{\cdot}06}|^{2} = 1{\cdot}1236$)100·(88·999644, ditto for the 2d year.
$\overline{1{\cdot}06}|^{3} = 1{\cdot}191016$)100·(83·961928, ditto for the 3d year.
$\overline{1{\cdot}06}|^{4} = 1{\cdot}26247696$)100·(79·209366, ditto for the 4th year.
$\overline{1{\cdot}06}|^{5} = 1{\cdot}3382255776$)100·(74·725817, ditto for the 5th year.

The ſum is the preſent worth, 421·236378 = £421 4 8·731, as above.

(2.) Required the preſent worth of an annuity of 80l. to continue 9 years, at 5 per cent. per annum, compound intereſt?

(3.) Required the preſent worth of an annuity of 560l. to continue 7 years, at 5 per cent. per annum, compound intereſt allowed for diſcount.

The following examples are performed by Logarithms.

(4.) What is the preſent worth of an annuity of 356l. payable half-yearly, to continue 9 years, allowing 6 per cent. compound intereſt for diſcount to the purchaſer?

Here $n = \frac{356}{2} = 178$, $r = 1{\cdot}03$, (*by table* I. *page* 245.) and $t = 18$.

$$\log.\ r = \log.\ 1{\cdot}03 = 0{\cdot}0128372$$
$$18$$
$$\log.\ r^t = 0{\cdot}02310696$$

; this, ſubtracted from the *log.* of $1 = 0$, gives $-1{\cdot}7689304$ for the *log.* of $\frac{1}{r^t}$; the number anſwering to which is $\cdot5873946$; then $1 - \cdot5873946 = \cdot4126053 = 1 - \frac{1}{r^t}$.

$$\log.\ 1 - \frac{1}{r^t} = \log.\ \cdot4126053 = -1{\cdot}6155348$$
$$\log.\ n = \log.\ 178 = +2{\cdot}2504200$$
$$+1{\cdot}8659548$$
$$\log.\ r - 1 = \log.\ \cdot03 = -2{\cdot}4771213$$

log. of the preſent worth $+3{\cdot}388335$ the number anſwering to which is $2448{\cdot}1253 = £2448\ 2\ 6{\cdot}072$ anſwer.

(5.) What is the preſent worth of an annuity of 350l. payable half-yearly, to continue 4 years, allowing 4 per cent. for diſcount?

(6.) What is the preſent worth of an annuity of 70l. payable quarterly, to continue 5 years, allowing 5 per cent. per annum compound intereſt for diſcount?

Examples to Prop. 2. Theo. II.

(7.) What annuity, to continue 5 years, at 6 per cent. diſcount, will be worth 421l. 4s. 8·731d.?

Firſt, $£421\ 4\ 8{\cdot}731 = £421{\cdot}23637916' = p$, and $421{\cdot}23637916' \times {\cdot}06 = 25{\cdot}27418275 = \overline{r-1} \times p$, dividend.

$\underline{1{\cdot}06 \times 1{\cdot}06 \times 1{\cdot}06} \times 1{\cdot}06 \times 1{\cdot}06 = 1{\cdot}3382255776 = r^{t}$.

$1 - 1 \div 1{\cdot}3382255776 = 1 - {\cdot}7472581725 = 2527418275$, diviſor.

Then $25{\cdot}27418275 \div {\cdot}2527418275 = £100 = n$, the annuity required.

(8.) What annuity, to continue 9 years, will be worth 568l. 12s. 6·177d. allowing the purchaſer 5 per cent. compound intereſt for ready money?

(9.) What annuity, to continue 7 years, will 2842l. 7s. 9·0183d. ready money, purchaſe, allowing 5 per cent. per annum compound intereſt for diſcount?

The following examples are performed by Logarithms.

(10.) What annuity, (payable half-yearly,) to continue 9 years, will 2448l. 2s. 6·072d. purchaſe, allowing compound intereſt at 6 per cent.?

Here $t = 18$, $r = 1{\cdot}03$, and $£2448\ 2\ 6{\cdot}072 = 2448{\cdot}1253 = p$.

By example 4, *page* 269, $1 - \dfrac{1}{r^{t}} = {\cdot}4126053$.

$$\begin{array}{rr} \log.\ r - 1 = \log.\ {\cdot}03 = & -\ 2{\cdot}4771213 \\ \log.\ p = \log.\ 2448{\cdot}1253 = & +\ 3{\cdot}3888336 \\ \hline & +\ 1{\cdot}8659549 \\ \log.\ 1 - \dfrac{1}{r^{t}} = \log.\ {\cdot}4126053 = & -\ 1{\cdot}6155348 \\ \hline \end{array}$$

log. of ½ the annuity $= +\ 2{\cdot}2504201$, the number anſwering to which is 178; hence the annuity is 356l.

(11.) What annuity, (payable half-yearly,) to continue 4 years, 1268l. 5·08s. ready money, purchaſe, allowing compound intereſt at 4½ per cent.?

(12.) What annuity, (payable quarterly,) will 307l. 19s. 9·6d. ready money, purchaſe, allowing the purchaſer 5 per cent. per annum compound intereſt, or diſcount, for his money?

Examples to Prop. 3. *Theo.* III.

(13.) In what time will an annuity of 100l. be worth 421l. 4s. 8·731d. preſent money, if it continues in arrears; *or, which is the ſame thing, if it be received annually,* allowing 6 per cent. diſcount?

£421

$$
\begin{array}{rll}
£421\ 4\ 8{\cdot}731 = & 421{\cdot}23687916' & = p. \\
 & 100{\cdot} & = n \\
 & \underline{421{\cdot}23687916'} & = p \\
 & 521{\cdot}23687916' & = n+p \\
421{\cdot}23687916' \times 1{\cdot}06 = & \underline{446{\cdot}511091916'} & = pr \\
 & 74{\cdot}72578725 & = \overline{n+p}-pr.
\end{array}
$$

$100 \div 74{\cdot}72578725 = 1{\cdot}338226 + = \overline{1{\cdot}06}^{t}$; *whence the value of t may be found by repeated divisions*, or rather by logarithms.

(14.) In what time will an annuity of 80l. be worth 568l. 12s. 6·177d. allowing the purchaſer 5 per cent. per annum for preſent payment?

(15.) How long may a perſon enjoy an annuity of 560l. if he pays 2842l. 7s. 9·0183 ready money, and is allowed 5 per cent. per annum diſcount?

The following examples are performed by Logarithms.

(16.) How long muſt an annuity of 356l. (payable half-yearly,) continue, at 6 per cent. to be worth 2448l. 2s. 6·072d. ready money?

$$
\begin{array}{rll}
\text{Firſt, } £2448\ 2\ 6{\cdot}072 = & 2448{\cdot}1253 & = p. \\
 & 178{\cdot} & = n \\
 & \underline{2448{\cdot}1253} & = p \\
 & 2626{\cdot}1253 & = n+p \\
2448{\cdot}1253 \times 1{\cdot}03 = & \underline{2521{\cdot}569059} & = pr \\
 & 104{\cdot}55624 & = \overline{n+p}-pr \\
\log. n = & \log. 178 & = 2{\cdot}2504200 \\
\log. \overline{n+p}-pr = & \log. 104{\cdot}55624 & = \underline{2{\cdot}0193508}
\end{array}
$$

$\log. r = \log. 1{\cdot}03 = 0{\cdot}0128372\ )\ 0{\cdot}2310692\ ($ 18 payments; hence the time is 9 years.

(17.) How long muſt an annuity of 350l. (payable half-yearly) continue to be worth £1268 5·08 ready money, allowing the purchaſer 4½ per cent. diſcount for his money?

(18.) How long muſt an annuity of 70l. (payable quarterly) continue to be worth £307 19 9·6 ready money, allowing 5 per cent. per annum diſcount?

Examples to Prop. 4.

(19.) An annuity of 100l. to continue for 5 years, was purchaſed for £421 4 8.731: what rate per cent. diſcount was the purchaſer vllowed for his ready money?

$$\overline{\pounds 421\ 4\ 8\cdot 731}=421\cdot 23637416'=p.$$

$$\text{Then, } \frac{100+421\cdot 23637416'}{421\cdot 23637416'}\times r^{5}-r^{5+1}=\frac{100}{421\cdot 23637416'}.$$

$$\text{Or, } 521\cdot 23637416'\,r^{5}-421\cdot 23637416'\,r^{6}=100.$$

By note 2, Double Poſition, I find $r=1\cdot 06$, very near.

(20.) An annuity of 80l. to continue 9 years, was ſold for 568l. 12s. 6·177d. ready money; what rate per cent. diſcount was the purchaſer allowed?

PRESENT WORTH *of Annuities in Reverſion, at Compound Intereſt.*

Here t = the *reverſion*, or the time the purchaſer holds the annuity, T = the time which muſt elapſe before he enters upon it; r, n, &c. as before.

Propoſition 1. *To find the preſent worth of an annuity in reverſion at compound intereſt.*

Rule. Find the preſent worth of an annuity (for the time of its continuance) as though it were to be entered on immediately, (by Prop. 1, page 267,) then find what principal put to intereſt, at the ſame rate per cent. for the time between the purchaſe and commencement of the annuity, will amount to *that* preſent worth, (by Prop. 2, page 246,) and it will be the anſwer.

Or, *Theo.* I. $\dfrac{\overline{r^{t}-1}}{\overline{r-1}\times r^{T+t}}\times n=p$, the preſent worth, when T, t, r, and n, are given.

Logarithmically. $\log.\,\overline{r^{t}-1}+\log.\,n-\overline{\log.\,r\times\overline{T+t}+\log.\,\overline{r-1}}=\log.\,p.$

Prop. 2. *Given* T, t, r, *and* p, *to find* n.

Theo. II. $\dfrac{\overline{r-1}\times r^{T+t}}{r^{t}-1}\times p=n.$

Logarithmically. $\overline{\log.\,r\times\overline{T+t}+\log.\,\overline{r-1}+\log.\,p}-\log.\,\overline{r^{t}-1}=\log.\,n.$

Examples to Propoſition 1.

(1.) The *reverſion* of a leaſe of 175l. per annum, to continue 11 years, which commences 9 years hence, is to be ſold; what is it worth, allowing the purchaſer 6 per cent. per annum diſcount for his ready money?

$1+1{\cdot}06+\overline{1{\cdot}06}\rceil^{2}+\overline{1{\cdot}06}\rceil^{3}+\overline{1{\cdot}06}\rceil^{4}+\overline{1{\cdot}06}\rceil^{5}+\overline{1{\cdot}06}\rceil^{6}+\overline{1{\cdot}06}\rceil^{7}+\overline{1{\cdot}06}\rceil^{8}+\overline{1{\cdot}06}\rceil^{9}+\overline{1{\cdot}06}\rceil^{10}=\frac{\overline{1{\cdot}06}\rceil^{10}-1}{1{\cdot}06-1}+\overline{1{\cdot}06}\rceil^{10}=14{\cdot}97164264$, this, multiplied by 175, gives £2620·037462, amount of the annuity.

As $\overline{1{\cdot}06}\rceil^{11}=1{\cdot}898298558$: 1l. :: 2620·037462 : £1380·203052, preſent worth of the annuity, ſuppoſing it were to be entered upon immediately. Again,

As $\overline{1{\cdot}06}\rceil^{9}=1{\cdot}689478959$: 1l. :: 1380·203052 : £816·940065 = £816 18 $2\frac{1}{2}{\cdot}4624$, the preſent worth of the *reverſion.*

Or thus by the Theorem.

Here $t=11$, $r=1{\cdot}06$, $n=175$, and $T=9$.

$\overline{1{\cdot}06}\rceil^{11}=1{\cdot}898298558=r^{t}.$

$1{\cdot}898298558-\underset{t}{\underline{1=}}{\cdot}898298558=r^{t}-1$, and $\cdot898298558\times175=157{\cdot}20224765=r-1\times n$, dividend.

$r^{T+t}=\overline{1{\cdot}06}\rceil^{9+11}=\overline{1{\cdot}06}\rceil^{20}=3{\cdot}207135472$; this, multiplied by ·06, gives $\cdot19242812833=\overline{r-1}\times r^{T+t}$ diviſor.

Hence $157{\cdot}20224765\div\cdot19242812833=816{\cdot}940065$ = £816 18s. $9\frac{1}{2}{\cdot}4624$d. as before.

(2.) What is the preſent worth of the *reverſion* of a leaſe of 50l. per annum, to continue for 5 years, but not to commence till the end of 3 years, allowing 5 per cent. for preſent payment?

(3.) What ought a perſon to pay in ready money for the reverſion of 1000l. a year, to continue 20 years, on a leaſe which cannot commence till the expiration of 5 years, allowing the purchaſer compound intereſt at 5 per cent?

Examples to Prop. 2. Theo. II.

(4.) What annuity, or yearly rent, to be entered upon 9 years hence, and thence to continue 11 years, may be bought for 816l. 18s. $9\frac{1}{2}{\cdot}4624$d. ready money, allowing the purchaſer 6 per cent.?

£816 18 $9^{1}{\cdot}4624=816{\cdot}94005=p$, $1{\cdot}06-1=\cdot06=r-1.$

$r^{T+t}=\overline{1{\cdot}06}\rceil^{9+11}=\overline{1{\cdot}06}\rceil^{20}=3{\cdot}207135472.$ $\cdot06\times3{\cdot}207135472\times816{\cdot}940065=137{\cdot}2022476575=\overline{\overline{r-1}\times r^{T+t}\times p}$, dividend. $\overline{1{\cdot}06}\rceil^{11}-1=\cdot8982985583=r^{t}-1$, diviſor. Hence $157{\cdot}2022476575\div\cdot8982985583=$ £175, the annuity required.

(5.) What annuity, or yearly rent, to be entered upon 3 years hence, and then to continue 5 years, may be bought for £187 0 0¾·4176 ready money, at 5 per cent.?

(6.) The reverſion of a leaſe, to be entered on 5 years hence, and then to continue 20 years, was ſold for £9764 9 4¼·088, allowing the purchaſer 5 per cent. what ought the yearly rent to be?

PURCHASING FREEHOLD-ESTATES, OR PERPETUAL ANNUITIES, TO BE ENTERED ON IMMEDIATELY.

Propoſition 1. *Given the annual rent of any perpetual annuity, or freehold-eſtate, to find the value thereof, allowing the purchaſer any aſſigned rate per cent. for his money.*

Rule. Divide the rent by the ratio leſs 1, and the quotient will be the preſent worth of the eſtate.

Or, *Theo.* I. $\frac{n}{r-1} = p$, when n and r are given. If the rents are to be paid either $\frac{1}{2}$-yearly or quarterly, as is generally the caſe, then the ratio, or r, muſt repreſent the amount of £1 for that time, and the annuity, or n, muſt be divided by 2, 4, &c. to repreſent the $\frac{1}{2}$, $\frac{1}{4}$, &c. rent. —Here we may obſerve, that, though there be no ſuch thing as a *limited time* conſidered in the purchaſing of perpetual annuities, yet a due regard ought to be had to the *times* the annuities, or rents, are *paid*; for, it is evident the leſs the intervals between the payments of the rents are, the purchaſe is more valuable, and *vice verſa.*

Prop. 2. *When any ſum of money is propoſed to be laid out in a perpetual annuity or freehold-eſtate, to find what annual rent that ſum will purchaſe at any given rate per cent.*

Rule. Multiply the propoſed ſum to be laid out by the ratio leſs 1, and the product will be the yearly rent.

Theo. II. $p \times \overline{r-1} = n$, when p and r are given.

Prop. 3. *The annual rent of any perpetual annuity, or freehold-eſtate, and the ſum paid down for it, being given, to find what rate of intereſt per cent. is paid to the purchaſer.*

Rule. Divide the annual rent by the ſum that is paid for the purchaſe, the quotient, increaſed by an unit, will be the ratio, whence the rate per cent. may be found.

Theo. III. $\frac{n}{p} + 1 = r$, when p and n are given.

Examples

Examples to Propoſition 1.

(1.) An eſtate brings in £25 yearly rent; required the preſent worth thereof, allowing the purchaſer 4 per cent. compound intereſt for his money?

Firſt, $1{\cdot}04 - 1 = {\cdot}04$, the ratio leſs 1.
Then, $25 \div {\cdot}04 = £625$, the preſent worth required.

(2.) Suppoſe a freehold eſtate of 250l. yearly rent is to be ſold; what is it worth, allowing the buyer 6 per cent. compound intereſt for his money?

(3.) What is the preſent worth of a freehold eſtate of 250l. per annum, the rent payable half-yearly[*], allowing the purchaſer 4 per cent. for his money?

(4.) What is the preſent worth of a perpetual annuity of 2000l. payable quarterly, (viz. 500l. per quarter,) allowing the buyer 4½ per cent. compound intereſt for his money?

Examples to Prop. 2.

(5.) I propoſe to lay out 625l. in the purchaſe of a perpetual annuity, and to make 4 per cent. compound intereſt for my money; what ought the annuity to be?

$1{\cdot}04 - 1 = {\cdot}04$, the ratio leſs 1.
Then, ${\cdot}04 \times 625 = £25$, the annuity, or annual rent required.

(6.) A freehold-eſtate was bought for 4166l. 13s. 4d. what ought the yearly rent to be, allowing the buyer 6 per cent. compound intereſt for ready money?

(7.) A perſon is deſirous of laying out 1760l. in the purchaſe of a freehold-eſtate, ſo as to get 4½ per cent. compound intereſt for his money; what muſt be the annual income of ſuch an eſtate?

---

[*] It may not be improper to obſerve in this place, that, if the ratio be taken according to table I. page 245, it will make no difference whether the rents are payable yearly, half-yearly, or quarterly; but, if it be taken according to table II. page 246, the difference, *in this example*, will be £61 17s. 1¼d. this ſhews that the ſecond method, or table, is more accurate than the firſt; for it is certainly more advantageous to receive the rents half-yearly than yearly.

Examples to Prop. 3.

(8.) Suppoſe 625l. to be paid for a freehold-eſtate which yields 25l. per annum, what rate of intereſt has the purchaſer for his money?

625)25·00(·04
1·
——
1·04 the ratio; hence the rate per cent. is 4l.

(9.) Suppoſe a freehold-eſtate of 250l. per annum coſts 4166l. 13s. 4d. what rate of intereſt per cent. is allowed to the purchaſer?

(10.) A freehold-eſtate of 60l. a year rent was ſold for 1200l. what was the rate per cent. (compound intereſt diſcount) allowed the purchaſer for the ready money which he paid for the eſtate?

THE BUYING AND SELLING FREEHOLD-ESTATES TO BE ENTERED ON IMMEDIATELY, ACCORDING TO A NUMBER OF YEARS' RENT, OR INCOME, FOR THE PURCHASE-MONEY.

Propoſition 1. *The purchaſe-money, or preſent worth, of a freehold-eſtate being given, to find at what rent it muſt be let to clear itſelf in a given time.*

Rule. Divide the preſent worth by the propoſed time, and the quotient will be the annual rent.

Prop. 2. *Given the purchaſe, or preſent worth, of a freehold-eſtate, and the annual rent it lets for, to find in what time it will clear itſelf, or bring in the purchaſe-money.*

Rule. Divide the preſent worth by the annual rent, and the quotient will the time required.

Prop. 3. *Given the annual rent of a freehold-eſtate, and the time in which it will clear itſelf, to find the purchaſe, or preſent worth, of ſuch an eſtate.*

Rule. Multiply the rent by the time.

Prop. 4. *Given the time in which a freehold-eſtate brings in the purchaſe-money, or clears itſelf, to find what rate per cent. the purchaſer has for his money.*

Rule. Divide the time, more 1, by the time, and the quotient will be the ratio, whence the rate may be found.

Prop. 5. *When a perſon propoſes to lay out any ſum of money in a freehold-eſtate, ſo that he may make a certain rate per cent. of the money laid out, to find in what time the eſtate will clear itſelf.*

Rule. Divide an unit by the ratio leſs 1, and the quotient will be the time.

Examples to Propoſition 1.

(1.) A freehold-eſtate was purchaſed for 625l. At what rent muſt it be let that it may bring in the purchaſe-money in 15 years?

$$625 \div 15 = 41\tfrac{2}{3} = 41\text{l. } 13\text{s. } 4\text{d.}$$

(2.) A freehold-eſtate was purchaſed for 1200l. At what rent muſt it be let to clear itſelf in 20 years?

Examples to Prop. 2.

(3.) I purchaſed a freehold-eſtate for 4500l. which I let at 250l. per annum; in what time will it clear itſelf?

$$4500 \div 250 = 18 \text{ years, anſwer.}$$

(4.) I purchaſed a freehold-eſtate for 625l. which brings me in 25l. per annum; in what time will it clear itſelf?

Examples to Prop. 3.

(5.) If a freehold-eſtate, which lets for 40l. per annum, will clear itſelf in 20 years, what is its preſent worth?

$$40 \times 20 = \pounds 800, \text{ anſwer.}$$

(6.) If a freehold-eſtate, which lets for 25l. per annum, will clear itſelf in 25 years, what was it bought for?

Examples to Prop. 4.

(7.) If a freehold-eſtate be ſold for 20 years purchaſe, what rate per cent. compound intereſt is the purchaſer allowed for his money?

$$\frac{20+1}{20} = \frac{21}{20} = 1{\cdot}05, \text{ the ratio. Hence the rate is 5 per cent.}$$

(8.) If a freehold-eſtate be ſold for $22\frac{2}{9}$ years purchaſe, what rate per cent. is the buyer allowed for his money?

Examples to Prop. 5.

(9.) Suppoſe I want to lay out 1200l. in a freehold-eſtate, and to have 5 per cent. allowed me for my money; in what time will the eſtate bring in the purchaſe-money, or clear itſelf?

1 ÷ ·05 = 20 years, anſwer.

(10.) I wiſh to lay out 625l. in a freehold-eſtate, and to have 4 per cent. allowed me for my money; in what time will the eſtate clear itſelf?

PURCHASING FREEHOLD-ESTATES OR PERPETUAL ANNUITIES IN REVERSION.

Here n = the annuity, or yearly rent; T = the time before the annuity commences; p = the preſent worth; r = the ratio, &c.

Propoſition 1. *The yearly rent of a freehold-eſtate, and the rate per cent. being known, to find the preſent worth of the reverſion of ſuch an eſtate.*

Rule. Find the preſent worth for the *reverſion* (by Prop. 1, page 274.)

Then (by Prop. 2, page 246,) find what principal will amount to the full value of the eſtate for the time before it commences, and it will be the preſent worth required.

Theo. I. $\frac{n}{\overline{r-1}\times r^{T}} = p$, when n, r, and T, are given.

Logarithmically. Log. n — log. $r \times T$ + log. $\overline{r-1}$ = log. p.

Prop. 2. *The ſum given for the reverſion of a freehold-eſtate being known, to find the yearly income, allowing the purchaſer ſo much per cent. for his money.*

Rule. Find the amount of the purchaſe-money to the time when the *reverſion* begins, (by Prop. 1, page 246.) Then find the yearly income which that amount will purchaſe, (by Prop. 2, page 274,) and it will be the anſwer.

Theo. II. $\overline{r-1}\times r^{T}\times p = n$.

Logarithmically. log. $\overline{r-1}$ + log. $r \times T$ + log. p = log. n.

Examples to Propoſition 1.

(1.) The reverſion of a freehold-eſtate of 500l. per annum, to commence 5 years hence, is to be ſold; what is it worth in ready money, allowing the purchaſer 4 per cent. for his money?

500÷·04=12500l. value of the eſtate if entered on immediately.
1·04×1·04×1·04×1·04×1·04=1·2166529024, amount of 1l. for 5 years.
As 1·2166529024 : 1l. :: 12500 : 10274·088834=£10274 1 9¼·281, preſent worth of the reverſion.

Or thus by Theorem I.

Here $n = 500$, $r = 1{\cdot}04$, and $T = 5$.
$1{\cdot}04 \times 1{\cdot}04 \times 1{\cdot}04 \times 1{\cdot}04 \times 1{\cdot}04 = 1{\cdot}2166529024 = r^{T}$, and

$1{\cdot}2166529024 \times {\cdot}04 = {\cdot}048666116096 = \overline{r-1} \times r^{T}$.
Hence 500÷·048666116096=10274·088834=£10274 1 9¼·281 as before.

(2.) If a freehold-eſtate of 60l. 10s. per annum, to commence 10 years hence, is to be ſold, what is it worth, allowing the purchaſer 5 per cent. for preſent payment?

(3.) A freehold-eſtate of 290l. per annum, to commence 4 years hence, is to be ſold; what is it worth, allowing the purchaſer 4 per cent.?

Examples to Prop. 2.

(4.) A freehold-eſtate, to commence 5 years hence, is ſold for 10274l. 1s. 9¼d·281 allowing the purchaſer 4 per cent. for his money; what is the yearly rent?

Firſt, £10274 1 9¼·281=10174·088834.
1·04×1·04×1·04×1·04×1·04=1·2166529024.
1·2166529024 × 10274·088834=12500 (nearly) the amount of the purchaſe-money to the time the reverſion begins.
Then, 12500×·04=£500, the yearly rent.

By Theo. II.

$${\cdot}04 \times \overline{1{\cdot}04}|^{5} \times 10274{\cdot}088834 = \overline{r-1} \times r^{T} \times p = {\cdot}04 \times$$
1·2166529024 × 10274·088834=£500, (nearly) the annuity required.

(5.) If a freehold-eſtate, to commence 10 years hence, is ſold for 742l. 16s. 8¼d·8 allowing the purchaſer 5 per cent. what is the yearly rent?

(6.) If a freehold-eſtate, which commences 4 years hence, is ſold for 6197*l*. 6*s*. 5½*d*. allowing the purchaſer 4 per cent. for his money, what ought the yearly rent to be?

Thus I have given general rules and examples for calculating all kinds of annuities that do not depend upon *chance*: thoſe who wiſh for information on this ſubject, may conſult the works of Mr. *De Moivre*, Mr. *Simpſon*, Mr. *Dodſon*, Dr. *Price*, Mr. *Emerſon*, Mr. *Morgan*, or Baron *Maſeres*.

ON RATIOS.

1. RATIO is the relation which one quantity bears to another with reſpect to magnitude; and the compariſon is made by conſidering how often the one is contained in the other, or how often the one contains the other.

Thus the ratio of A to B is expreſſed by $\frac{A}{B}$, and the ratio of B to A by $\frac{B}{A}$: the former of theſe quantities, or the numerator, is called the antecedent; and the latter, or the denominator, is called the conſequent of the ratio.

2. When the antecedent is equal to the conſequent, viz. if $\frac{A}{B} = 1$, it is called a ratio of equality; if $\frac{A}{B}$ be greater than 1, we call it a ratio of greater inequality; and if $\frac{A}{B}$ be leſs than 1, it is called a ratio of leſs inequality.

3. The antecedent and conſequent are called the terms of the ratio, and the quotient of the two terms is called the index, or exponent of the ratio.

Thus, if $\frac{A}{B} = m$, then m is called the exponent of the ratio of A to B.

4. Compound ratio is made up of two or more ratios, by multiplying their terms and exponents together.

If $\frac{A}{B} = m$, and $\frac{C}{D} = n$, then $\frac{A}{B} \times \frac{C}{D} = mn = \frac{AC}{BD}$, ſo that the ratio of AC to BD, is ſaid to be compounded of the ratios of A to B, and C to D.

5. If a ratio be compounded of two *equal* ratios, it is called a duplicate ratio; if of three equal ratios, it is called a triplicate ratio, &c.

Thus, if $\frac{A}{B} = m$, $\frac{C}{D} = m$, then, $\frac{AC}{BD} = m^2$, hence the ratio of AC to BD is duplicate of the ratio of A to B, or of C to D. And if $\frac{A}{B} = m$, $\frac{C}{D} = m$, $\frac{E}{F} = m$, then, $\frac{ACE}{BDF} = m^3$, hence the ratio of ACE to BDF, is triplicate of the ratio of A to B, C to D, or E to F.

6. *If the terms of a ratio be prime to each other, no other quantities can be found in the ſame ratio, but what ſhall be ſome multiple[*] thereof.*

Let $\frac{A}{B} = m$, and $\frac{C}{D} = m$, where A and B are prime to each other, I ſay C ſhall be a multiple of A, and D a multiple of B. For $\frac{A}{B} = \frac{C}{D}$, multiply by D, then $C = \frac{DA}{B}$; now it is evident if B meaſures DA, it muſt meaſure D alone, becauſe A is prime to B: conſequently D is ſome multiple of B, therefore C is ſome multiple of A.

7. Cor. 1. * *The like multiples, or the like parts, of the terms of a ratio, have the ſame ratio as the terms themſelves.*

8. Cor. 2. *Numbers that are prime to each other, are the leaſt of all numbers in the ſame ratio.*

9. *Having the terms of a ratio given, in large numbers that are prime to each other, to find a ratio, nearly equivalent, whoſe terms are expreſſed by ſmaller numbers.*

This is performed by reducing the terms of the given ratio into a ſeries, of what are called *continued fractions.*

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[*] One number is ſaid to be a multiple of another, when it contains it ſome exact number of times. Thus, mB is a multiple of B, and $\frac{1}{n}A$ is a multiple of A.

Thus let the given ratio be expreſſed by $\frac{b}{a}$; and let a be contained in b, c times, with a remainder d; again let d be contained in a, e times, with a remainder f, and ſo on, we ſhall have

$$\begin{array}{l} a)\ b\ (c \\ \overline{d)}\ a\ (e \\ \overline{f)}\ d\ (g \\ \overline{h)}\ f\ (i \\ \overline{k)}\ h\ (l \\ \overline{m}\ \&c. \end{array}$$

$$\begin{array}{rcl} b & = & ac + d \\ a & = & de + f \\ d & = & fg + h \\ f & = & hi + k, \ \&c. \end{array}$$

Hence $\frac{b}{a} = \frac{ac+d}{a} = c + \frac{d}{a}$, but $a = de + f$,

Therefore $\frac{b}{a} = c + \frac{d}{de+f} = c + \cfrac{1}{e + \cfrac{f}{d}}$, but $d = fg + h$,

Therefore $\frac{b}{a} = c + \cfrac{1}{e + \cfrac{f}{fg+h}} = c + \cfrac{1}{e + \cfrac{1}{g + \cfrac{h}{f}}}$, but $f = hi + k$,

Therefore $\frac{b}{a} = c + \cfrac{1}{e + \cfrac{1}{g + \cfrac{h}{hi+k}}} = c + \cfrac{1}{e + \cfrac{1}{g + \cfrac{1}{i + \cfrac{k}{h}}}}$ &c.

Viz. $\frac{b}{a} = c + \cfrac{1}{e + \cfrac{1}{g + \cfrac{1}{i + \&c.}}}$ Then collecting the terms of this ſeries one after another, beginning at c, we continually approximate towards the ratio of $\frac{b}{a}$; and this approximation, is alternately leſs, and greater than the true ratio.

The firſt value is c, or $\frac{c}{1}$, the ſecond $c + \frac{1}{e} = \frac{ce+1}{e}$,

The third $c + \cfrac{1}{e + \cfrac{1}{g}} = c + \cfrac{1}{\cfrac{ge+1}{g}} = c + \frac{g}{ge+1} = \frac{cge+c+g}{ge+1} =$

$g \times ce+1$

$\frac{g\times\overline{ce+1}}{ge}$; $\begin{matrix}+\\+\end{matrix}\frac{c}{1}$. The fourth $c+\cfrac{1}{e+\cfrac{1}{g+\cfrac{1}{i}}}=c+\cfrac{1}{e+\cfrac{1}{\frac{ig+1}{i}}}=$

$$c+\cfrac{1}{e+\cfrac{i}{ig+1}}=c+\frac{1}{\frac{eig+e+i}{ig+1}}=c+\frac{ig+1}{eig+e+i}=\frac{ceig+ce+ci+ig+1}{eig+e+i}$$

$$=\left(\frac{g\times\overline{ce+1}}{ge};\ \begin{matrix}+\\+\end{matrix}\ \frac{c}{1}\right)\times\frac{i}{i};\ \begin{matrix}+\\+\end{matrix}\ \frac{ce+1}{e}.$$

Hence we deduce the following general rule.

1. Divide the greater term by the leſs, and that diviſor by the remainder, &c. as in Prop. 1, page 62, Vulgar Fractions. Then, if the antecedent be greater than the conſequent, the firſt quotient divided by 1, gives the firſt ratio; if leſs, an unit divided by the firſt quotient, will expreſs the firſt ratio.

2. Multiply the terms of the firſt ratio by the ſecond quotient, and add an unit to the numerator, or denominator, according as the antecedent of the original terms, is greater or leſs than its conſequent, and you will have the ſecond ratio.

3. Then, in general, multiply the terms of the ratio laſt found by the next ſucceeding quotient, and to the two products add the correſponding terms of the preceding ratio, and you will have the next ſucceeding ratio, &c.

Example 1. Let it be required to find a ſeries of ratios in leſs numbers, conſtantly approaching to the ratio of 314159 to 100000, which is nearly the ratio of the circumference of a circle to its diameter.

$100000)\ \underline{314159}\ (\ 3=c$

$d=14159\ )\ \underline{100000}\ (\ 7=e$

$f=887\ )\ \underline{14159}\ (\ 15=g$

$h=\underline{854}\ )\ 887\ (\ 1=i$

$k=33$ &c.

3 =

$3 = \frac{3}{1}$ the firſt ratio.

$\frac{3\times7+1}{1\times7} = \frac{22}{7}$ the ſecond ratio, being the approximation of *Archimedes*.

$\frac{22\times15}{7\times15}; \frac{+\,3}{+\,1} = \frac{333}{106}$ the third ratio.

$\frac{333\times1}{106\times1}; \frac{+\,22}{+\,7} = \frac{355}{113}$ the fourth ratio, the approximation of *Metius*.

Hence $\frac{314159}{100000} = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1}}}$ &c. in a continued fraction.

Example 2. Let it be required to find a ſeries of ratios in leſs numbers, conſtantly approaching to the ratio of 7853981633 to 10000000000, which is nearly the ratio of the area of a circle to the ſquare of its diameter.

7853981633) 10000000000 (1

2146018367) 7853981633 (3

1415926532) 2146018367 (1

730091835

730091835) 1415926532 (1

685834697) 730091835 (1

44257138) 685834697 (15

20977627) 44257138 (2

2301884, &c.

$1 = \frac{1}{1}$ firſt ratio.

$\frac{1\times3}{1\times3+1} = \frac{3}{4}$ ſecond ratio.

$\frac{3\times1}{4\times1}; \frac{+\,1}{+\,1} = \frac{4}{5}$ third ratio.

$\frac{4\times1}{5\times1}; \frac{+\,3}{+\,4} = \frac{7}{9}$ fourth ratio.

$\frac{7\times1}{9\times1}; \frac{+\,4}{+\,5} = \frac{11}{14}$ *fifth ratio.*

$\frac{11\times15}{14\times15}; \frac{+\,7}{+\,9} = \frac{172}{219}$ ſixth ratio.

$\frac{172\times2}{219\times2}; \frac{+\,11}{+\,14} = \frac{355}{452}$ *ſeventh ratio*

ON PROPORTION.

10. Proportion, is an equality of ratios.

Thus, if $\frac{A}{B}=m$, and $\frac{C}{D}=n$; then if m be equal to n, the ratios are equal; that is, A has the ſame ratio to B, which C has to D, and the four quantities are ſaid to be proportional; viz. $A : B :: C : D$ or $\frac{A}{B}=\frac{C}{D}$.

If m be greater than n, then A has to B a greater ratio than C has to D, and the four quantities, are not proportional.

If m be leſs than n, then A has to B a leſs ratio than C has to D, and the four quantities are not proportional.

If m and n are each equal to an unit, then the ratios of A to B, and C to D, are ratios of equality.

11. *If four quantities be proportional, the rectangle, or product of the extremes, will be equal to the rectangle, or product of the means.*

For, if $A : B :: C : D$, then $\frac{A}{B}=\frac{C}{D}$ by the definition. The fractions $\frac{A}{B}$ and $\frac{C}{D}$ reduced to a common denominator, will be $\frac{AD}{BD}$ and $\frac{BC}{BD}$, but when two *equal* fractions have the ſame denominator, their numerators are equal, therefore $AD=BC$; A and D being the extremes, and B and C the means.

12. *If the product of two quantities be equal to the product of two others; the four quantities may be turned into a proportion, by making the terms of one product the means, and the terms of the other the extremes.*

Thus, if $AD=BC$, divide each by BD, then $\frac{AD}{BD}=\frac{BC}{BD}$ viz. $\frac{A}{B}=\frac{C}{D}$, or $A : B :: C : D$.

13. *If four quantities be proportional, they ſhall alſo be proportional when taken inverſely*, viz. if $A : B :: C : D$, then $B : A :: D : C$. For $AD = BC$, mult. by $\frac{1}{AC}$, then $\frac{AD}{AC} = \frac{BC}{AC}$, or $\frac{B}{A} = \frac{D}{C}$, hence, INVERTENDO, $B : A :: D : C$.

14. *If four quantities be proportional, they ſhall alſo be proportional when taken alternately:* viz. if $A : B :: C : D$, then $A : C :: B : D$. For, $\frac{A}{B} = \frac{C}{D}$, mult. by $\frac{B}{C}$, then $\frac{BA}{BC} = \frac{BC}{DC}$, or $\frac{A}{C} = \frac{B}{D}$, hence, ALTERNANDO, $A : C :: B : D$.

15. *When four quantities are proportional, the firſt together with the ſecond, is to the ſecond; as the third together with the fourth, is to the fourth.*

Thus, if $A : B :: C : D$, then
COMPONENDO, $A+B : B :: C+D : D$.
For $AD = BC$ (article 11) add DB to each,
then $AD+DB = BC+DB$; or $\overline{A+B} \times D = \overline{C+D} \times B$, therefore (art. 12.) $A+B : B :: C+D : D$.

16. *If four quantities be proportional, the difference between the firſt and ſecond, is to the ſecond; as the difference between the third and fourth, is to the fourth.*

Thus, if $A : B :: C : D$, then
DIVIDENDO, $A-B : B :: C-D : D$,
For $AD = BC$ (art. 11.) take DB from each,
then $AD-DB = BC-DB$; or $\overline{A-B} \times D = \overline{C-D} \times B$, therefore (art. 12.) $A-B : B :: C-D : D$.

17. *If four quantities be proportional; the firſt, is to the difference between the firſt and ſecond, as the third, is to the difference between the third and fourth.*

Thus, if $A : B :: C : D$, then
CONVERTENDO, $A : A-B :: C : C-D$.

For $\frac{A-B}{B} = \frac{C-D}{D}$, art. 16th, and $\frac{B}{A} = \frac{D}{C}$, art. 13th,

Hence, $\frac{A-B}{B} \times \frac{B}{A} = \frac{C-D}{D} \times \frac{D}{C}$, or $\frac{A-B}{A} = \frac{C-D}{C}$,
that is, $A-B : A :: C-D : C$. and INVERTENDO,
$A : A-B :: C : C-D$.

18. *If ſeveral quantities be proportional, as one of the antecedents is to its conſequent; ſo is the ſum of all the antecedents, to the ſum of all the conſequents.*

Thus, if $A : B :: C : D :: E : F :: G : H$, &c.
Then $A : B :: A+C+E+G : B+D+F+H$.

For $\frac{A}{B} = \frac{A}{B} = \frac{C}{D} = \frac{E}{F} = \frac{G}{H}$, hence $AB = BA$, $AD = BC$, $AF = BE$, $AH = BG$, therefore $AB + AD + AF + AH = BA + BC + BE + BG$, or $A \times \overline{B+D+F+H} = B \times \overline{A+C+E+G}$; by art. 12, $A : B :: A+C+E+G : B+D+F+H$.

19. *Cor. As any antecedent, is to its conſequent; ſo is any other antecedent to its conſequent.*

20. *If four quantities be proportional, and if any antecedent and its conſequent, or the two antecedents and their conſequents, be both multiplied or both divided by the ſame quantity, the four quantities will ſtill be proportional.*

Thus, if $A : B :: C : D$
Then $mA : mB :: C : D$

$$A : B :: \frac{C}{n} : \frac{D}{n}$$

$$mA : \frac{B}{n} :: mC : \frac{D}{n}$$

For in each caſe, $\frac{A}{B} = \frac{C}{D}$.

21. *If there be four proportional quantities in one rank, and four more in another; or if there be ſeveral ſuch ranks; the products of the correſpondent terms will be proportional.*

Thus, if $A : B :: C : D$
$E : F :: G : H$
$I : K :: L : M$, &c.

Then $AE : BF :: CG : DH$
Or $AEI : BFK :: CGL : DHM$, &c.

For $\frac{A}{B} = \frac{C}{D}$, $\frac{E}{F} = \frac{G}{H}$, $\frac{I}{K} = \frac{L}{M}$, &c. Hence $\frac{AE}{BF} = \frac{CG}{DH}$ and $\frac{AEI}{BFK} = \frac{CGL}{DHM}$.

22. Cor. *The like powers, or the like roots, of proportional quantities, are proportional.*

Thus, if A : B :: C : D, then

$$A^m : B^m :: C^m : D^m; \text{ or } A^{\frac{1}{m}} : B^{\frac{1}{m}} :: C^{\frac{1}{m}} : D^{\frac{1}{m}}$$

This is obvious, by ſuppoſing A, E, and I, equal to each other; alſo B, F, K.

23. *If there be any number of quantities in one rank, and an equal number of quantities in another rank; ſo conſtituted that the firſt is to the ſecond in the firſt rank, as the firſt is to the ſecond in the ſecond rank; or the ſecond is to the third in the firſt rank, as the ſecond is to the third in the ſecond rank, &c.; then ſhall the firſt be to the laſt in the firſt rank, as the firſt is to the laſt in the ſecond rank. And any four of theſe quantities, in the form of a ſquare, or parallelogram, ſhall be proportional. The ſame is general let the number of ranks be ever ſo many.*

Thus, if A : B : C : D : E ::
F : G : H : I : K ::
L : M : N : O : P ::
Q : R : S : T : V :: &c.

Then, EX ÆQUO ORDINATA, A : E :: F : K, or A : E :: Q : V. For,

A : B :: F : G
B : C :: G : H
C : D :: H : I
D : E :: I : K. Hence, art. 21,

ABCD : BCDE :: FGHI : GHIK. Conſeq.

$\frac{ABCD}{EBCD} = \frac{FGHI}{KGHI}$, or $\frac{A}{E} = \frac{F}{K}$, or A : E :: F : K, and ſo on for any other two ranks.

From this demonſtration it follows, that the ratio of A to E, is compounded of the ratios of A to B, B to C, C to D, and D to E.

24. *If there be any number of quantities in one rank, and an equal number of quantities in another rank; ſo conſtituted that the firſt is to the ſecond in the firſt rank, as the laſt but one in the ſecond rank is to the laſt; and the ſecond of the firſt rank is to the third, as the laſt but two in the ſecond rank is to the laſt but one, &c.; then ſhall the firſt be to the laſt in the firſt rank, as the firſt is to the laſt in the ſecond rank.*

Thus, if A · B · C · D · E be the firſt rank,
And F · G · H · I · K the ſecond rank.
Then EX ÆQUO PERTURBATA, A : E :: F : K.

For, A : B :: I : K
B : C :: H : I
C : D :: G : H
D : E :: F : G by the propoſition.

Hence, by compounding the terms as in art. 23, A : E :: F : K.

ON NUMBERS, ODD AND EVEN.

25. *If any number of even numbers be added together, the ſum will be an even number.*

For, let 2A, 2B, 2C, &c. be even numbers. Then will $2A+2B+2C$, &c. be their ſum. But this ſum can be divided by 2, therefore it is an even number, Defin. 7, p. 2.

26. *If any even number of odd numbers be added together, the ſum will be an even number.*

For, let $2A+1$, $2B+1$, $2C+1$, $2D+1$, &c. repreſent the odd numbers, then $2A+2B+2C+2D$ is an even number, and $1+1+1+1$ is alſo an even number, that is, an even number of units is an even number; it is therefore obvious that the whole is even.

27. *If an odd number of odd numbers be added together, the ſum will be an odd number.*

This is evident from above, for $2A+1; +2B+1; +2C+1 = 2A+2B+2C+3$, an odd number.

28. *If an even number be taken from an even number, the remainder will be even.*

For, ſince 2A and 2B are even numbers, if A be greater than B; $2A-2B$, being diviſible by 2, is an even number.

29. *If an odd number be taken from an odd number, the remainder will be even.*

Let $2A+1$ and $2B+1$ be odd numbers, where A is greater than B; $2A+1-\overline{2B+1}=2A-2B$ an even number.

30. *If an even number be taken from an odd number, or an odd number from an even one, the remainder will be odd.*

Let $2A$, $2B$, be two even numbers, and $2C+1$, $2D+1$, two odd numbers, where C is greater than A, and B greater than D. Then $\overline{2C+1}-2A$ and $2B-\overline{2D+1}$ are evidently odd numbers.

31. *If an odd number be multiplied by an odd number, the product will be odd.*

Let $2A+1$ and $2B+1$ be any two odd numbers, their product $4AB+2B+2A+1$ is evidently an odd number.

Cor. *The quotient of an odd number by an odd number, is an odd number.*

32. *If an even number be multiplied by any number whatever, the product will be even.*

Let $2A$ and $2B$ be any even numbers, and $2C+1$ an odd number, $2A\times 2B=4AB$ an even number, alſo $2A\times\overline{2C+1}$ and $2B\times\overline{2C+1}$, are even numbers.

Cor. *If an even number, contain any number whatever a certain number of times, the quotient will be an even number. Hence alſo, an even number cannot be contained an exact number of times in an odd number.*

Other particular properties of numbers are given at page 6, 10, 15, 62, 64, 92, 93, &c.

ON SQUARE AND CUBE NUMBERS, &c.

33. *The ſum of any number of terms of the ſeries of odd numbers,* 1. 3. 5. 7. 9. 11. *&c. is equal to the ſquare of that number.*

1 · 2 · 3 · 4 · 5 · 6 number of terms.
1 · 3 · 5 · 7 · 9 · 11, &c. ſeries of odd numbers.

Then $1+3=2^2$; $1+3+5=3^2$; $1+3+5+7=4^2$; and ſo on as far as you pleaſe.

34. *If* N *be any given number or power whatever, whoſe root is ſought, r the neareſt rational root*; or r^n *the neareſt rational power to* N, *whether greater or leſs.* Then will

$$\frac{\overline{n+1}\times N;+\overline{n-1}\times r^n}{\overline{n-1}\times N;+\overline{n+1}\times r^n}\times r=N^{\frac{1}{u}};$$ See the inveſtigation in Dr. Hutton's Tracts, page 46, &c.

Hence for the cube root $\frac{2N+r^3}{N+2r^3}\times r=N^{\frac{1}{3}}$, and this rule is *exactly* the ſame as I have given at page 182, deduced from Mr. Emerſon's algebra; only expreſſed in a different manner.

35. *If to the ſum of any number of terms of the ſeries of ſquares* 1. 4. 9. 16. 25. 36. 49. *&c. you add the ſquare of half the ſum of the ſame number of terms, and increaſe that ſum by an unit, the laſt ſum will always be a ſquare number.*

Thus, $1+4+\overline{\frac{1+4}{2}}\Big|^2+1=1+4+6\cdot25+1=12\cdot25.$

$1+4+9+\overline{\frac{1+4+9}{2}}\Big|^2+1=1+4+9+49+1=64$

$1+4+9+16+\overline{\frac{1+4+9+16}{2}}\Big|^2+1=1+4+9+16+225+1=256$

Hence may be found as many ſquare whole numbers as you pleaſe, whoſe ſum ſhall univerſally be a ſquare number.

36. *In a ſeries of ſquares proceeding from an unit, the ſecond differences will be equal to each other; in cubes the third differences; in biquadrates the fourth differences, &c.*

Thus, 1 · 4 · 9 · 16 · 25 · 36, &c. ſeries of ſquares.
3 · 5 · 7 · 9 · 11, &c. firſt differences.
2 · 2 · 2 · 2, &c. ſecond differences.

And, 1 · 8 · 27 · 64 · 125 · 216, &c. ſeries of cubes.
7 · 19 · 37 · 61 · 91, &c. firſt differences.
12 · 18 · 24 · 30, &c. ſecond differences.
6 · 6 · 6, &c. third differences.

In the ſame manner the fourth order of differences in the ſeries of biquadrates, 1. 16. 81. 256. 625. 1296. &c. will be 24.

37. *The ſum of any two ſquare numbers whatever; their difference, and twice the product of their roots; will expreſs the three ſides of a right angled triangle in rational numbers.*

Let 4 and 9 be the two ſquares, then $4+9=13$ their ſum, $9-4=5$ their difference, and $\sqrt{4}\times\sqrt{9}\times2=12$, twice the product of their roots; hence the three ſides of the triangle will be 13, 5, and 12, and $12^2+5^2=13^2$.

38. *The cube of any number divided by 6 will leave the ſame remainder as the number itſelf when divided by 6. Or, the difference between any number and its cube, will divide even by 6.*

Let 47 be propoſed, the cube of which is 103823; each of theſe numbers divided by 6 will leave 5 for a remainder.

Or, $103823-47$ will divide by 6 without a remainder.

39. *The ſum of any number of the cubes of the natural ſeries 1. 2. 3. 4. 5. &c. taken from the beginning, always makes a ſquare number; the roots of theſe ſquares are 1. 3. 6. 10. 15. 21. &c. whoſe differences are 2. 3. 4. 5. 6. &c.*

Let 1. 8. 27. 64. 125. 216. 343. &c. be a ſeries of cubes; the ſum of the two firſt is 9, the ſum of three 36, of four 100, &c. whoſe roots are 3. 6. 10. &c.

40. *An even ſquare number will divide by 4 and leave no remainder, but an odd ſquare number divided by 4 will leave a remainder of 1.*

Since the ſquare of an odd number muſt be an odd number, let $2R+1$ expreſs the root; the ſquare of which is $4RR+4R+1$, this divided by 4 leaves 1 for a remainder; and the firſt term of it, viz. the ſquare of $2R$, is diviſible by 4.

Other particular properties of numbers may be ſeen in INVOLUTION, EVOLUTION, PROGRESSION, &c.

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FINIS.